Analytical and numerical studies on reducing lateral restraints in conventional & all steel Buckling Restrained Braces

Mohamad Hosein Mortezagholi a,b, Seyed Mehdi Zahrai a,*

a School of Civil Engineering, College of Engineering, The University of Tehran, P.O. Box 11155-4563, Tehran, Iran
b Univ. of Ottawa, Canada

A R T I C L E   I N F O

Keywords:
Buckling restrained braces
Discontinuous restraint
Tangent and double modulus theories
Romberg-Osgood behavioral model
Abaqus software

A B S T R A C T

A typical Buckling Restrained Brace (BRB) consists of a steel core and a mortar-filled tube to prevent buckling of the core. In BRBs, the mortar used in a casing considerably increases their weight. In another type of such braces, to reduce the weight, a steel casing is used to avoid buckling of the steel core. In this paper, discontinuous restraint is investigated analytically and numerically to manage buckling behavior of the core along the steel tube.

In the analytical section, equations are proposed to determine a suitable distance between the restraints in rectangular and cruciform sections using inelastic buckling, tangent and double modulus theories and their combination with the Romberg-Osgood behavioral model. In the numerical section, first the cyclic behavior of 9 Conventional BRB models with continuous and discontinuous mortar casing is evaluated using the Abaqus software. Then, the appropriate distance between restraints is determined for the numerical model and compared to those from analytical equations. Results show that the tangent modulus theory is more compatible with the finite element model. Moreover, in cruciform cores, it is possible to reduce the amount of mortar up to 64% without any changes in the cyclic behavior of bracing. Finally, two all-steel BRB models are exposed to cyclic loading where steel plates prevent core bucking. Those results demonstrate that the cumulative ductility exceeds the minimum value defined by AISC.

1. Introduction

Steel braces demonstrate an ideal behavior under tension and dissipate a major part of energy by yielding, while they are prone to buckling and instability under compressive loads. The buckling of these braces under compression interrupts the function of structure and significantly reduces energy dissipation. On the other hand, an increased cross-sectional area of braces to prevent buckling under compression totally changes the structural behavior and increases the strength and stiffness of the braces under tension imposing a severe force on other members [1–3]. Various approaches to prevent buckling of the braces or self-centering braces reducing the residual deformation have been studied [4–6]. Buckling Restricted Brace, BRB, has been introduced to prevent buckling of the braces under compression and reduce the level of tension. Generally, this type of bracing systems consists of a steel core (like rectangular, cruciform, tubular sections) which bears tensile and compressive forces and a set to control the buckling of the steel core. The restraining set, which can consist of a concrete casing or a set of various steel plates and profiles, does not play a role in bearing the tensile and compressive forces and only prevents the instability of the steel core [7–9].

BRBs with concrete- or mortar-filled tubes consist of three components: steel core, outer steel tube and mortar or concrete casing. It is difficult to manufacture BRBs in steel factories due to the use of the mortar which leads to more cost and time of production, increased weight and size of lateral cross sections in comparison with conventional steel bracing. Also, the high dead weight and large size of lateral sections restrict the application of the BRBs in lightweight steel structures, compared to conventional steel braces [10,11]. Their use in bridges is controlled by stricter criteria due to highly variable environmental conditions and large dimensions of the bridges increasing the weight of the BRBs leading to higher initial deformations in addition to more difficult construction. Therefore, the weight reduction for the BRBs can have a good impact in design and implementation processes [12].

These disadvantages have attracted the attention of researchers to the introduction and investigation of all-steel BRBs [13] where a set of steel members is used to maintain core stability instead of concrete casing. The geometry of these restraining members is also of great
importance in the stability of the whole system, resulting in extensive research on the shape of the core and restraining members [14-18]. Naghavi et al. investigated cyclic behavior of four types of BRB frames with ABAQUS [19]. Li et al. studied the experimental and analytical performance of composite frames with BRBs [20].

Bolted connections used to assemble the encasing members of all-steel BRBs not only simplify the construction process, but also allow for inspection of the core and its replacement after earthquake due to the disassembly conditions [21-23]. An idea proposed to decrease the weight of all-steel BRBs is to reduce their length. The main consequence of this approach is the creation of higher strains in comparison with conventional models, which may exacerbate the failure in the core [24, 25]. Another idea proposed to reduce the weight of the BRBs is the application of lightweight materials for their components. Dusicka et al. studied the bracing performance of a glass fiber reinforced polymer (GFRP) tube to restrain an aluminum core [26].

The global buckling of the BRBs before core yielding is a great concern, as confirmed by the results of many previous studies. Watanabe et al. experimentally studied the cyclic performance of the BRBs with mortar casing [10]. They found that as the dimensions (the moment of inertia) of casing fall from a certain amount, the global buckling occurs in the casing and the core. In all-steel BRBs, there are short distances between the restraining members and the center of the core due to their interface for prevention of core buckling. In such systems, reducing moment of inertia of the steel casing may raise this concern [27, 28].

Another important concern about the behavior of the BRBs is the boundary conditions between the core and concrete or steel restraining member or the boundary conditions between the core and the frame. In recent years, factors such as the type of debonding material between the core and concrete casing, details of core-gusset plate connections, design process have been studied experimentally and numerically for the BRBs. A small gap (less than 2 mm) between the core and the concrete or steel encasing allows the core to move freely inside the casing and to experience longitudinal strain along directions perpendicular to axial loads (due to the effect of Poisson’s ratio). As the dimensions of the gap rises, the forces caused by collisions between the core and the restraining encasing increases due to higher buckling modes, which is not favorable [29, 30]. Hoveidae and Rafezy studied the effect of details of all-steel BRBs on local buckling by parametric study and numerical modeling [31]. They assessed parameters such as coefficient of friction (COF), empty space and thickness of debonding material between the core and the restraining member. Wang et al. examined the experimental and numerical performance of the BRBs in different connection conditions [32].

In previous studies, a combination of inelastic buckling theories and performance of the BRBs has been less considered. Wu et al. examined the effect of higher modes on the performance of a new type of all-steel BRBs [33]. Lin et al. studied the force applied to restraining member due to local buckling of the core [34]. A fundamental question raised for the BRBs is that whether it is necessary to fill the whole tube with mortar to deactivate buckling modes of the steel core and whether it is also necessary to place restraining members throughout the entire length of the core for all-steel BRBs. If not, some advantages both in term of cost efficiency and construction can be concerned:

1. The total weight of a BRB decreases due to the less use of mortar (Fig. 1a) or steel restraints results in ease of transport and installation of braces and less initial deformation leading to better compressive strength. On the other hand, the decreased interface of mortar and steel core reduces the cost of debonding materials.

2. Reducing the amount of mortar can encourage designers to employ more expensive materials with higher strength, lower weight and weaker tangent behavior. The increased strength prevents the restraint material from damage in severe earthquakes and creates stable cycles in the behavior of bracing system.

3. In all-steel BRBs, the distance between restraining tube and the core is short due to prevention of core buckling. In such a case, the tube should have a larger cross-section leading to required moment of inertia to prevent global buckling of the bracing system. In case of discontinuous restraints, it is possible to place the tube further away from the core to increase its moment of inertia. For example, steel plates may be used to provide lateral support for the core. (Fig. 1b).

The modified BRB model with mortar casing is created by placing the mortar in specific lengths and filling the gaps by Expanded Poly Styrene (EPS). The mortar can be placed step by step to construct this model. For example, the mortar is injected into the steel tube by 10 cm in length, a specific distance is then filled with EPS and this process continues until the brace is accomplished (Fig. 1a). To prevent slippage of concrete segments, a shear key should be used for each segment to manage the application of the modified conventional BRB in construction. This process is simpler for all-steel BRB; the details shown in Fig. 1b can be implemented inside two channels and then those channels are bolted or welded to each other.

In this paper, it is attempted to determine an appropriate distance for restraints by developing inelastic buckling theories, so that higher buckling modes can be achieved. In this study, tangent and double modulus theories are discussed based on the Romberg-Osgood behavioral model. Then numerical models prepared by Abaqus software are used to compare the results with analytical ones. In this modeling, the cyclic behavior of the BRBs with square and cruciform cores is evaluated. The reliability of modeling and extraction of numerical results is investigated by comparing the results of reference experimental model and those of numerical model prepared via Abaqus software. It must be noted that the results of hardening models proposed by Prager, Chaboche and Romberg-Osgood are compared to

![Fig. 1. Primary design of the proposed BRB: (a) filled with mortar; and (b) all-steel BRB.](image-url)
2.1. Romberg-Osgood behavioral model

The tangent modulus behavior curve for steel is an effective parameter in determining the restrained length based on the tangent and double modulus theories. In the Romberg-Osgood behavior model, the strain of steel is defined in terms of stress as follows [36]:

\[
\varepsilon = \varepsilon_c + \varepsilon_p = \frac{\sigma}{E} + \left( \frac{\sigma}{H} \right)^n
\]

(1)

where \( H \) and \( n \) are constant parameters obtained according to the steel type and its behavioral characteristics. The range of plastic strain is used to determine these parameters. To solve the problem numerically, it is assumed that the steel SS400 is used, for which the elastic modulus, yielding stress, ultimate stress and ultimate strain are 200 GPa, 288 MPa, 400 MPa and 0.22, respectively. The plastic strain and its equivalent yield and ultimate stresses are used to determine the parameters. It should be noted that if plastic strain at the point of yielding is zero, it is not possible to determine these parameters and, therefore, \( \varepsilon_p \) is considered 0.002.

\[
\varepsilon_p = \left( \frac{\sigma}{H} \right)^n, \quad \log(\varepsilon_p) = \frac{1}{n} \log(\frac{\sigma}{H}) \quad i = 1, 2
\]

\[
\frac{1}{n} = \frac{\log(\varepsilon_p)}{\log(\sigma_p/\sigma_1)} \quad (2)
\]

The strain and stress values of steel SS400 are substituted in the equation as follows:

\[
1 = \frac{\log \left( \frac{\frac{289 \times 10^6}{600}}{400} \right)}{\log \left( \frac{200}{600} \right)} = 14.289
\]

(3)

\[
\log(\varepsilon_p) = \frac{1}{n} \log(\frac{\sigma}{H}) \quad \Rightarrow \quad H = 444.918 \text{ MPa}
\]

(4)

and finally:

\[
\varepsilon = \frac{\sigma}{200 \times 10^6} + \left( \frac{\sigma}{444.918} \right)^{1.289}
\]

(5)

2.2. Tangent modulus theory

In the tangent modulus theory, the tangent modulus \((E - d\sigma / d\varepsilon)\) is used instead of the elastic modulus \((E)\), in contrast to the Euler’s equation \((P_{cr} = \pi^2 EI / (KL)^2)\). In this theory, the basic assumption is the increase of axial force at the moment of buckling \((P \rightarrow P + \Delta P)\). The result of this assumption is that compressive stress must increase in all cross-sections, so that the tangent modulus can be applied for all axes in the cross-section [38,39].

\[
P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad \Rightarrow \quad \sigma_{cr} = \frac{\pi^2 E}{(KL)^2}
\]

(6)

where \( I \), \( L \) and \( k \) are the moment of inertia, the radius of gyration, the length and effective length factor, respectively. The tangent modulus is derived from the Romberg-Osgood behavioral model as follows:

\[
E_{cr} = \frac{d\sigma}{d\varepsilon} = E + 1 \frac{\sigma}{nH} \frac{1}{H} \quad \Rightarrow
\]

(7)

By substituting above equation in Eq. (6), it is deduced:

\[
\sigma_{cr} = \frac{\pi^2 E}{(KL)^2} \quad \Rightarrow \quad \sigma_{cr} = \left( \frac{1}{E} + 1 \frac{\sigma}{nH} \frac{1}{H} \right) = \frac{\pi^2 E}{(KL)^2} \quad \Rightarrow
\]

(8)

\[
L = \frac{\pi r}{k \sqrt{\frac{E}{\pi} + \frac{1}{n} \frac{\sigma}{H}}}
\]

The required spacing for the restraints to achieve critical stress is determined based on the tangent modulus theory using Eq. (7).

2.3. Double modulus theory

The double modulus theory has a more impeccable logic than the tangent modulus. In this theory, it is assumed that the compressive axial load remains constant over the buckling. Accordingly, due to the bending occurred at the moment of buckling, the stress rises on the concave side of column (compression region) and declines on the convex side of column (tension region). The stress continues to increase in the concave part with the slope \((E_1)\), but it drops in the convex part (unloading region) with the slope \((E)\). In this theory, the critical force is also determined based on the Euler’s equation through replacing the elastic modulus by the reduced modulus in the buckling equation [40]. Therefore, the general solution of the problem for different boundary conditions is:

\[
P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad \Rightarrow \quad \sigma_{cr} = \frac{\pi^2 E}{(KL)^2}
\]

(9)

In order to calculate the reduced modulus, it is necessary to determine the position of neutral axis of the cross-section by Eq. (10), where
$Q_c$ and $Q_t$ are the first moment of section for concave and convex parts, respectively.

\[ E_c Q_c + E_T Q_T = 0 \]  \hspace{1cm} (10)

The reduced modulus will be obtained using Eq. (11).

\[ E_c I_c + E_T I_T = E I \]  \hspace{1cm} (11)

where $I_c$ and $I_T$ are the moments of inertia for the compression and tension regions around the neutral axis obtained from Eq. (10) and $I$ is the moment of inertia for entire section around its central axis.

According to Fig. 2, $E_t$ for a rectangular section is equal to:

\[ \frac{1}{E_t} \frac{dE_t}{d\sigma} = \frac{1}{E} + \frac{1}{I} \left( \frac{\sigma}{I} \right)^{-1} \]  \hspace{1cm} (13)

The reduced tangent modulus of a rectangular section is substituted in the critical stress equation as follows:

\[ \sigma_{cr} = \frac{\pi^2 E_r r^4}{(kL)^2} \Rightarrow L = \frac{2\pi r}{k \sqrt{\sigma_{cr} \left( \frac{E}{I} + 1 \right)}} \]  \hspace{1cm} (14)

By substituting $E/E_t$ in Eq. (14),

\[ L = \frac{2\pi r}{k \sqrt{\frac{E}{E_t} \left( \frac{1}{r} + \frac{\sigma_{cr}}{E} \right) + \frac{E_{cr}}{E_t}}} \]  \hspace{1cm} (15)

The required spacing to achieve critical stress will be obtained from the equation above, based on the double modulus theory in the rectangular section. The calculations of double modulus theory are more complicated and time-consuming for cruciform sections. In this case, there are two probable positions for the neutral axis (Fig. 3a and b). Each position depends on the growth of strain and the amount of tangent modulus; one position along the large restrained length where the tangent modulus does not reduce significantly and another position along the small restrained length. In the following, the reduced modulus will be calculated for each state.

Assuming that $c/b = X$, $a/b - 2 = Y$ and $E/E_t = Z$, it is concluded that:

\[ E_c \frac{\left[ \frac{a}{b} \right]^2 + \left( \frac{a}{b} \right) \left( \frac{c}{b} - 1 \right) \left( \frac{c}{b} - \frac{a}{b} - 1 \right) \right]}{Z} = E \left[ \frac{a}{b} \right]^2 + \left( \frac{a}{b} \right) \left( \frac{c}{b} - 1 \right) \left( \frac{c}{b} + 1 \right) \left( - \frac{c}{b} \right)^2 \]  \hspace{1cm} (16)
\[ X^2 + Y \left( X - \frac{1}{2} Y \right)^2 = Z \left( Y - X + 1 \right)^2 + Y \left( \frac{1}{2} Y + 2 - X \right)^2 \]  

(17)

The values of \( c \) and \( t \) are obtained by solving the quadratic equation above and \( E_r \) is eventually determined by the equations below. It is worth mentioning that the acceptable root of this equation should satisfy the inequality \( a < c < a + b \).

\[ E_r = \frac{E_r I_c + E_r I_r}{I} \]

\( I_c = \frac{bc^3}{3} + \frac{(a-b)b}{12} \left\{ \left( \frac{c-a}{2} \right)^2 \right\} \), \hspace{1cm} I_r = \frac{bt^3}{3} \]

(22)

Finally, when \( E_r \) is determined for both states, the restrained length can be obtained from Eq. (23).

\[ L = \pi r \sqrt{\frac{E_r}{\sigma_c}} \]

(23)

Obviously, numerical solutions are required for the cruciform section, opposed to the rectangular section. Double modulus theory proposes an upper bound of buckling resistance due to the constant \( E_r \) despite the strain variations in the buckling moment [41]. Therefore, distance for the restraints in Eqs. (15) and (23) is expected not to be predicted conservatively.

3. Analytical results

In this section, the reduction of mortar volume is investigated numerically in the cyclic behavior of BRBs. The volume of mortar is reduced in the experimental model presented in the validation section [10]. Here the analytical results of inelastic buckling theories are described. Since the increased moment of inertia in the core may impose a significant impact on the results, square and cruciform sections with constant areas are used for the core modeling. The dimensions of these equivalent sections are given below (Fig. 4): where \( A_R, A_S \) and \( A_C \) are the area of rectangular, square and cruciform sections, respectively, and \( I_R, I_S \) and \( I_C \) are the moment of inertia for the rectangular, square and cruciform sections, respectively. As can be observed, the moments of inertia of square and cruciform sections increase by more than 4.5 and 9 times that of rectangular section through the conversion, respectively. After determining the dimensions of the core, the restrained length must be calculated using the tangent and double modulus equations for square and cruciform sections.

3.1. Square section

The restrained length of square section 41 mm long is presented based on fixed-end boundary conditions and behavior curved defined for the steel, the tangent modulus theory in Eq. (28) and the double

\[ A_R = A_S = A_C \]

![Fig. 4. Equivalent square and cruciform sections.](image)

\[ a = 19 \times 90 \Rightarrow a = 41 \text{ mm} \]  

(24)

\[ I_r = \frac{4}{3} A_r = 4.58 \]

(25)

\[ A_C = A_R \Rightarrow 2ab - b^2 = 19 \times 90 \Rightarrow b = 12 \text{ mm} \hspace{1cm} a = 77.3 \text{ mm} \]  

(26)

\[ I_C = \frac{77.3 \times 12^2}{12} = 9.14 \]  

(27)
modulus theory in Eq. (29):

\[
\begin{align*}
L &= \frac{\pi \times 0.041 \times \sqrt{3}}{0.5 \times \sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \quad \text{Eq 8} \\
&\Rightarrow L = \frac{\pi \times 0.041}{\sqrt{200000 + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \\
&\Rightarrow L = \frac{2\pi \times 0.041}{\sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \\
&\Rightarrow L = \frac{\pi \times 0.04166}{\sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \\
&\Rightarrow L = \frac{\pi \times 0.04166}{\sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \quad \text{Eq 15}
\end{align*}
\]

(28)

(29)

3.2. Cruciform section

According to the dimensions and fixed-end boundary conditions in the cruciform section, the answer of quadratic equations of cruciform section based on the double modulus theory. Given the dimensions of cruciform section, the restrained length is calculated using Eq. (30) in accordance with the tangent modulus theory.

\[
\begin{align*}
L &= \frac{\pi r}{k \left(\frac{a}{2} + \frac{b}{2}\right)^{1/2}} \quad \text{Eq. (31)} \\
&\Rightarrow L = \frac{\pi \times 0.0166}{0.5 \times \sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}} \\
&\Rightarrow L = \frac{\pi \times 0.0166}{0.5 \times \sqrt{\frac{3\sigma_{cr}}{200000} + 42.866 \left(\frac{\sigma_{cr}}{444.918}\right)^{14.289}}}
\end{align*}
\]

where the Gyration radius \( r \) of cruciform section is obtained by Eq. (31).

\[
r = \sqrt{\frac{I_c}{A_c}} = \sqrt{\frac{a^3 + (a - b)b^2}{12(2a - b)}} = \sqrt{\frac{77.3^3 + (77.3 - 12) \times 12^2}{12(2 \times 77.3 - 12)}} = 16.6 \, \text{mm}
\]

A numerical solution is required to determine the restrained length of cruciform section based on the double modulus theory. Given the dimensions of cruciform section, the answer of quadratic equations presented in section 3.3 is as follows:

\[
\begin{align*}
c &= 4.141080E - 2.296420E + 1.213592 \sqrt{3.627406EE - 0.658503(E^2 + E^3)} \\
&\Rightarrow c = 6.4375(E + E) - E \pm \sqrt{76.445313(EE + E^3) - 11.875E^3} \\
\end{align*}
\]

Eqs. (32) and (33) are the first and second modes of probable position of neutral axis in cruciform section, respectively.

Given the dimensions of cruciform section, the roots of first and second equations are acceptable only in ranges of 38.625 mm \( < c < 44.625 \, \text{mm} \) and 44.625 mm \( < c < 77.25 \, \text{mm} \), respectively. It should be noted that the positive roots under root symbol never fall in these ranges for both equations and, therefore, only negative roots under root symbol are acceptable.

In Fig. 5, the critical stress and strain are shown in terms of variations in the restrained length for square and cruciform sections, where S and C represent the square and cruciform sections, respectively, and TM and DM are the abbreviations of the tangent modulus and double modulus, respectively. As observed, the restrained length must be reduced in order to increase the amount of critical stress or strain.

For example, according to the curves in Fig. 7, the restrained length required for prevention of bucking at strain of 0.04 in a square core is 10 and 19 cm, respectively, based on the tangent and double modulus theories. However, these values are 14 and 33 cm for cruciform section at the same amount of strain, respectively, due to the increase in moment of inertia compared to the square section.

4. Numerical study

4.1. Result verification

The result validation is conducted here for numerical analysis. The BRB proposed by Watanabe et al. [10] is modeled by Abaqus finite element software for result verification (Fig. 6) comparing the experimental and numerical results of cyclic performance.

4.1.1. Modeling cyclic plasticity

It is so important to select the suitable plastic behavior model for steel members under cyclic loading. A number of behavior models have been already proposed by researchers [42,43], among which the Armstrong-Frederick model is the most important [44]. A few years later, this behavior model was modified by Chaboche to be applied for the prediction of behavior of different metals. As a basis for their studies, many researchers employed this behavior model included also in numerous commercial software packages [45,46]. In general, three fundamental factors considered to describe the plastic behavior of metals are yield function (mostly Von Mises yield criterion), flow rule (mostly associated plastic flow) and hardening rule. This last rule encompasses three conditions of isotropic hardening (uniform increase of yield surface in all directions of stress space), kinematic hardening.
(displacement of yield surface in stress space by back stresses) and combined hardening. The non-linear kinematic hardening model proposed by Chaboche can predict the behavior of metals under cyclic loading with appropriate accuracy. In this model, the material parameters of $C_i$ and $\gamma_i$ are required in addition to initial yield stress, where $i$ represents the number of back stresses. The combination of non-linear kinematic and isotropic models (the material parameters of $Q_\infty$ and $b$) can improve the accuracy for the prediction of behavior of metals.

Each parameter is calibrated by tests on metal coupons. Jia and Kavamora calibrated the material parameters of SS400 steel for the
Prager (linear kinematic hardening) and Chaboche (nonlinear kinematic hardening with/without isotropic hardening) behavior models [47]. Table 1 presents the SS400 steel calibrated parameters used in this article.

### 4.1.2. FE modeling for result verification

The BRB consists of a core with a rectangular section $90 \times 19$ mm and a tube with a box section 150 mm long and 4.5 mm thick. The core and outer tube are made of steel SS41 (JIS) with a yielding stress of 288 MPa and steel TSK50 (JIS) with a yielding stress of 370 MPa.

Since mechanical properties of the core governs the hysteresis curve of experimental specimen, it is necessary to apply an appropriate and accurate plastic behavior model for its simulation. Since the mechanical properties of steel SS41 is not available, the steel SS400 is used as its equivalent. Three behavior models given in Table 1 as well as model presented in Section 3.1 are utilized for the simulation in order to choose an alternative most compatible with experimental results. Since the behavioral properties of filling mortar are not introduced, the concrete with strength of 25 MPa is used. It should be noted that the hysteresis curve shows no sensitivity to the strength of concrete, according to the result validation. The plastic damage behavioral model is utilized for the concrete to address the potential of failure. In this behavior model, the failure of concrete is simulated using decreased hardness of elements, caused by crushing under compression or cracking under tension.

Boundary conditions of the BRBs are considered fixed on one side of the core and a controlled displacement is applied to the other side along the degree of axial freedom while other degrees of freedom are restrained. Based on experimental model presented in Ref. [10], distance between the steel core and the concrete in the restrainer is 3 mm in the finite element models. It is very important to consider the influence of imperfections on the inelastic buckling [48]. To activate the buckling modes, a buckling eigenvalue analysis is performed and a deformation similar to the first mode of buckling with the maximum amplitude of 3 mm is imposed on the core as the imperfection [19]. The elements are of the type C3D8R (8-node linear brick, reduced integration, hourglass control) and both normal (hard contact) and tangential (penalty) behaviors with coefficient of friction of 0.15 are employed to define the interaction between the core and the concrete casing.

The value selected for the coefficient of friction leads to the most compatibility with test results. As the coefficient of friction rises, the interaction between the core and the concrete casing.

Five BRB models with square steel core and different mortar contents are studied. In the first model, the mortar casing is used along the entire length of the tube (Fig. 8a) and in models 2 to 5, the number of mortar segments is reduced from 13 to 9 (Fig. 8b–e). The first and last mortar segments are 15 cm long while the middle segments are 10 cm long. The length of the mortar segment should somehow be determined not to get damaged from the impact force, as well as to provide the core with fixed boundary conditions. The reduced volume of mortar and back-to-back distances between segments (restrained length) are obtained for the model with 13 mortar casing segments using Eqs. (34) and (35), respectively.

\[
R = \frac{V_T - V_C}{V_T} = \frac{329A - 2 \times 154 - 13 \times 10A}{329A} = 0.51
\]  
(34)

\[
L = \frac{329 - 2 \times 15 - 13 \times 10}{14} = 12.07 \text{ cm}
\]  
(35)

where $V_T$ and $V_C$ are the total volume of tube and the volume of mortar, respectively. In Table 2, the reduced volume of mortar and restrained lengths are given for each model.

For the cruciform section, a model with entire mortar casing (Fig. 8a) and 3 models with 8, 9 and 11 mortar casing segments are simulated and evaluated (Fig. 8c, e and f). The decreased number of segments for the cruciform section is because of the increase in its moment of inertia and stability compared to the square section. For instance, Fig. 9 illustrates the interior of proposed finite element BRB model with a square core.

As expected, the behavior of models does not change under tension and they all withstand a certain level of forces, while compressive strength declines as restrained length increases in the compression region. In the case with square core and 9 mortar segments (64% reduction in mortar volume), the strength loss equals 24% (Eq. (36)) and in the case with cruciform core and 8 mortar segments (67% reduction in mortar volume), the strength decreases by 8% (Eq. (37)). In the case with square core and 11 and 13 mortar segments and also the case with cruciform core and 9 mortar segments, no strength loss occurs and the

Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prager’s hardening rule</th>
<th>Chaboche models</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$, $C_0$</td>
<td>255.9, 1429.0</td>
<td>With Isotropic Hardening</td>
</tr>
<tr>
<td></td>
<td>255.9</td>
<td>Without Isotropic Hardening</td>
</tr>
<tr>
<td>$C_1$, $C_2$, $C_3$</td>
<td>97.2, 9.72, 3763.0</td>
<td>$r_1$, $r_2$, $r_3$</td>
</tr>
<tr>
<td>255.9</td>
<td>26.9, 1617.2, 26.9</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Units of $\sigma_0$, $C_0$, $C_1$, $C_2$, $C_3$ and $Q_{\infty}$ are MPa, $r_1$, $r_2$, and $r_3$ and b are dimensionless.
results can be considered the same as those for braces with completely mortar-filled tube.

\[
\frac{P_{M\text{-All}} - P_{M\text{-Part}}}{P_{M\text{-All}}} = \frac{57.9 - 43.8}{57.9} \times 100 = 24\% \quad (36)
\]

\[
\frac{P_{M\text{-All}} - P_{M\text{-Part}}}{P_{M\text{-All}}} = \frac{57.9 - 53.2}{57.9} \times 100 = 8\% \quad (37)
\]

In Fig. 11a and b, the inelastic buckling is shown for the case with square core and 9 mortar segments and the case with cruciform core and 8 mortar segments, respectively.

According to Table 2, the restrained length for BRB with 11 mortar segments is 15.75 cm. Given the changes in last cycle of loading and its corresponding strain \(\epsilon = \delta / L = 4.723 / 319 = 0.01481\), this value is obtained 17.1 cm for the square section from the tangent modulus theory. Moreover, the restrained length for 9 mortar segments is 20.09 cm, while this value is obtained 24 cm for the last cycle strain in the cruciform section based on the tangent modulus theory. However, the restrained length equals 31.3 and 53.3 cm for square and cruciform sections, respectively, at this amount of strain. Obviously, despite the use of more impeccable arguments in the double modulus theory, the results of more conservative tangent modulus theory seems more compatible with reality. This is due to the lack of factors such as imperfection, eccentricity of forces and residual stresses in these theories. The reports also suggest that the tangent modulus theory better justifies the behavior of short columns. On the other hand, the uniform distribution of axial strain along the core is not a reasonable assumption and strain concentration occurs in some parts of the core because of high buckling modes in the core and its collision with the casing. Therefore, it seems more reliable to apply the tangent modulus theory which has a higher safety factor.

The results demonstrate that when restrained length exceeds the values proposed by the tangent modulus theory, strength loss begins in
the cycles. As restrained length increases, the strength loss becomes more severe in the compression region. One of the main factors in determining the restrained length is the core strain demand, so that as the core experiences larger strains, the tangent modulus declines more and strength loss increases. For example, in cycles with a displacement of 2.4 cm which is equivalent to a strain of 0.073 in the bracing system, no strength loss happens in the BRBs with square and cruciform cores and various numbers of mortar segments. Given the tangent modulus theory, the restrained length of square core is 25.77 cm for this amount of strain, which is more than the length of 20.09 cm considered for 9 mortar segments. Given the direct relationship between the drift and the strain in bracing system, a reasonable estimation of drift demand may have a direct effect on the estimation of distances between the restrainers. On the other hand, as effective length of the core increases, ultimate strain decreases and the design becomes more economical.

As explained previously, the uniform distribution of axial strain
along the core is not a correct assumption and can be only an initial criterion for estimating the distance between the constraints. Fig. 12 shows the maximum compressive axial strain versus the number of loading cycles for the modified BRBs. Typically, this strain is maximum in regions close to end of the core as shown in Fig. 8 with arrows for square and cruciform sections. Evidently, there is a significant difference between the assumptions of uniform distribution of strain along the core and each strain value read from the most critical element along the core. Since no damage model is considered via the software, the results are acceptable for the models where maximum strains do not exceed the strain corresponding to necking initiation, which equals 0.22 for SS400 steel [49]. As observed in the last cycle, the axial strain for the cases having a square core with 9 and 8 mortar segments and the case having a cruciform core with 8 mortar segments is more than the defined value and their results are not acceptable.

4.3. Modified all-steel BRBs

BRBs with discontinuous mortar segments pose problems such as difficult construction process and need for shear connectors to avoid the separation of discontinuous mortar segments. On the other hand, the failure of angular corners of discontinuous mortar segments may degrade the boundary conditions considered in analytical equations. Moreover, the use of some steel plates as a substitution for mortar segments can reduce the weight of the BRBs. To solve the problems, the steel plates welded to a steel tube are used instead of discontinuous mortar. The steel tube may consist of two channels connected by welding or bolting (when the plates are connected to its inner face). The steel plates are 1.5 cm thick and the geometry of the gap allows the cruciform core to be placed between them and according to Ref. [10] The distance between the core and the lateral plate is assumed to be 3 mm. The thickness of the lateral plate is obtained by a trial-and-error process, so that it transmits contact force to the restrainer without experiencing nonlinear behavior. The results of previous studies suggest that the non-uniform distribution of tensile and compressive strains increases and the shear rupture conditions occurs sooner in the core if it is connected to the casing at the end or if there is no connection [25]; thus, the core is connected to the casing using a stopper plate at the center for this model. We assume that the stopper plate is welded to the steel core and the restrainers. Fig. 13 illustrates the finite element model of this BRB.

\begin{table}[h]
\centering
\caption{Amplitude of loading cycles and cumulative ductility.}
\begin{tabular}{cccccccc}
  \hline
  Number of Cycle & 2 & 2 & 2 & 2 & 2 & 4 \\
  \hline
  Inter-story Drift (%) & 0.5 & 1 & 1.5 & 2 & 1.5 \\
  Displacement (cm) & 0.48 & 1.2 & 2.4 & 3.6 & 4.8 & 3.6 \\
  Cumulative Ductility & 0 & 12 & 44 & 96 & 168 & 272 \\
  \hline
\end{tabular}
\end{table}

Fig. 13. Proposed modified all-steel BRB.
In this all-steel BRB, both Romberg-Osgood and Chaboche behavior models are considered for the core and then subjected to cyclic loading in accordance with the AISC-341-16 loading protocol [50]. According to the protocol, not only the maximum amplitude of cyclic loading should be higher than inter-story drift of 2%, but also the cumulative ductility of BRB under cyclic loading must be higher than 200. The ductility in each loading cycle with the same amplitude $\mu_i$, will be estimated based on the following relation:

$$\mu_i = 4N_i \frac{\Delta u_e}{\Delta u_o}$$  \(38\)

Where $N_i$ is the number of cycles for the same loading amplitude and $\Delta u_e$ and $\Delta u_o$ are respectively the yield and ultimate displacement of each cycle. Table 3 represents the specification of cyclic loading applied to the BRB in accordance with the BRB dimensions given in reference [10]. As observed, since the cumulative ductility is smaller than 200 for 10 cycles equivalent to a drift of 2%, 4 cycles equivalent to a drift of 1.5% are added to the end of loading in order to increase the cumulative ductility to 272 (in the first two cycles, the value of 0.48 cm is equivalent to yield displacement of the BRB).

Fig. 14 shows the results of compressive and tensile axial strains for the most critical elements (the locations of axial true strains are shown in Fig. 15), hysteresis curves and energy dissipation by the core. According to Fig. 14a and b, the compressive and tensile axial strains are smaller than the necking strain. Given the high buckling modes in the compression region under loading, on the other hand, the level of compressive strain is higher than the level of tensile strain; so that the ratios of maximum compressive strain to maximum tensile strain for the Romberg-Osgood and Chaboche behavior models are $0.0845/0.0210 = 4.02$ and $0.1186/0.038 = 3.12$ respectively. The results also suggest that the IH applied in the Chaboche model makes the non-uniform distribution of strain more critical along the core in comparison with the Romberg-Osgood multi-linear model. Fig. 14c illustrates cyclic curve of the BRB for both behavior models. Obviously, given the characteristic of isotropic hardening in Chaboche behavior model, as loading cycles increase in terms of number and amplitude, the level of generated forces is higher and the hysteresis loops are larger than those for the Romberg-Osgood model. Given the area under the cyclic curves and Fig. 14d which indicates the amount of energy dissipated by nonlinear behavior of the core, it is found that more energy dissipation occurs in the chaboche model in spite of more non-uniform distribution of strain, compared to the Romberg-Osgood behavior model (about 20%).

In Fig. 15, the inelastic buckling and maximum compressive and tensile axial true strains are shown for the modified all-steel BRB.

5. Conclusion

In this study, the impact of discontinuous casing on the behavior of conventional and all-steel BRBs with square and cruciform cores was investigated. Thus, equations were derived to determine the distance between restrainers using the inelastic buckling, tangent and double modulus theories. After the result validation based on experimental results, 11 models were simulated using Abaqus finite element software and evaluated under cyclic loading. Prager, Romberg-Osgood and Chaboche behavior models were also compared to each other to select the appropriate plasticity behavior for the core. Finally, the results of numerical analysis were compared to those of analytical equations.

![Fig. 14. Numerical results: (a) compressive axial true strain; (b) tensile axial true strain; (c) hysteresis curves; (d) accumulated amount of dissipated energy.](image-url)
1. Despite small differences in cyclic curves for the validation section, the results suggest that the Romberg-Osgood and Chaboche behavior models simulate the experimental results more accurately than the Prager model. But the results of all-steel BRBs indicate that as the cycles increase in number, the difference between Romberg-Osgood and Chaboche behavior models increases.

2. Results of inelastic buckling theories show that the double modulus theory presents higher values for the distance between the restraints at a given strain level, while the finite element results are more compatible with those of the tangent modulus theory.

3. Finite element models indicate that it is possible to reduce the mortar volume up to 57% for a steel core with square section, while compressive strength of the bracing system remains unchanged compared to that of the entirely mortar-filled tube. Moreover, for the cruciform section, the reduction of mortar volume up to 64% does not lead to strength loss in cyclic behavior of the bracing system.

4. If the spacing between restrainers exceeds the values presented based on the tangent modulus theory in finite element models, the cyclic load results in strength loss in the compression region. In the case with square core and 10 mortar segments, the restrained length is 18.09 cm. This value is obtained 17.1 cm for the square section and the tangent modulus theory showing a decline in compressive strength.

5. According to the Romberg-Osgood behavioral model, the tangent modulus decreases continuously as strain rises. Therefore, as strain level increases, the critical buckling stress and spacing between restrainers decline. For example, the spacing between restrainers equals 17.1 and 25.77 cm for strain levels of 1.5% and 0.73%, respectively, based on the tangent modulus theory.

6. The Romberg-Osgood and Chaboche behavior models for all-steel BRBs were chosen for plastic behavior of the core, as the most compatible cases with the experimental results. Each BRB was subjected to cyclic loading that could satisfy the cumulative ductility of 272 for acceptable strains. The results show that the distribution of axial strains along the core is more non-uniform for the Chaboche model compared to the numerical model leading to a high level of compressive and tensile strains in the core. Given the isotropic hardening in the Chaboche model, the increased level of forces results in greater hysteresis curves and higher energy dissipation.

Acknowledgement

The authors appreciate the INSF (Iran National Science Foundation) for supporting this research.

References


