A tutorial and open source software for the efficient evaluation of gravity and magnetic kernels

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ABSTRACT

Fast computation of three-dimensional gravity and magnetic forward models is considered. When the measurement data is assumed to be obtained on a uniform grid which is staggered with respect to the discretization of the parameter volume, the resulting kernel sensitivity matrices exhibit block-Toeplitz-Toeplitz-block (BTTB) structure. These matrices are symmetric for the gravity problem but unsymmetric for the magnetic problem. In each case, the structure facilitates fast forward computation using two-dimensional fast Fourier transforms. The construction of the kernel matrices and the application of the transform for fast forward multiplication, for each problem, is carefully described. But, for purposes of comparison with the non-transform approach, the generation of the unique entries that define a given kernel matrix is also explained. It is also demonstrated how the matrices, and hence transforms, are adjusted when padding around the volume domain is introduced. The transform algorithms for fast forward matrix multiplication with the sensitivity matrix and its transpose, without the direct construction of the relevant matrices, are presented. Numerical experiments demonstrate the significant reduction in computation time and memory requirements that are achieved using the transform implementation. Thus, it becomes feasible, both in terms of reduced memory requirements and computational time, to implement the transform algorithms for large three-dimensional volumes. All presented algorithms, including with variable padding, are coded for optimal memory, storage and computation as an open source Matlab code which can be adapted for any convolution kernel which generates a BTTB matrix, whether or not it is symmetric. This work, therefore, provides a general tool for the efficient simulation of gravity and magnetic field data, as well as any formulation which admits a sensitivity matrix with the required structure.

1. Introduction

Fast computation of geophysics kernel models has been considered by a number of authors, including calculation within the Fourier domain as in Li et al. (2018), Pilkington (1997), Shin et al. (2006) and Zhao et al. (2018), and through discretization of the operator and calculation in the spatial domain as in Chen and Liu (2018) and Zhang and Wong (2015). Pilkington (1997) introduced the use of the Fast Fourier Transform (FFT) for combining the evaluation of the magnetic kernel in the Fourier domain with the conjugate gradient method for solving the inverse problem to determine magnetic susceptibility from measured magnetic field data. Li et al. (2018) considered the use of the Gauss FFT for fast forward modeling of the magnetic kernel on an undulated surface, combined with spline interpolation of the surface data. Their work focused on the implementation of the model in the wave number domain and only applied the method for forward modeling. The Gauss FFT was also used by Zhao et al. (2018) for the development of a high accuracy forward modeling approach for the gravity kernel. Moreover, in earlier work, Shin et al. (2006) designed a Fortran code for fast forward and inverse modeling of the gravity model using the Fourier domain method using the FFT for achieving fast computation. On the other hand, Gómez-Ortiz and Agarwal (2005) provided a Matlab code for computing the geometry of a density interface related to a known gravity anomaly by also employing the FFT to achieve fast computation, but which is not related to forward modeling for gravity models. Bruun and Nielsen (2007), and subsequently, Zhang and Wong (2015) introduced the use of the Block-Toeplitz–Toeplitz-Block (BTTB) structure of the modeling sensitivity matrix for fast three-dimensional inversion of three-dimensional gravity and magnetic data. For a matrix with BTTB structure, it is possible to embed the information

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