Eccentricity fault diagnosis indices for permanent magnet machines: state-of-the-art

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Abstract: Eccentricity fault with 10% severity is probably the only existing fault in a brand new healthy electrical machine that is acceptable as a manufacturing tolerance. This fault can be developed due to a continuous pull between stator and rotor, even in a de-energized machine. So, it must be diagnosed, and its severity monitored. The authors introduced and criticized various indices for eccentricity fault diagnosis. Advantages, drawbacks, and ambiguous points of each index and potential ground for improvement of these techniques are addressed. Moreover, different eccentric PM machine modelling techniques are reviewed and environmental factors affecting the eccentricity fault detection process are discussed. This survey covers all direct current and alternating current PM machines. It provides information about the researches performed since 1986 and it is helpful to newcomers to this field, artisans, and protection system designers. Considering the given information, the readers achieve enough understanding of the weaknesses and strong points of each eccentricity fault diagnosis index. Then, these indices are evaluated through assigning weight coefficients under various scenarios. Interested individuals can modify these weight coefficients based on the specifics of the application and decide which index is the most appropriate.

Nomenclature

\( v_d \) direct-axis voltage component  
\( v_q \) quadrature-axis voltage component  
\( V_d \equiv \) direct-axis voltage command  
\( V_q \equiv \) quadrature-axis voltage command  
\( L_d \equiv \) direct-axis inductance component  
\( L_q \equiv \) quadrature-axis inductance component  
\( L_{d,low} \equiv \) direct-axis inductance of lower half of magnetic circuit  
\( L_{d,up} \equiv \) direct-axis inductance of upper half of magnetic circuit  
\( L_{d,e} \equiv \) direct-axis inductance of eccentric motor  
\( f_s \equiv \) synchronous frequency  
\( f_f \equiv \) fault harmonic components frequency  
\( f_{eccc} \equiv \) eccentricity fault harmonic frequency  
\( f_r \equiv \) fault harmonic frequency of the direct-axis inductance  
\( s \equiv \) slip  
\( P \equiv \) number of pole pairs  
\( R \equiv \) number of rotor slots  
\( n_n \equiv \) integer coefficient  
\( n_f \equiv \) integer coefficient  
\( n_{ws} \equiv \) integer coefficient  
\( k \equiv \) integer coefficient  
\( F_f \equiv \) field component of MMF  
\( F_a \equiv \) armature component of MMF  
\( F_{fa} \equiv \) combination of field and armature MMF  
\( E_f \equiv \) field component of back-emf  
\( E_a \equiv \) armature component of back-emf  
\( E_{fa} \equiv \) combination of field and armature back-emf  
\( A_i \equiv \) amplitude of the \( i \)th harmonic  
\( f_i \equiv \) frequency of the \( i \)th harmonic  
\( \theta_i \equiv \) initial phase of the \( i \)th harmonic  
\( T_S \equiv \) sampling interval  

1 Introduction

Permanent magnet (PM) machines are widely used in various industrial, automotive, and aerospace applications [1–4]. Similar to other types of electrical machines, PM machines are not fault-free. Unbalanced load, misalignment, improper mounting, and bent shaft can cause rotor eccentricity. Comprehensive study and review of indices for a number of fault types in PM machines have been done, including turn-to-turn short circuit fault in [5] and demagnetisation fault in [6]. Such a survey has not been yet carried out for eccentricity fault.

Redundancy is a major tool in the fault diagnosis process which can be pursued in two different ways: hardware redundancy and software redundancy. Hardware redundancy is defined as an act of having a separate system similar to the one being monitored, fed with the exact inputs and is operating at the identical condition [7]. Although hardware redundancy is a reliable and accurate way of extracting reference outputs of the apparatus, it requires extra space to house the redundant hardware, and also, impose extra expenses to the entire process. Software redundancy can be considered as modelling the monitored system. Modelling techniques are the major tools in emulating the behaviour of faulty systems and are a dominating means in the design process of fault detection schemes. Modelling techniques can be fast but accompanied by simplifying assumptions which degrade the accuracy of the output results. To achieve a higher accuracy, more detailed models are introduced that have long computation time. Modelling techniques can be classified as analytical, numerical, and hybrid methods. We would see in the next sections of this paper that fault diagnosis indices are classified as parameter-based indices and signal-based indices. Most of the time, the only information one has about a rotating PM machine is its line currents, voltages, and frequency. Extracting and/or estimating fault indicator parameters in parameter-based indices is usually done through utilising these analytical, numerical, and hybrid models to relate the measured signals of the motor to the fault indicator parameter. Therefore, modelling is an undeniably important aspect of the fault diagnosis process which needs attention. That is why in the rest of this section, the available modelling techniques in the literature are discussed.
Analytical modelling techniques have low computational burden which is mostly achieved through a set of assumptions and simplifications. A radial flux PM brushless direct current (PM-BLDC) machine has been modelled through solving the governing magnetic field equations in polar coordinates at the annular airgap/magnet region of multi-pole motor [8–11] assuming uniform radial magnetisation of PMs and constant relative recoil permeability. The model introduced in [8–11] is used in [12] to define magnetic field distribution of a surface-mounted PM-BLDC motor (suffering from asymmetrical magnetisation pattern of magnets and also rotor eccentricity) and calculate the resultant radial force waves applied to the stator core.

A two-dimensional (2D) analytical model based on the perturbation method has been introduced to analyse the behaviour of the eccentric motor in [13, 14]. The field equations have been solved in moving coordinate system and then they have been transferred to a fixed coordinate system for simpler problem solving, as this transfer decreases the asymmetrical boundary conditions.

A magnetic lumped-parameter-based model has been presented in [15] where the static eccentricity (SE) and dynamic eccentricity (DE) faults in PM-BLDC motors have been applied and the impact of these faults on the motor performance from the unbalanced magnetic pull (UMP) point of view has been investigated. However, the saturation effect has been ignored because saturation makes difficult to model the machine under eccentricity fault. The model introduced in [8] has been extended to fractional-slot PM machines [16, 17], where the model solves governing Laplacian/quasi-Poissonian field equations in PM and airgap regions with no pre-assumption and simplification on relative recoil permeability of PMs.

Perturbation method can be combined with the subdomain approach in order to model rotor eccentricity fault analytically [18, 19]. Magnetic field domain is divided into three normal and simple subdomains including PM area, airgap region, and stator slots. Analytical solution has been obtained by solving the governing magnetic field equations in each subdomain and applying boundary conditions to interfaces between the subdomains. Eccentricity fault in an interior permanent magnet synchronous motor (IPMSM) has been modelled in [20] using the magnetic equivalent circuit. This modelling method has advantages of short computation time and reasonable accuracy, and core saturation has been considered.

The superposition method is used in [21, 22] in order to calculate the airgap magnetic field in eccentric fractional-slot PM machine. The eccentric machine is divided into airgap sections along the circumferential direction. For each section, a concentric model is used. In this concentric piecewise model, in fact, an equivalent airgap length to predict the airgap magnetic field using the subdomain method has been assumed. Airgap magnetic field distribution of the eccentric model is synthesized from these concentric models.

Finite element method (FEM) as a well-established numerical modelling approach can consider geometrical complexities of all motor parts, stator winding spatial distribution, and core materials non-linearity. Both 2D-FEM [23] and 3D-FEM [24] have been used for modelling eccentric PM machines.

A hybrid method for UMP estimation in surface-mounted PM and consequent-pole rotors type PM-BLDC has been presented in [25, 26] where flux wave is defined by the FEM and then is fed as input to the analytical model for UMP calculation. Both radial and tangential components of force wave are used to estimate and analyse UMP. A multi-physics analysis has been applied and acoustic behaviour of a permanent magnet synchronous motor (PMSM) under SE and partial demagnetisation fault have been studied in [27] where FEM has been substituted by finite reconstruction method. This method uses the field generated by a single slot/phase of the stator, along with the field generated by the PMs on the rotor to determine the magnetic field distribution and electromagnetic force components in a PMSM.

Another characteristic aspect of fault diagnosis indices is whether they are extracted actively or passively. Active fault index extraction involves enhancing the detectability of a potential fault and improving the fault index extraction by injecting a suitably designed input signal into the system. During intervals allocated for health monitoring and testing, the fault detection scheme injects a specific signal into the system. The test signal is designed to hold the fault features and enhance their detectability. However, their main drawback is that they are mostly offline methods and the testing intervals might not be as frequent as needed. Passive fault diagnosis indices do not inject any artificial/engineered signal into the system as their detectability is satisfactorily enough.

This paper deals with the presented indices of eccentricity fault diagnosis for PM machines and categorising them into parameter-based indices and signal-based indices. In Section 2, a detailed discussion of various intelligent techniques is presented. Section 3 sets the ground for the effect of eccentricity fault in various applications and presents the criteria for index evaluation. Sections 4 and 5 review the fault diagnosis indices for radial flux PM (RPDM) and axial flux PM (APDM) machines, respectively. Section 6 compares various indices reviewed in Sections 4 and 5, using the criteria defined in Section 3. Section 7 provides a complementary discussion on the other important aspects of eccentricity fault indices and Section 8 concludes the paper.

2 Intelligent algorithms

Usually, there is a large amount of data available for the monitored system. This data carries valuable knowledge on the system variables that define its health condition. Such knowledge can be extracted and utilised to train an intelligent algorithm with the aim of more effective fault diagnosis. This section focuses on intelligent algorithms applied for eccentricity fault detection in PM machines.

K-nearest neighbour (K-NN): This is a tool for non-parametric estimation [28, 29] that locates K-nearest pre-marked/labelled training samples and estimates the class densities. If most of the samples among these K neighbour points, found in a predefined radius from the data point under classification, belong to a certain class, then data point under classification belongs to that class too. This involves partitioning the independent axes and calculating the probability density function on each of these partitions. If the number of training sample points is large enough, one can use the simplest form of the algorithm known as NN where K = 1. If the sample points of different classes do not overlap with each other, then this method can be used with high reliability. However, if different classes overlap on each other and there is no distinct border between different distribution regions, K = 1 fails to give reliable classification performance and higher values of K should be used. The K-NN has been employed in the diagnosis process of PM machines in [30-34].

Support vector machine (SVM): The SVM performs supervised classification duties with generalisation capabilities [35]. They use mathematical optimisation approaches to incorporate a derivative threshold of statistical learning theory in the system to train the machine and improve its classification capability [36]. Training the SVM can be seen as a mathematical problem of optimising the convex quadratic function. In this method, the margin between the training data set is maximised to achieve distinctive decision boundary by hyperplanes [32]. For further elaboration, the input data sample points are mapped in feature space (using kernel function), with hyperplanes separating the classes and the algorithm maximises the distance between the decision boundary and the closest subset patterns which are called support vectors. The basic SVM can only be employed for two-class problems. For multi-class classifying, the ‘one-against-all’ method can be used which distinguishes one class from all the other ones (more information on the one-against-all method can be found in [37]). This method is utilised in [32] to incorporate the eccentricity fault severity in PMSMs.

Fuzzy SVM (FSVM): This is an improved version of the SVM in terms of sensitivity to the outline data and provides larger margins (which yields to stronger generalisation ability) [38]. Its principle is similar to the SVM in which it maximises the margin between the decision boundary and the training data. The difference between FSVM and SVM is that different training sample data give different contributions to the output data.
set has been fuzzified. Based on the importance of each data sample, each is given a membership value. Same as SVM, the basic FSVM can only classify and solve two-class problems, and for multiclass problems, the one-against-all method is utilised to empower FSVM in handling multiclass problems [37]. In [33], FSVM is used to evaluate the eccentricity fault severity in a PMSM.

4.1 Linear discriminant analysis (LDA): The LDA operates based on maximising a ratio that has between-class variance in the numerator and the within-class variance in the denominator [39, 40]. Such maximisation is achieved by means of defining a hyperplane in the input space that maximises the between-class variance and minimises the within-class variance [35]. Defining this hyperplane and maximising that ratio serves the goal of getting the maximum separation between the feature sets in each of the classes [41]. An unknown sample is classified through calculating the linear discriminant function for the unknown sample several times, each time using only the coefficients of one class. The class that yields to the greatest value for the linear discriminant function claims the unknown sample as its member. The LDAs have been used for the eccentricity fault detection and condition monitoring of PMSM [41].

4.2 Artificial neural network (ANN): The ANN is the general nonlinear function approximators inspired by the human brain [42]. Approximation task is achieved through the construction of relevant networks, each assigned an appropriate connection weight. One can categorise ANNs based on their topology into radial basis networks, backpropagation networks, extension networks, and recurrent dynamic networks [43]. The finalisation of the training process is governed through mathematical error thresholds such as square error ($E(K)$) and mean square error ($E_M$) [44]. The squared error calculates the instantaneous sum of the squared errors of the output layer neurons in relation to the $K$th training pattern. Also, the mean square error is defined by the sum of the squared errors of all input patterns in the training set. The synaptic weight matrix is adjusted in the training process with the aim of minimising the $E_M$. A three-layer perceptron ANN is used to estimate the eccentricity fault severity in a PMSM [30, 31, 45].

3 Fault index evaluation criteria

Nowadays, PMSMs are used in electric vehicles (EVs) and hybrid EVs (HEVs) [46] as they bring the highest driven range for the same input power and can provide good performance in the extended speed range (using flux weakening mode). In these applications, PMSMs are designed for current ratings higher than the conventional values to achieve their distinctive features (high power and torque density) [47]. This increases the failure probability. In EV and HEV applications, motor fault can result in noise and vibration which degrades the pleasant driving experience [48] and in severe cases, can put human lives at risk. Another safety critical application is air transportation in which, the PMSMs can be used as the propeller or as a generator linked to the jet engine [49, 50]. Due to PMs in the rotor, the back-emf is there as long as the rotor rotates. Under an undetected fault condition, this can lead to various destructive events such as uncontrolled generating mode. These machines also have wide usage in renewable energy applications. They are used for power generation (permanent magnet synchronous generator (PMSG) connected to a wind turbine) [51] and power storage (flywheels) [52]. Without ensuring their reliable operation through fault diagnosis, managing the connected grid can be challenging. Throughout the rest of this paper, various fault diagnosis indices are reviewed in details and compared with each other. The criteria that will be used in the comparison process are listed below:

$A1$: The index is able to distinguish DE fault and partial demagnetisation fault.
$A2$: The index is robust against noise.
$A3$: There is no need to fix additional sensors for index extraction.
$A4$: The index is able to estimate the fault severity.
$A5$: The index is able to precisely diagnose the fault under load-level variations.

$A6$: The index is able to precisely diagnose the fault under speed variations.
$A7$: The index is independent of parameters of the motor.
$A8$: Computationally fast index.
$A9$: The index is on-line.
$A10$: The index is able to diagnose the fault in radial and axial flux machines.
$A11$: The index is non-invasive.
$A12$: Saturation has no impact on the index.
$A13$: No need to dismantle the motor for extracting the index.
$A14$: The index is usable for different topologies of the stator winding.
$A15$: The index is validated experimentally.
$A16$: The index is usable for the diagnosis of SE, DE, and mixed eccentricity (ME) faults.
$A17$: In addition to fault diagnosis, the index is able to classify it intelligently.

4 Eccentricity fault diagnosis indices for radial flux PM machines

Eccentricity fault indices for RFPM machines are categorised into two distinctive categories and studied in the following subsections.

4.1 Parameter-based indices

In parameter-based indices, one or more of the machine or control system parameters, affected by eccentricity fault, are evaluated and utilised as a fault indicator. These indices may be extracted actively or passively, which will be stated for each studied index.

4.1.1 d-Axis apparent inductance (first index): It has been shown in [53] that by increasing the eccentricity fault severity, $d$-axis inductance is decreased due to the changes in the magnetic saturation degree. As a result, $L_d$ which is apparent inductance in nature has been introduced as eccentricity fault diagnosis index which is extracted actively. The inverter supplying the motor can be used for automatic measurement of $L_d$ under standstill conditions. To extract $L_d$ in [53], a very small ac current is injected to the stator winding in $d$-axis direction (no DC component is injected). In such a way, $L_d$ is measured independent of load variation and/or load oscillation which is perfect for fault diagnosis purposes. It is assumed that under open-circuit condition, only PMs generate flux in the machine, and the machine is designed to be below the knee and in the linear region of the $B-H$ curve. If the $d$-axis is excited by a very small Magneto-Motive Force (MMF), and it can be assumed that $L_d$ is proportional with the slope of the permeance curve at that operating point. This slope is the inverse of the equivalent magnetic circuit reluctance observable in Fig. 1. $L_d$ is the healthy machine $d$-axis inductance.

For the concentric motor of Fig. 2a, the $d$-axis flux in the upper and lower magnetic circuit is the same. However, for the eccentric machine (Fig. 2b), the flux density of the magnetic circuit of the upper part of the motor in the stator and rotor cores increases due to the airgap length reduction. On the other hand, the flux density in the magnetic circuit of the lower part of the motor decreases due to the increase of the airgap length. Airgap length reduction (decreasing reluctance) in the upper half of the machine can be modelled by increasing the slope of the permeance curve, and increase of the airgap length in the lower half of the motor may be modelled by reducing the permeance curve slope. In the eccentric motor, operating points of magnetic circuits for the upper half and lower half are displaced in the opposite directions. The $d$-axis inductance of the lower half of magnetic circuit $L_{d,low}$ decreases due to the permeance curve slope reduction, and the $d$-axis inductance of the magnetic circuit of the upper half of motor, $L_{d,up}$, decreases because the core enters to the saturation region. This means that the overall $d$-axis apparent inductance of the machine decreases under eccentricity fault. Reduction of the $d$-axis inductance ($L_d$) has been used for eccentricity fault diagnosis. The $L_d$ of the eccentric motor is called $L_d,e$. The variation level of the
introduced index strongly depends on the design of the motor and motor design and direction of eccentricity axis relative to the inductance particularly non-linear magnetic core saturation characteristics. In the first index for extracting apparent inductance, the $d$-axis is only excited by a very small AC signal, but in [55], the current excitation signal is a DC signal plus an AC signal.

The AC signal (in the $d$-axis direction) is superimposed on the DC current signal (also in the $d$-axis direction). Injected DC current generates a flux in the $d$-axis direction which elevates the resultant flux and MMF from the open-circuit level. This increase of flux means the core saturation, decrease of the permeance curve slope, and inductance. If the injected DC current is in the $d$-axis direction, it is called $i_d$. The extracted apparent inductance in $d$-axis in the 1st index can be calculated as follows:

$$L_d = \frac{\Delta \lambda}{\Delta i_d},$$

where $\Delta \lambda$ is the total flux in $d$-axis, $\lambda_m$ is the PM flux. Incremental $d$-axis inductance symbolised by $L'_d$ is also called the $d$-axis differential inductance which is the instantaneous slope of $\lambda$–$i$ curve and calculated as follows:

$$L'_d = \frac{\partial \lambda}{\partial i_d}.$$  

Fig. 3 by the injected DC current in the $d$-axis. The trend of inductance $L'_d$ variations is not uniform and for different values of injected DC current component to the stator by inverter leads to different $|\Delta L'_d|$. Amplitude of the DC current $i_d$ that maximises $|\Delta L'_d|$ can be used as an index for DE fault diagnosis. In fact, two following indices have been introduced in [55].

First index: Amplitude of $L'_d$ variations for a given and pre-defined value of $i_d$.

Second index: $i_d$ value in which $|\Delta L'_d|$ is maximised.

Advantages of the introduced indices are the low cost, because no additional sensor and/or a special type of hardware is needed for extraction of these two indices. Also, high sensitivity and independency of the operation condition and parameters/modelling uncertainties and noise are the weaknesses of this method of eccentricity fault detection, along with the fact that cannot provide a continuous and on-line monitoring of the machine.

4.1.3 Noise source black box parameters (third index): A very fast and efficient index called as black box method has been introduced in [56] for determining and diagnosing abnormal noise sources of PMSM. Based on [57], box modelling theory can be categorised into three classes: white box, grey box, and black box. In the white box theory, the model takes into account physical governing equations of the process and a deep knowledge of the system is necessary. In the grey box theory, the system is modelled using prior and/or auxiliary data of the system. Such auxiliary knowledge can be in the form of steady-state data. As for the black box theory, the model is extracted using only the data obtained from the process during a dynamic test. In this case, no other source of knowledge is utilised in the modelling process [58]. Both the white box and grey box theories require some information on the internal parameters of the PMSM, which are usually not shared by the manufacturers. This is the reason why the black box theory is chosen in this research. The first stage is a black box testing theory model in which rotation speed signal is its input, and noise frequency is its output. The main sources of unnatural noises of PMSM have been considered in this model. These sources include...
current harmonics, PWM switching frequency, eccentricity fault, sliding bearing, and resonance.

The second stage is a black box testing experiment of noise which is used under acceleration of PMSM and the sound coming out of the machine is analysed with objective acoustic indicators as well. The acoustic noise of a PMSM is analysed in three aspects: linearity (how noise curve with the rotational speed variation resembles a straight line), loudness (attribute of auditory sensation on a scale extending from quiet to loud), and sharpness [proportional relation of the high-frequency (HF) noise and the low-frequency noise]. During the third stage, sources of the identified abnormal noises are determined using black box testing experiment. The introduced method considers the existing abnormal noise source and determines whether the source of this noise is eccentricity fault or not and is implemented passively.

4.1.4 Fluctuation of HF d-axis inductance (fourth index): Inductance of PMSM oscillates under eccentricity fault caused by the variations of the airgap, and this generates fault harmonics in the stator current spectrum. An on-line method for eccentricity fault diagnosis and distinguishing it from partial demagnetisation has been presented in [59]. This passively extracted index is \( f_i \) harmonic component of the \( d \)-axis inductance as follows:

\[
 f_i = \frac{f_s}{P_f}, \quad (3)
\]

where \( f_s \) is the synchronous frequency. This inductance is estimated by the inductance high-frequency mathematic model. To distinguish eccentricity and partial demagnetisation fault, a combination of the \( f_i \) component of the estimated inductance, and estimated PM flux-linkage is used. Inductance oscillations occur due to airgap alternating variations which occur exclusively due to the eccentricity fault. Consequently, \( f_s \) component of the estimated inductance appears only if eccentricity fault occurs. In the case of the partial demagnetisation, no harmonic is observed. On the other hand, PM flux-linkage decreases due to demagnetisation, but eccentricity fault has no impact on this component. So, the existence of each fault is determined by these two indices. This has been presented for an IPMSM and the results have been experimentally validated. The question that can be raised is how the PM flux-linkage is measured, and it should be noted that the accuracy of PM demagnetisation fault is highly affected by the temperature changes in the machine.

4.1.5 DQ0 frame command voltage error (fifth index): If eccentricity fault occurs, the machine is divided into two regions and the eccentricity axis is perpendicular to the border of these regions. One of the regions, with a shorter airgap than that of the healthy machine, is more saturated and another region with longer airgap keeps a distance from saturation. The overall effect of these two causes more saturation in the machine with SE compared to the healthy machine. A rise in the magnetic flux density means the increase of \( d \) and \( q \) axes flux-linkages, it means that \( V_{q0} \) decreases and \( V_{d0} \) increases. In fact, this change in the system means the movement of the operating point voltage deep in the second quadrant noted in [34] for static fault diagnosis in IPMSM. The trend of the fault diagnosis is that first, the machine is characterised and command voltages (\( \hat{V}_{d0}, \hat{V}_{q0} \)) are estimated. Then, the actual command voltages of the machine are read from the controller and compared with the estimated command voltages. Judging on the difference between the estimated and real command voltages, and using classifier algorithms, presence of the fault, fault type, and fault severity are determined passively. Three different classifiers have been used in [34]. These methods are of the KNN, quadrature discriminant analysis, and LDA.

4.2 Signal-based indices

In signal-based indices, the index is extracted from a signal associated with the motor that is affected by the fault. In other words, the motor malfunction results in the fault signature to appear in the processed signal. These indices may be extracted actively or passively, which will be stated for each studied index.

4.2.1 Stator current fast Fourier transform (FFT) sideband amplitudes in ABC reference frame (sixth index): Eccentricity fault excites particular harmonic components of stator current. These components for induction motors are following [60]:

\[
 f_i = \left( n_s R \pm n_p \left( \frac{1-s}{p} \pm n_{ds} \right) \right) f_s, \quad (4)
\]

where \( f_i \) is the frequency of current components which is monitored to diagnose the eccentricity fault, \( n_s \) is an integer (generally 1), \( n_d \) is \( 1, 2, 3, \ldots \), and \( s \) is the slip, \( p \) is the number of pole pairs, \( n_{ds} \) is the order of time harmonics of the supply driving the motor, \( n_{ds} = \pm 1, \pm 3, \pm 5, \ldots \), and \( R \) is the number of rotor slots. Adjusting \( n_p, n_{dh}, \) and \( n_{ds} \) leads to proper frequencies which are monitored for DE fault diagnosis. By substituting \( n_{ds} = 0 \), the fundamental slot harmonics are obtained which are monitored for SE fault diagnosis. However, these frequency components and expressions cannot be applied to PMSMs, because there is no slot on the rotor.

DE fault generates harmonic components of \( F = p \) and its integer multiples appear in the stator phase current. Through evaluating the amplitude of these harmonics, the fault occurrence is recognised. A four-pole PMSM over different speeds and loads has been modelled in [61, 62] in which the current signal in steady-state mode and ABC reference frame is sampled and Fourier transform is applied to the signal. It is observed that the DE fault affects the harmonics with an order of 0.5 and 1.5. In a healthy motor, these harmonics are not zero. The order 0.5 is chosen for DE fault diagnosis. Occurrence of the DE fault causes the increase of the amplitude of this harmonic. The measurement is carried out under different loads and speeds. It is concluded that the threshold value of fault harmonic varies. Stator current frequency spectrum in ABC frame has been investigated and observed that the harmonic order of 0.5 in the motor with SE fault is lower than that of the healthy motor. This phenomenon has been also observed in simulation results, but it has not explained the reason behind it.

Following equation has been introduced for sideband components of the fault current, where their amplitude can reveal fault existence and its severity [30–32, 45, 63]:

\[
 f_{\text{side}} = \left( 1 \pm \frac{2(K - 1)}{p} \right) f_{s}, \quad K = 1, 2, 3, \ldots \quad (5)
\]

In [45], an ANN has been used for fault diagnosis and its severity classification. This ANN is a multilayer perceptron type. A three-layer feed-forward perceptron ANN system by propagation learning is trained to serve for fault severity classification. Also, white Gaussian noise has been added to the measured current signal to evaluate the robustness of the index against environmental changes; so, it is a desirable index when low sensitivity to parameters changes can be claimed for it. The sensitivity of the introduced index against different white noise levels is also investigated. In [30], this index has been employed for SE, DE, and ME fault diagnosis and they can be distinguished through the incremental rate of change in amplitude of side band component (ASBC) of the mentioned harmonics. This rate is larger in SE than DE and its value in ME is larger than the other two. Also, simulation has shown that ASBCs are not affected by load variations. This may be only true in this motor as its analytical proof has not been provided. It has been deduced from the simulation results that the harmonic components of the fault with the pattern presented in (5) will not be excited by demagnetisation faults and short and open circuit fault and these harmonics are only excited by eccentricity fault and exclusive to this fault type. It is noted that all machines addressed in [30–32, 61, 62, 64, 65] are integer slot distributed machines. DE fault may generate \( 1/p \) order fault harmonic in the flux-linkage signal and also machine inductances \([61, 62, 65]\). The reason is that the minimum airgap
rotates with the frequency of \( fs/p \). Also, these harmonics are not observed in the current, flux-linkage, and inductances of the machine in SE fault case, because the mean airgap length is fixed and not displaced. However, as reported in [30–32, 64], such harmonics appear in the stator current and can be used for fault diagnosis. In addition, [61, 62, 65] show that the harmonic components with the pattern presented in (5) are excited due to the angle of the fundamental component of the current signal. Also, torque (e.g. reciprocating compressors) generates sidebands with harmonics similar to the harmonics caused by eccentricity fault in the stator current; therefore, inductances of phases are non-identical and under faulty conditions; their amplitudes are increased by the effect of slots and poles number of machines in eliminating these fault harmonics. This has been shown in [30] that these harmonics are not excited in the stator current frequency spectrum due to partial demagnetisation, short circuit, and open-circuit fault. All the theoretical results in [30, 61, 62, 65] have been experimentally validated. The reason for disappearance of the fault harmonics in the stator current under partial demagnetisation fault may be explained by the effect of slots and poles number of machines in eliminating these fault harmonics. However, it has been shown in [30] that the elimination of SE fault harmonics in PMSMs can be proposed in the future.

The previously extracted index is not suitable for fault diagnosis under non-stationary conditions. This index is on-line and non-invasive and does not need an additional sensor. Determination of the fault severity from the mentioned harmonics is not possible in noisy conditions. It is noted that pulsating loads can generate harmonics similar to the harmonics of SE fault. Therefore, in the stator current spectrum and distinguishing these two through this index is impossible. Also, position-depending oscillating load torque (e.g. reciprocating compressors) generates sidebands with a frequency pattern identical to the above-mentioned fault frequency pattern, but with higher values which makes it even harder to use machine current signature analysis method for reliable fault detection, because there is no clear way of distinguishing these two phenomena. Also, fault diagnosis using FFT is not clear at low speeds.

4.2.2 Stator current resampled FFT correlation dimensions (seventh index): Synchronous resampling technique (SRT) is used to handle non-stationary current signal in variable speed direct drive wind generator [67]. In fact, fault diagnosis in [67] is based on the machine current signal and its Fourier transform. The difference is that this method is used to transform the non-stationary current signal to a stationary one. The SRT method is a non-uniform sampling method for signal processing. In the SRT technique, it is necessary to have the signal instantaneous phase angle for the estimation and determination of each SRT’s position. A forward–backward filter and Hilbert transform-based method have been employed in [67] to evaluate the instantaneous phase angle of the fundamental component of the current signal. Also, linear interpolation has been utilised to obtain the sampling time of the corresponding synchronous resampling points. Then, fault characteristics are extracted passively from the stator current signal using the application of SRT on it. These characteristics are used to build a new signal. After using the Grassberger–Procaccia algorithm and its implementation on the signal, correlation dimensions related to the signal are extracted and used to diagnose the fault in direct drive PMSG.

4.2.3 Seventh harmonic of stator current negative sequence (eighth index): The airgap length under SE fault differs at various positions; therefore, inductances of phases are non-identical and can be used for SE fault diagnosis. In the healthy machine, these are fifth and seventh harmonics in stator currents in the ABC-reference frame. Due to the existing difference in the inductance of phases under SE fault, there will be a difference in the sequence components of harmonic currents between the healthy motor and the motor with SE fault. Normally, fifth and seventh harmonics are presented in the stator current signal which is in nature, sequence components, and under faulty conditions; their amplitudes are affected by the existing difference between the inductances. The seventh harmonic is inherently a positive sequence component. If the measured seventh harmonic of phase current is resolved into positive, negative, and zero sequence components, the existing negative sequence component extracted from the seventh harmonic is in conflict with the positive sequence nature of seventh harmonic and it is there because it is induced by the fault and its amplitude rises for higher fault severity. So, by examination of the negative sequence component of the seventh harmonic of stator current, which is extracted passively, the SE fault is diagnosed. In [61, 62], the seventh harmonic of the phase current is measured and converted to sequence components. Then, the negative sequence components of the seventh harmonic of the current in healthy motor and motor with SE fault are inspected. Based on the results reported in [61], there is a difference between the negative sequence of the seventh harmonic of the stator current in the healthy motor and motor with SE fault under different operating conditions. The amplitude of these components rises by increasing the severity of the SE fault. This method needs further study, because sequence components of the voltage must be also taken into account. It must be examined whether other faults have a similar impact on this component or not. Also, the presented index is not a proper one under non-stationary conditions. This index is sensitive to the asymmetry of the motor and it diagnoses the generated unbalance due to the fault. However, any asymmetry arose from the motor structure and/or unbalanced supply can affect the accuracy of the fault diagnosis.

4.2.4 Window Fourier ridge (WFR) transform of stator current (ninth index): Many applications are considered to be non-stationary. However, fault diagnosis in such operating conditions is very challenging, because a very complicated signal processing is required. Short-time Fourier transform (STFT) for fault diagnosis under non-stationary conditions is commonly used in various electrical machines (of course this method has not been so far used in PM machines) and it is one of the common methods for fault diagnosis under non-stationary conditions. Moreover, this algorithm is based on the assumption that speed and load vary slowly and there are sufficient intervals along which operation of the machine can be assumed stationary. Such an assumption is not always correct. Also, the STFT method is used to prevent having a trade-off between time and frequency resolution that has to be made when using the FFT method. Nevertheless, a fixed length window can cause inconsistent treatment of different current frequencies, and speed variations of the motor lead to a harder determination of harmonic orders. The WFR algorithm passively extracts the fault characteristics from the spectrogram of an adaptively filtered non-stationary motor current signal by local maxima estimation. It is noted that fault frequencies also appear under non-stationary operation conditions. In fact, WFR method calculates instantaneous frequencies using local maxima. This algorithm has been firstly used in [68] in order to analyse musical sounds. The WDR has been utilised in [69] to diagnose eccentricity fault in PM-BLD machines. First, a spectrogram of the filtered current signal has been extracted. Then, Fourier ridges (amplitude of the instantaneous frequency) are applied on it. These characteristics are used to build a new signal. After using the Grassberger–Procaccia algorithm and its implementation on the signal, correlation dimensions related to the signal are extracted and used to diagnose the fault in direct drive PMSG.

4.2.5 Wigner–Ville distribution (WVD) of stator current (tenth index): The WVD method considers time, frequency, and amplitude which are enough to describe transient events and occurrences in the signal. This method is widely applicable in fault diagnosis of mechanical systems. In fact, this method gives a time–frequency 3D distribution of the signal. Under non-stationary conditions, WVD is calculated by extracting the relationship between the power spectrum and the autocorrelation function of the event. There are some undesirable terms that are side products of this algorithm known as cross terms that can disturb the interpretation of the distribution. The
biquadratic WVD generates many of these cross terms. Their amplitudes are larger than the actual spectral components in the signal. Various versions of WVD have emerged to mitigate some disturbing effects of these cross terms.

The WVD algorithm has been used in [70] to diagnose eccentricity fault in PM-BLDC motor which is done passively. First, the filtered current is transformed into an analytic signal using Hilbert transformation. Then, the WVD is implemented over a specific time range on Hilbert transformed analytic current signal. Finally, the rms of the fault components for fault diagnosis is estimated. In this algorithm, the performance quality of the disturbing effects of these cross.

In this algorithm, the performance quality of the WVD is very sensitive to the type and length of the data window. The reason is that the choice of these two parameters is a compromise between the output time and frequency resolution. This method is able to diagnose the fault under non-stationary conditions and is on-line and non-invasive. This method is very costly due to its long computation time and hardware requirements. So, using this method in low-cost drives is very difficult. Position-depending oscillating load torque (reciprocating compressors) generates the sideband having the frequency pattern identical with the fault frequency pattern, but with larger amplitudes which is a major disadvantage.

4.2.6 Wavelet transformation of stator current (11th index): Wavelet transform (WT) is a means of decomposing signals, especially the non-stationary ones. This transformation takes a wide-band non-stationary signal at its input and generates a time–frequency distribution of the input as the final product of the transformation [5]. Through this transformation, the HF components concentrate over short time ranges and low-frequency components over long time ranges [6]. The most important step in applying a WT on a signal is to choose which wavelet function to use as it must be used up to the end of the analysis. Therefore, choosing the right wavelet function is not easy. The use of adaptive wavelets to address this issue can be a good research topic. Continuous WT (CWT), discrete WT (DWT), wavelet packet transform (WPT), and un-decimated DWT (UDWT) are among various types of available WT which are explained below:

\[
\text{CWT}(s, \tau) = \frac{1}{\sqrt{s}} \int x(t) \psi \left( \frac{t - \tau}{s} \right) dt,
\]

where \( \psi \) is the wavelet function that is both scaled and shifted in time in (6). Therefore, \( s \) and \( \tau \) are the scale and time-translation parameters, respectively. The CWT formulation is the Fourier transform equation in which the WT has replaced the sine and cosine functions. The \( s \) and \( \tau \) parameters result in signal transformation into a 2D plane. Applying CWT on a signal provides freedom in choosing how information is extracted (redundancy) as \( s \) and \( \tau \) parameters can be changed by design. This redundancy is beneficial but comes at the cost of the computational burden. It is well proved that dyadic scales, \( s = 2 \) and \( \tau = k2^j \) can provide a trade-off between computational burden and good information extraction. Such a discretisation gave birth to DWT which can be implemented using a pair of low-pass and high-pass wavelet filters known as quadrature mirror filters. The DWT can be calculated using the following equation:

\[
\text{DWT}(j, k) = \sqrt{2^{j}} \int x(t) \psi \left( \frac{t - k2^j}{2^j} \right) dt.
\]

UDWT [74–76]: In general, the DWT of the shifted (downsampled) signal is not exactly similar to the shifted version of the DWT of the original signal. That is due to the fact that DWT does not possess the shift-invariant property. Lack of this feature can cause issues in the fault classification applications. The UDWT, also known as stationary WT, is an improved version of the DWT algorithm that has shift-invariant property. In UDWT, the down-sampling operation is not conducted at each level. Instead, the filters used at each level are up-sampled versions (by two) of the filter used at the previous level.

\[
WPT [77, 78]: \text{The WT does not process the detail signals of higher frequencies. Therefore, it has a low-frequency resolution at HF components. The WPT improves the frequency resolution for higher frequency components by further decomposing the detail signals of higher frequency. The time resolution, which is a measure of detailed information of a signal, is changed by filtering the operations. The frequency is changed by down sampling operations. The WPT algorithm begins by passing a signal \( x \) through a high-pass filter and a low-pass filter. The outputs of high-pass and low-pass filters constitute one level of signal decomposition. Signal at the first level is down sampled by two to get a better frequency resolution. After down sampling, each signal is again decomposed into low-pass and high-pass filters. The WPT is suitable for the detection of HF components superimposed on the fundamental frequency of a signal. It should also be mentioned that in PM machines, there might be a high level of noise in the analysed signal by the WT, which can get amplified due to the fault occurrence. Therefore, it is important to know how WT handles these noise disturbances in the signal. One way of handling a noisy signal by WT for the fault diagnosis purposes is denoising it through thresholding principle. Thresholding technique can be categorised into hard thresholding [79] and soft thresholding [80]. Denoising a noisy signal by means of thresholding technique imbedded into WT core has two steps given as follows [81]:

i. Using WT to analyse the noisy signal and decompose it into \( N \) levels and obtain approximation and detailed coefficients.

ii. Thresholding of the coefficients.

The second step is different under soft and hard thresholding. During the second step, a threshold is set in the denoising process. Differences in soft and hard thresholding methods appear in this step. Under hard thresholding, coefficients that have an absolute value lower than the defined threshold are set to zero, while the coefficients that possess higher values than the threshold are untouched and preserved. However, under soft thresholding, in addition to setting the coefficients lower than the threshold to zero, the threshold value is subtracted from the absolute value of the coefficients larger than the threshold. Hard thresholding results in a smaller mean square error, while soft thresholding mitigates the issues associated with spurious oscillations. Another approach to improve the robustness of WT against noise is through active noise cancellation. This involves imbedding a lean mean square adaptive filter to denoise the signal that is going to be then processed by the WT in the fault detection scheme [82].

Daubechies 6 has been used as the mother wavelet in [83], and according to [84], it has the best adapted global signal properties (at least for fault diagnosis purposes) and is the best choice for signal general decoupling. The WT has been used for time-frequency decomposition of a PMSM stator current (which is extracted actively) under eccentricity fault [83]. Then, KNN method is used to classify the fault type. The fault severity has been determined using the F SVM. Efficiency and accuracy of the method have been investigated by adding noise to the original signal (to imitate the measurement noise) and detecting fault from the signal containing noise which led to good results. It means that this index is capable of detecting eccentricity fault even in extremely low severities (below 10%).

4.2.7 Park transformation of stator current (12th index): Fault characteristic in the stator current is not always clear and in many cases is shadowed by the fundamental component. The reason is that the fault harmonics are much small particularly compared to the fundamental component, especially when the fault severity is not high. Different methods have been pursued to eliminate the dominant effect of the fundamental component upon the fault harmonics. One method is to use \( q \)-axis current which was adopted for the estimation of mechanical unbalances [75, 85]. By using Park’s vector approach, the fundamental component is removed from the Fourier spectrum of the current, and then, fault
components are analysed using DWT. Extracting the eccentricity fault index from PMSM has been carried out under non-stationary conditions [86]. This passive index is on-line, non-invasive, and independent of machine parameters and has no need for an additional sensor. However, simultaneous utilisation of two signal processing methods leads to a high computational burden.

4.2.8 Search coil fundamental voltage component (13th index): A fault diagnosis method which recognises different types of the fault using search coils has been introduced in [46]. The search coils measure passively a vector summation of the flux due to PMs and flux generated by the machine armature coils. In this measurement, it is assumed that there is no saturation. To analyse the reason for unbalanced flux, it is necessary to decouple these two components. Under no-load conditions, and rotor rotating with synchronous speed, \( E_1 \) voltage is induced by MMF of the PMs in each search coil. The MMF distribution can be described as space vector and induced back emf in each search coil is a time phasor. Superposition of MMF due to armature MMF (known as armature reaction) generates a combined MMF called \( \mathbf{E}_g \) which is the vector summation of \( \mathbf{E}_f \) and \( \mathbf{E}_a \). This MMF is responsible for generating the airgap resultant flux which induces back emf in search coils under on-load conditions. The measured voltage in the search coil is decoupled into two components as follows:

\[
E_{g}(t) = E_{f}(t) + E_{a}(t)
\]

\[ (8) \]

where \( f_a \) and \( f_a \) are the field, armature, and their combination, respectively. Since PMSM is always supplied by an inverter, its phase current is not a perfect sinusoidal waveform. For extracting each harmonic component, Fourier transformation is applied to both sides of (8) and their fundamental components are used for later analyses as follows:

\[
E_{g-1} = E_{f-1} + E_{a-1} \]

\[ (9) \]

Then, the field and armature fundamental components of the induced voltage in the search coils wound around the stator teeth are plotted in a separate polar graph. To diagnose some of the faults, the component due to the field and for some others the component due to the armature must be examined. For example, for turn-to-turn short circuit fault, the component due to armature must be considered. The impact of any fault on the set of two polar diagrams is unique and any fault can be diagnosed and located without signal processing and/or pattern recognition algorithm. These coils are wound around armature teeth, and consequently, this method is invasive. As a result, these search coils must be mounted during building the machine. In this method, the fixed search coils sense the magnetic flux around the stator and only the fundamental component of the coil voltage is used for fault diagnosis. Therefore, this method is immune to HF harmonics and it is suitable for application in motors and generators connected to power converters with switching devices. In addition, this method does not need parameters of the machine and it locates the minimum airgap location under the SE.

4.2.9 Additional winding normalised back-emf (14th index): The search coils can also be used for the DE fault diagnosis [87]. In this method, an additional search coil with a three-slot pitch is wound within the machine and back-emf of this coil, normalised by the rotation speed, is used for eccentricity fault diagnosis. For example, the additional coil has a leg in slot #1 and a returning leg in slot #4. The prerequisite of the method is that there is an even number of poles over three-slot pitches. In other words, the number of poles per slot should be 0.677 or integer multiples of this number. Speed of the machine can be determined from zero crossing of the back-emf signal. This index is extracted passively and is independent of the speed, because the measured voltage is normalised by the speed and is useable under non-stationary conditions. This method uses no signal processing algorithm and diagnoses the fault using the amplitude of the normalised voltage. This invasive signal is zero in the healthy motor.

4.2.10 Linear discriminant analysis of voltage and current (15th index): The problem of using subharmonics with the frequency given in (5) is that different faults generate the same sideband patterns in the stator current spectrum. Also, the amplitude of the fault subharmonics depends on the speed and load level. In non-stationary operating conditions over low speeds, correct and precise detection of these harmonics is difficult. Moreover, the number of stator slots and machine poles affect the amplitude of the subharmonics in the stator current spectrum. The index has been used in [41] for passive fault diagnosis in PMSMs, and also, fault type and severity. In this method, two LDA classifiers have been used. The first classifier determines the existence of fault and its type. The second classifier estimates the fault severity. In the first step of this trend, current or voltage harmonic up to 15th harmonic measured in the ABC-reference frame. Then, FFT is applied to the measured signal and the amplitude of the main harmonics (fs, 2fs, 3fs …) is extracted and the first 15 harmonics (the fundamental harmonic up to 15th harmonic) are given as input to the first LDA to define the type and severity of the fault. The LDA algorithm has different types such as classical LDA, Foley–Sammon LDA, and uncorrelated LDA. Different LDAs have been compared in [88].

4.2.11 Amplitude of side band components (ASBCs) of torque profile (16th index): An index, extracted passively, based on ASBCs of the fault with the pattern presented in (5) in the torque spectrum of PMSM machine has been introduced in [31, 45]. To address the capability of the introduced index in the eccentricity fault diagnosis, the correlation coefficient analysis has been performed which evaluates the linear dependency of the extracted index from the torque signal to the fault severity. This extracted index has a higher linear dependency on the fault severity compared to the index ASBCs of the current spectrum. In the following, an ANN is used for the fault classification and determination of its severity which considers the non-linear dependency of the ASBCs in the torque spectrum to the fault severity. This ANN is a multilayer perceptron-type system. The three-layer feed-forward perceptron ANN is trained by propagation learning to have a good performance in the classification of the fault severity. The obtained results indicate that the extracted index from the torque using ANN performs better than that extracted from the current in term of determination of the fault severity. The reason is that ASBCs in the torque spectrum have a stronger non-linear dependency on the fault. It is noted that this index is very sensitive to pulsating torques, and also, the required sensor for machine torque extraction is not cheap and its fixing and calibrating is only economical for large machines.

4.2.12 Cogging torque peak and average value (17th index): The impact of rotor eccentricity on the cogging torque of a PM generator coupled to a small wind turbine has been studied in [89]. The generator is a radial flux type and it has been designed as such that the start-up has been improved over low-speed winds through the cogging torque reduction, in other words, it has the low start up wind speed. The FEM has been used for modelling the generator. It has been observed that the DE fault can be diagnosed passively by measuring the cogging torque waveform. It has been shown in [89] that the DE fault leads to non-zero mean cogging torque and positive and negative peaks of the cogging torque also change. The use of the cogging torque has been suggested for the DE fault. However, no organised and aimful fault diagnosis using this cogging torque signal can be observed [89] and it is simply a suggestion made to somehow use the changes in the peaks and mean value of the cogging torque. Furthermore, this method is offline and the cogging torque is a small quantity which makes its precise measurement difficult. Fixing torque sensor on all machines is not possible and rating of the sensor for on-load torque measurement is much higher than the required ratings for the cogging torque measurement.
4.2.13 Acoustic noise harmonics (18th index): Vibration and noise in the PM machine are related to the radial forces and mechanical behaviour of the machine. An analytical electromagnetic vibro-acoustic model has been developed in [83, 84] to investigate the eccentricity fault effect upon the generated electromagnetic noise in PMSM. The passively obtained results indicate that eccentricity fault produces a series of low-mode radial forces. Moreover, this fault in PM machines affects certain characteristic frequency components in the airgap magnetic field which influences the vibration and sound power level spectrum of the machine. Fault harmonic components appear in the estimated noise spectrum and can be used for eccentricity fault diagnosis. It has been suggested in [83, 84] that the DE fault can be detected through acoustic noise monitoring. This technique is not possible to be applied to the motors fixed on the fan and the disk due to the combination of acoustic noise caused by eccentricity and aerodynamic noise produced by the fan and the disk.

4.2.14 Slot passing harmonics (19th index): The UMP under eccentricity fault contains different harmonics where 17th (number of slots minus 1) and 19th (number of slots plus 1) harmonics have the highest amplitudes [25]. The amplitudes of the above-mentioned harmonics rise with higher severity and they can be used for fault diagnosis. These harmonics are also called slot passing vibration harmonics. These harmonics are the dominant components in the UMP vibration signal. It should be noted that it is very difficult to measure precisely the UMP signal. It has been shown in [90] that the airgap magnetic flux density power spectral density in the case of SE fault excites the sidebands with the frequency pattern of (2). These harmonics exhibit harmonic force on the core which transfers vibrations with the same frequency pattern to the stator core surface. Since this vibration signal consists of sideband components caused by SE and DE faults, monitoring these harmonics leads to diagnosing the SE and DE faults. Eccentricity fault also generates sideband components in radial magnetic stress at frequencies with the pattern given in (5). Distinguishing between SE and DE faults can be accomplished using the incremental rate of the amplitude of side-band components where this rate is higher for the SE fault. Moreover, there is a need to study the effect of other faults on these UMP harmonic components. In addition, these harmonics in the UMP could be created by the other components of the system coupled to the machine shaft. It should be mentioned that this fault indicator is obtained passively.

5 Eccentricity fault diagnosis indices for axial flux PM machines

Rotor disk polar moment of inertia in the axial flux machines is much larger than that of the shaft. Also, the mechanical design integrity of the contact location between the rotor and the shaft is more compromised in such machines due to the fact that for the same rated power, the contact surface between the rotor and the shaft is smaller [91]. These special structural features in the axial flux machines enhance the probability of eccentricity fault. A very complete definition of eccentricity fault for AFPM machine has been presented in [92] which covers static angular misalignment, dynamic angular misalignment, SE misalignment, and DE misalignment. Various studies have focused on analytical [93–101], numerical [103–105], and hybrid [106] modelling aspect of eccentric RFPM machines. The same has been done for AFPM machines [92, 107–109]. In this section, all the eccentricity fault diagnosis indices available for AFPM machines are analysed.

5.1 Parameter-based indices

5.1.1 Parametric spectral estimation of current and vibration signatures (20th index): A passive index has been introduced in [110] for distinguishing and diagnosing static misalignment and turn-to-turn short circuit in AFPM machines. This method is based on the combined current and vibration analysis. The FFT technique generally needs a long measurement duration to present a good and acceptable frequency resolution. However, in many applications, extracting a stationary signal over a long time is not possible. In addition, the required hardware for storing huge data is too expensive. To solve this problem and achieve a desirable frequency resolution during short signal measurement, estimation of signal parameters via rotational invariant technique (ESPRIT) is employed for current and vibration analysis. This technique has been applied to fault diagnosis in induction machines [111]. The general principle of this method is as follows. If a signal depends on a finite set of parameters, all statistical properties of this signal, including its power spectrum, can be expressed versus these parameters. This is possible using parametric models which connects the eigenvector of decomposition of the correlation matrix and estimates the discrete part of the spectrum. If the measured vibration current and signal are presented by \( x(n) \), the measured signal \( x(n) \) can be expressed as follows:

\[
x(n) = \sum_{i=1}^{N} A_i \cos(2\pi f_i n T_s + \theta_i), \quad n = 1, 2, 3, \ldots, N,
\]

where \( A_i \), \( f_i \), and \( \theta_i \) are the amplitude, frequency, and the initial phase of the \( i \)th harmonic, respectively, \( T_s \) is the sampling interval, and \( p \) is the number of harmonics. Parametric models related to the eigenvector decomposition of the correlation matrix can be developed from the current or vibration signal and used to estimate the discrete part of the spectrum. In order to extract the fault harmonics, an algorithm based on (10)–(15) has been written as follows:

\[
y(n) = x(n + 1)
\]

\[
X(n) = \begin{bmatrix} x(n) & x(n+1) & \ldots & x(n+m-1) \end{bmatrix}^T
\]

\[
Y(n) = \begin{bmatrix} y(n) & y(n+1) & \ldots & y(n+m-1) \end{bmatrix}^T
\]

\[
R_{XX} = E[X(n)X^H(n)]
\]

\[
R_{XY} = E[X(n)Y^H(n)].
\]

Parametric models relating to the eigenvector decomposition of the correlation matrix developed from the current or vibration signal are used to estimate the discrete part of the spectrum by first measuring the \( X(n) \) signal for \( 3 \) s, and then using (13) and (14), a transition matrix of \( X(n) \) is constructed. In the next step, the correlation matrix and mutual-correlation matrix are put together using (14) and (15). Here, \( E \) and \( H \) represent the mathematical expectation and conjugation, respectively. The full details of the ESPRIT method have been described in [112].

5.2 Signal-based indices

5.2.1 Individual coil’s no-load voltage (21st index): Static angular misalignment fault does not affect the phase back-emf signal in axial flux machines. The machine has been studied in [113] which has four coils in each phase. The back-emf of all four coils of phase A has been extracted using FEM and also by experiments. Under the fault, the back-emf varies at least in two of the four coils. If a coil is located where the airgap length has decreased due to the fault, the back-emf increases and the back-emf reduces where the airgap length has increased. It is clear that those coils which do not sense any airgap length variation have no back-emf change. The technique for fault diagnosis is the use of fundamental and third harmonic component (extracted by FFT) of back-emf. One of the routines in the design of electrical machines is adjusting the airgap length on a base value for reduction and minimisation of the third harmonic content. If SE fault occurs, airgap deviates from the base value and the third harmonic as well as the fundamental component of back-emf of four coils of phase A vary. The index can be seen as the investigation of the fundamental and 3rd harmonic of the back-emf which is extracted passively. This index is only presented for the topology and winding pattern of the case study given in [113] and a general form index is absent.
So, a comprehensive and general method for different types of windings must be introduced. Also, it must be investigated whether it is possible to use a smaller number of coils. It is noted that the method is off-line.

5.2.2 Search coil induced voltage (22nd index): This index is the improved version of the 21st index extended to other types of machines [114]. The method presented in [113] is based on the analysis of the induced back-emf in the stator phase coils under a no-load condition which is an off-line method. In this passive index, use of the main coils has been abandoned and search coils are fixed inside the machine. Induced voltage in these search coils has been studied. Several choices on the number and position of the search coils are available. In choosing these two parameters, two points must be considered. The first is the minimum number of search coils must be utilised, and the second is the number of search coils must be sufficient enough for estimation purposes in order to determine not only SE fault, but also define the minimum airgap position. There are always two positions in the airgap region in which the airgap length does not vary under eccentricity fault. So, if two search coils are used, it is possible that the two search coils get placed on these two blind spots and the search coils lose their capability to detect the fault. To avoid such problems, three search coils uniformly distributed in the airgap in 120 mechanical degrees distance in the stator space is the best choice. These search coils indirectly measure the airgap flux through the induced voltage in the search coils. This index is on-line and it provides the possibility of estimation of the eccentricity fault severity and the minimum airgap position. Despite fault diagnosis methods based on the stator current spectrum which are under severe influence of winding configuration, the present method is independent of the winding layout and applicable on all axial flux machines.

5.2.3 Frequency content of start-up transient and steady-state signal (23rd index): The SE fault in AFPM machines having concentrated winding (a type of winding with rich current harmonic content) has been diagnosed using vibration analysis [68]. The SE fault has a direct relation with vibration level, but transducer positioning is very important in the effective and resolute measurement of vibration. An experimental approach for precise and secure isolation of electromagnetic vibration from other existing types of vibration has been introduced in [68]. The accuracy and quality of the extracted electromagnetic data from vibration monitoring in [68] are very high and it is appropriate for SE fault diagnosis. In steady conditions, the FFT of vibration signal is used, and from the frequency content, harmonics sensitive to the SE fault is determined and used for fault diagnosis. Under non-stationary operation, the ZAM distribution method is utilised for the analysis of the vibration signal. In fact, ZAM distribution was used to analyse the start-up transient vibration characteristics and time-varying spectral energy plot of the vibratory harmonics in the time–frequency domain. The fault diagnosis index is robust against a load-level variation, and vibration sensors can be fixed in a non-invasive manner. This index is extracted passively and can be obtained through off-line or on-line measurements. The measured signals can be analysed under steady state and/or transient conditions using standard signal processing methods.

5.2.4 Amplitude of side-band harmonic components of stator phase current (24th index): The impact of SE fault on the current harmonics and torque ripples in the AFPM machines with fractional-slot concentrated winding has been addressed in [115]. Also, the effect of single-sided and double-sided and cogging torque minimisation techniques on the current harmonics has been investigated. Finally, the amplitude of the fault harmonic components of stator current spectrum with the following frequency patterns has been introduced as an appropriate index for SE fault diagnosis in AFPM machines:

\[
f_{se} = \left(1 \pm \frac{2K - 1}{P}\right)f_{se}, \quad K = 3, 8, 13, \ldots
\]

This index is calculated passively and is not disturbed by the techniques applied for cogging torque reduction.

6 Complementary discussion

To summarise the discussion on the fault diagnosis indices, a tree chart is presented in Fig. 4. One may ask if the environmental factors can affect these index performances, and also, if the provided indices provide the same performance for various PM drive systems. The answer to the first question is YES. Take for instance, vibration and noise signals can be extracted via contact devices and sensors for diagnosis purposes. In environments which are greasy, the contact maybe slippery and loose [116]. Moreover, ambient noise, temperature, and humidity may damage or disturb the sensor performance [117]. In addition, PM machines generate heat due to their losses. This heat is evacuated towards the machine surroundings via two thermal processes: convection and radiation [118]. Various environmental factors can compromise the heat evacuation process. Extreme ambient temperature can disturb the heat convection. An environment with acidic chemicals can damage the cooling fan, or dust clogging/covering the motor casing/fins [119]. Any obstruction of heat flow results in temperature rise and irreversible, if not irreversible partial demagnetisation. As partial demagnetisation and DE have the same fault frequency pattern [62], indices may raise false DE alarm. Moreover, dust, mixed with the bearing lubricant, forms a strongly abrasive substance disturbing the ball bearings [120] and emulates the similar fault signatures to eccentricity fault.

As for the second question, the answer is NO. Various properties of a PM drive system can affect the performance of a given index. Take the control system as an example. The appearance of fault harmonic components given in (2) depends strongly on the bandwidth of the current and speed controllers and weather the motor is in control is open loop or closed loop. Based on [121], in the open-loop control the current control loop (torque) with no external speed control loop. If the bandwidth of the current controller is infinite, fault harmonic components due to SE and DE fault does not appear in the phase current.

In the case of closed-loop control, since speed reference command and subsequently d-axis reference current command is fixed, the harmonics caused by the DE fault appear in electromagnetic torque. Assuming a constant load torque, the existing harmonics in the electromagnetic torque cause speed oscillations \(\Delta\omega_c\). Now, these speed oscillations are feedback to the speed controller. The bandwidth of current controllers is infinite; it means that the current command fully coincides with the feedback current. Now, there are two cases for the speed controller:

Low bandwidth of speed controller: In this case, the speed controller is unable to compensate for the existing speed.
oscillations. Therefore, the speed controller generates a torque command at its output which causes sinusoidal currents in the abc reference frame and DC currents in the DQ reference frame. As a result, the fault harmonics with the mentioned order appear in the voltage spectrum of the machine [65].

High bandwidth of speed controller: In this case, the feedback speed oscillations to the speed controller are compensated by the torque (speed controller output) command. Consequently, fault harmonics appear in the motor currents, and no fault harmonic is there in the voltage spectrum of the machine.

Other design choices such as different PM arrangement [23], the existence of parallel paths in stator winding [122–125], and having single- or double-sided rotors (in AFPM machines) [115] can influence the reaction of the machine to eccentricity fault and therefore, affect the extracted indices.

7 Score-based comparison of indices

In Sections 4 and 5, a total of 24 indices for SE, DE, and ME fault diagnosis in RFPM and AFPM machines were introduced and discussed and the details of each index were investigated. This section addresses the comparison of these indices under different conditions and scenarios using the 17 criteria set forward in Section 3.

Each evaluation statement $A_i$ ($i = 1, 2, 3, …, 17$) is assigned a weighting coefficient $K_{n}$ ($n = 1, 2, 3, …, 17$). The weighting coefficients associated with the evaluation statements specify how important each evaluation statement is and are based on the specifics of the application and the designer’s judgement. To elaborate how the blocks of Table 1 are populated, one can say that if a statement ($A_i$) is satisfied by an index, then the sign + is considered for it, while if the statement is not satisfied for an index, then the sign − is assigned for it. When the status of an index against a statement is unknown (not considered in the existing papers), the sign * is assigned to it. In fact, the sign * is the question which has not been so far considered and can be a topic of future researches.

After populating every cell of Table 1 with the correct sign, we can start our comparison process. The symbol ++ stands for the positive score associated with each index and the symbols − and ? resemble the negative score and the floating score for each index. Comparison can be performed under two different logics. In the first logic, the ambiguous points are not seen as a good or bad thing and they are ignored in the comparison process. Therefore, the following conversion is applied to the recorded evaluation statements:

$$\begin{align*}
&\text{If } A_{n,m} = + \Rightarrow A_{n,m} = + 1 \\
&\text{If } A_{n,m} = - \Rightarrow A_{n,m} = - 1, \\
&\text{If } A_{n,m} = * \Rightarrow A_{n,m} = 0
\end{align*}$$

where $A_{n,m}$ stands for the $n$th evaluation statement value with regards to the $m$th index. The alternative logic is to see the ambiguous points as a bad thing. Therefore, the following conversion is applied to the recorded evaluation statements:

$$\begin{align*}
&\text{If } A_{n,m} = + \Rightarrow A_{n,m} = + 1 \\
&\text{If } A_{n,m} = - \Rightarrow A_{n,m} = - 1 \\
&\text{If } A_{n,m} = * \Rightarrow A_{n,m} = 0
\end{align*}$$

where $A_{n,m}$ stands for the $n$th evaluation statement value with regards to the $m$th index. In the next step, the score for each index is calculated as below:

$$S_m = K_1A_{1,m} + K_2A_{2,m} + K_3A_{3,m} + \ldots + K_{17}A_{17,m}$$

where $S_m$ is the total score calculated for the $m$th index. As a simple case study, assume that all the evaluation statements possess equal importance $K_1$, $K_2$, $K_3$, …, $K_{17} = 1$. After calculating the index scores under these logics, we see that under both logics, the 11th index is the most powerful one. Under the first logic, the 22nd index is the weakest. However, using the second logic, indices 19th and 22nd are the weakest. As mentioned before, the comparison process can be customised based on the specifics of the application. This is a very appealing feature of the proposed comparison method as there could be a situation where some performance capabilities are prerequisite and must be satisfied. For example, an

| Statement | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ | $A_{11}$ | $A_{12}$ | $A_{13}$ | $A_{14}$ | $A_{15}$ | $A_{16}$ | $A_{17}$ | ++ | − | ? |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1         | +    | +    | +    | +    | +    | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 13 | 4  | 0 |
| 2         | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | +    | 14 | 3  | 0 |
| 3         | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | +    | +    | +    | +    | +    | −    | 9  | 4  | 4 |
| 4         | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 3  | 1  | 1 |
| 5         | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | +    | +    | +    | +    | +    | +    | 11 | 4  | 2 |
| 6         | −    | −    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 10 | 3  | 3 |
| 7         | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 3  | 1  | 3 |
| 8         | −    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 15 | 2  | 0 |
| 9         | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 13 | 4  | 0 |
| 10        | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 12 | 4  | 1 |
| 11        | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 9  | 6  | 2 |
| 12        | −    | +    | +    | −    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | +    | −    | 9  | 5  | 3 |

Table 1. Comparison of 24 indices
index can be a very powerful one but is not able to perform on-line fault detection. If the fault detection process must be performed on-line, utilising the mentioned index is out of the picture. Under such applications, the weighting coefficient associated with those prerequisite capabilities must be modified to a higher value.

To give further clarification, three different scenarios are considered for the special application of eccentricity fault diagnosis. The first scenario is fault diagnosis under non-stationary conditions. The second scenario is the capability of distinguishing between the DE and partial demagnetisation fault. Finally, the third scenario is a computational burden. To reflect the prerequisite capabilities in the evaluation process, the weighting coefficients are modified for the three given scenarios as below:

First scenario: \( K_1 - K_2 = 1 \) and \( K_1 - K_{17} = 1 \), \( K_6 = 17 \)

Second scenario: \( K_1 = 17 \), \( K_2 - K_{17} = 1 \)

Third scenario: \( K_1 - K_2 = 1 \) and \( K_1 - K_{17} = 1 \), \( K_6 = 17 \).

We start the comparison with the first logic. Considering the first scenario, the 11th index is the most powerful and 23rd index is the weakest one. As for the second scenario, the 2nd index is the most powerful and 20th index is the weakest. Finally, in the third scenario, the 2nd index is the most powerful and the 22nd index is the weakest.

Considering the second logic, in the first scenario, the 11th index is the most powerful one, and 23rd and 20th indices are the weakest. In the second scenario, 14th index is the most powerful, while 3rd and 20th indices are the weakest. Finally, in the third scenario, the 2nd index is the most powerful while the 19th and 22nd indices are the weakest.

8 Conclusion

This paper addressed various aspects of eccentricity fault in PM machines. Different analytical, numerical, and hybrid modelling approaches, capable of simulating the performance of eccentric PM machine, were proposed. Also, different applications of PM machines were explained and the impact of undetected fault in the motor structure was given. Moreover, the most common intelligent algorithms utilised in the eccentric PM motors were described in comparison made in this paper was a simple example of how different indices should be evaluated based on the machine scenario. The 11th index is the most powerful and 23rd index is the weakest one. As for the second scenario, the 2nd index is the most powerful, while the 19th and 22nd indices are the weakest.

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