A comparative reliability study of corroded pipelines based on Monte Carlo Simulation and Latin Hypercube Sampling methods

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ABSTRACT

Internal corrosion is categorized as one of the most destructive phenomena for pipeline services. This study compares the reliability analysis of pipelines against internal corrosion based on Monte Carlo Simulation (MCS) and Latin Hypercube Sampling (LHS) methods. Three different failure mechanisms (pitting perforation, local burst and rupture) have been considered for single corrosion defect and only local burst has been investigated for multiple corrosion defects. Furthermore, the effects of correlation between the defect depth and defect length for single defects as well as the correlation between the neighboring segments for multiple defects on the failure probability of the pipelines, have been comprehensively studied. The results show that LHS method could lead to quite acceptable accuracy with considerably less computational effort. Regarding multiple corrosion defects, the correlation between the defect depth or defect length of the neighboring segments can change the system failure probability without any monotonic trend.

1. Introduction

Pipelines might be considered as one of the most reliable means of transporting vital products such as water, oil, and gas. New pipeline projects are being built such as the very long one (4100 km) that connects Africa to Europe from the delta site production in Nigeria to Italy and Spain crossing Niger, Algeria and the Mediterranean sea [1]. On the other hand, gas and oil pipelines especially the offshore ones could experience deterioration and degradation followed by catastrophic environmental and financial consequences. Such severe damages can be caused by different reasons such as 1) corrosion 2) mechanical and structural 3) third party 4) natural hazards 5) operating errors [2]. Among all these reasons, corrosion has been recognized as the most underlying and cost bearing defect mechanism which can substantially reduce the long term reliability and integrity of pipelines. Considering different sources of aleatoric and epistemic uncertainties, it is more reasonable to evaluate the corrosion defects in the scope of reliability and statistical processes [3–7]. In this regard, some probabilistic procedures have been developed based on modified ASME B31G-91 and DNV (2010) [8,9].

In 1996, Ahammed and Melchers [10] proposed a probabilistic method for the evaluation of the suitability of corroded pipelines under pressure loading. They developed a statistical limit state model from the deterministic failure equations and utilized the advanced First Order Second Moment (FOSM) method to estimate the reliability. In another study, Caleyo et al. [11] investigated the reliability assessment and remaining life prediction of underground pipelines containing active corrosion defects using FOSM iterative reliability method, Monte Carlo (MC) integration technique and the first order Taylor series expansion of the Limit State Function (LSF). Teixeira et al. [12] studied the reliability of pipelines against corrosion defects by First Order Reliability Method (FORM). They defined a limit state function based on the results of small-scale experiments and three dimensional nonlinear finite element analysis of burst pressure of intact and corroded pipelines. Also, they prioritized the influence of the uncertain parameters on the probability of burst pressure of corroded pipelines by an efficient sensitivity analysis. Li et al. [13] worked on predicting corrosion remaining life of underground pipelines with a mechanically-based probabilistic model by taking the impacts of random uncertainties into account. They used MC Simulation (MCS) technique to estimate the remaining life of the corroded pipelines and their Cumulative Distribution Functions (CDFs). They did not consider the effects of correlation between some random variables such as defect depth and defect length for simplicity. The results of their study revealed that the corrosion defect depth and radial corrosion rate are the key factors influencing pipeline failure probability and remaining life. Also, the estimated CDF can characterize the pipeline

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failure probability more appropriately, compared to Probability Density Function (PDF). Zhou [14] presented a methodology to analyze the time dependent system reliability of a pipeline segment under stochastic internal pressure loading considering three different failure modes caused by corrosion as small leak, large leak and rupture. The results of his study showed that the spatial variability of pipeline properties has a negligible influence on the system reliability. On the contrary, the spatial variability of the internal pressure, initial defect sizes and defect growth rates can significantly affect the system reliability. In another paper, Leira et al. [15] studied the system reliability analysis of corroded pipelines employing enhanced MCS method which is an efficient approach with less computational expenses compared to the crude MCS technique [16]. They computed the probability of failure caused by single and multiple corrosion defects considering the correlation between defect components by enhanced MCS method. More specifically, Zelmati et al. [1] worked on assessing the effects of correlation between axial corrosion rate, yield and tensile strength) on reliability analysis of pipelines against corrosion considering three imperative failure modes as 1) corrosion perforation 2) local burst 3) rupture. This study aims to perform a reliability assessment of corroded pipelines considering single and multiple (uncorrelated and correlated components) corrosion defects by two efficient sampling methods as 1) MCS method [19] 2) Latin Hypercube Sampling (LHS) technique [20]. It is declared that MCS has been frequently applied in several relevant studies. But, the main purpose of this study is to compare the efficiency of these two sampling methods (MCS and LHS) considering three failure modes as 1) corrosion perforation 2) local burst 3) rupture for single corrosion defect along with local burst for multiple corrosion defects. The reason is that the computational cost of LHS method is much less than MCS method. Moreover, it is intended to inspect the effects of the correlated components on reliability evaluation of corroded pipelines using LHS technique. To this objective, Cholesky decomposition method can be applied if the correlation matrix is positive definite [21]. Otherwise, some other algorithms such as Simulated Annealing (SA) [22] can be utilized to optimize the correlation matrix of the randomly generated realizations. Nonetheless, extensive time and computational expenses of this approach especially for large correlation matrices makes it very difficult to find the best realization configuration with optimal correlation matrix. For multiple corrosion defects, since the correlation matrix is too large and the matrix might be non-positive definite, neither one (Cholesky decomposition and SA) is applicable. Consequently, another optimization technique called Iterative Spectral Method (ISM) [23] is implemented to find the minimum required perturbation for changing the non-positive definite correlation matrix to a positive definite one.

It should be added that computational cost is not the first priority in mega oil and gas projects such as pipelines. Nevertheless, the viability and accuracy of the proposed approach should be quantitatively inspected. Therefore, the precision of the applied methodology is compared with a benchmark like MCS method using a well-known statistical criterion such as Root Mean Square Error (RMSE). Hence, the RMSE between the failure probability of the corroded pipeline based on MCS and LHS methods are calculated for different number of LHS simulations to investigate the tradeoff between the precision and computational costs for the employed approach.

In the remainder of this paper, a brief description of the mentioned corrosion limit states (pitting perforation, local burst and rupture) and the concept of MCS and LHS approaches are explained. Afterward, the process of reliability evaluation considering single and multiple corrosion components of the pipeline as well as the ISM procedure are discussed. Finally, the main results of the conducted assessments are presented followed by the conclusions.

2. Limit state function for corrosion defect

Regarding morphology of defects, the damages due to corrosion can be divided into 3 classifications as [18]:

(A) Pitting Corrosion: when there is a pitting corrosion in the local region of the pipeline surface, but the rest is not corroded. The limit state function for Pitting Corrosion (PC) mode \( g_{PC}(X) \) is defined by the following equation [14,18]:

\[
g_{PC}(X) = 0.8t - d
\]

where \( t \) and \( d \) denote pipeline wall thickness and defect depth, respectively.

(B) Local Burst: it happens in certain parts of the pipeline metal surface and it is more destructive than the previous type despite the fact that its weight loss is smaller than uniform corrosion. The limit state function for Local Burst defect \( g_{LB}(X) \) is determined by Eq. (2) [8].

\[
r_{LB}(X) = \left\{ \begin{array}{ll}
2m_{s} \left( \frac{s}{D} \right) \left( 1 - \frac{t}{t_{a}} \right) - p_{a} & \text{if } s < t_{s} \\
0 & \text{if } s \geq t_{s}
\end{array} \right.
\]

In this equation, \( m_{s}, s, t, D, p_{a} \) and \( F \) represent multiplying factor, yield stress, pipeline wall thickness, defect depth, pipeline diameter, the internal pressure acting at the relevant cross section and Folias factor, respectively. Folias factor is calculated by Eq. (3) [24], and \( L \) is the corrosion defect length.

\[
F = \left\{ \begin{array}{cc}
1 + 0.6275 \frac{L^{2}}{Dt} - 0.003375 \frac{L^{4}}{D^{2} t^{2}} & \text{if } \frac{L^{2}}{Dt} \leq 50 \\
0.03 \frac{L^{2}}{Dt} + 3.3 & \text{if } \frac{L^{2}}{Dt} > 50
\end{array} \right.
\]

(C) Uniform Corrosion (rupture): occurs on the pipeline surface and the metal loss is considerable so that the rest of the pipeline wall thickness cannot afford the task of transporting media. The limit state function for Uniform Corrosion defect \( g_{UC}(X) \) is obtained by Eq. (4) [18].

\[
r_{UC}(X) = p_{R} - p_{a}
\]

In this equation, the fracture failure pressure of the pipeline with defects \( p_{R} \) can be calculated by Ref. [25]:

\[
p_{R} = \begin{cases}
1.8 \frac{s_{m}}{tF} & \text{if } s_{m} > 241 \text{ MPa} \\
2.3 \frac{s_{m}}{tF} & \text{if } s_{m} \leq 241 \text{ MPa}
\end{cases}
\]

\( s_{m} \) and \( s_{m} = 359 \) [16] are the tensile strength of the pipeline material.
and the rated minimum yield strength of pipeline in Mpa.

In fact, for each of the 3 considered limit state functions \( P(g(X) < 0) \) is equal to failure probability and \( P(g(X) \geq 0) = 1 - P(g(X) < 0) \) equals the reliability of the corroded pipeline. The statistical properties of the effective random variables as well as their corresponding limit state functions are tabulated in Table 1. In this table, \( a_i : \Delta a_i : a_{Si} \) is a vector which increases from \( a_i \) to \( a_{Si} \) with an increment of \( \Delta a_i \).

3. Reliability evaluation of single corrosion defect

3.1. Monte Carlo Simulation

As mentioned earlier, MCS [19] has been employed to compute the failure probability of the corroded pipeline in this study, which demands high computational effort for large scale problems. For instance, to estimate a probability of the order of \( 10^{-4} \), it is required to perform about \( 10^{9-10} \) to \( 10^{10-11} \) simulations [19]. So, in this study \( 10^6 \) simulations are randomly generated based on the statistical properties of the random variables introduced in Table 1, considering the correlation between the defect depth and defect length (the only correlated variables) of the corrosion defect [1] by Multivariate Normal Random function (mvrnd) in Matlab software. It should be emphasized that since both \( d \) and \( L \) are normal variables, it is possible to either utilize “mvrnd” function or perform the following steps to generate all the random realizations [26]:

1) Generate the initial standard normal realization matrix with \( N_{\text{var}} \) rows and \( N_{\text{Sim}} \) columns. \( N_{\text{var}} \) and \( N_{\text{Sim}} \) are the number of random variables and simulations, and the standard normal random variables are denoted by \( Z_i \) Normal(0, 1) (\( i = 1, 2, ..., N_{\text{var}} \)).

2) Considering \( C \) as the target covariance matrix, \( A \) is determined by Cholesky decomposition as \( C = A A^T \); where, \( A \) is the lower triangular matrix and \( A^T \) is its transposed matrix. It is noted that the Cholesky decomposition cannot be applied unless the matrix \( C \) is positive definite (symmetric with positive eigenvalues).

3) The multivariate normal samples are easily derived by the transition \( X = \mu + A Z \). In this equation, \( \mu \) is the mean vector and \( Z \) is the generated standard normal realization matrix containing \( Z_1, Z_2, ..., Z_n \).

For the other uncorrelated Gaussian or non-Gaussian random variables, since \( N_{\text{Sim}} \) is large enough (\( N_{\text{Sim}} = 10^6 \)) and no predefined target correlation matrix is assigned to the random generator function, the correlation coefficients between all the random variables other than \( d \) and \( L \) automatically become zero. In fact, for generating uncorrelated univariate normal and lognormal samples, “normrnd” and “lognrnd” Matlab functions are employed, respectively. It is advantageous to declare that for considering the correlation between the non-Gaussian random variables some other techniques such as the one described in Ref. [27] should be implemented.

Finally, the failure probability \( P_l \) can be calculated as:

\[
P_l = P(g(X) < 0) = \frac{N_l}{N_{\text{Sim}}}
\]

\( N_{\text{Sim}} \) is the number of generated simulations and \( N_l \) is the number of failure cases \( g(X) < 0 \).

3.2. Latin Hypercube Sampling

The main advantage of LHS method is that its computational expense is much less than MCS technique [20]. In fact, it utilizes stratification of the probability distribution of the random variable \( X_i \) to generate \( N_{\text{Sim}} \) realizations. The \( i \)th sample value of the \( i \)th random variable \( x_{ij} \) can be obtained by the following equation (see Fig. 1):

\[
x_{ij} = F^{-1}_i(p_{ij}) = F^{-1}_i\left(\frac{j - 0.5}{N_{\text{Sim}}}\right), \quad i = 1, 2, ..., N_{\text{var}}, \quad j = 1, 2, ..., N_{\text{Sim}}
\]

In this Equation \( F_i^{-1} \) is the inverse CDF of the \( i \)th random variable and \( p_{ij} \) represents the CDF of the \( i \)th random variable, which is a uniformly distributed random variable in the interval (0, 1).

In this sampling technique, there is an undesired correlation between the random variables when the realizations are generated. Hence, a target correlation matrix \( \rho \) should be considered when the random realizations are being generated. To this end, for the correlation matrix of single corrosion defect with small sample sizes two different approaches can be applied. The first one is based on Cholesky decomposition method [21] if the matrix \( \rho \) is positive definite. It is added that the aforementioned condition (positive definiteness of the correlation matrix) is an essential requirement even for generating the MCSs by “mvrnd” function. If the target correlation matrix is not a positive definite matrix, the second procedure which is based on Simulated Annealing (SA) [22] optimization technique, can be utilized instead, to correct the undesired existing correlation values between the random variables despite the fact that implementation of this approach is more complicated than the previous one. After these steps, the failure probability is computed based on the same equation (Eq. (6)).

4. Reliability evaluation of multiple corrosion defects

When there are multiple corrosion defect components, the failure probability of the pipeline can be derived by system reliability methods [19]. Since for a corroded pipeline the occurrence of failure for each corrosion component can result in its total failure, a series system is a reasonable probabilistic model for this case [17]. According to Ref. [19],

![Fig. 1. A schematic sampling procedure in LHS technique [28].](image-url)
the Lower Bound (LB\(_{P,d}\)) and Upper Bound (UB\(_{P,d}\)) of the failure probability for a series system with \(N_{\text{Cor}}\) defect components (\(P_{d}\)) is determined based on the failure probability of single components (\(P_{d,i}\)) by the following inequality:

\[
LB\(_{P,d}\) = \max(P_{d,i}) \leq P_{d} \leq 1 - \prod_{i=1}^{N_{\text{Cor}}} (1 - P_{d,i}) = UB\(_{P,d}\)
\]

It is mentioned that \(P_{d,i}\) is equal to 1 - \(\prod_{i=1}^{N_{\text{Cor}}} (1 - P_{d,i})\) if all the random variables (\(N_{\text{Var}}\)) are mutually uncorrelated.

Performing MCS for a case of corroded pipeline with 100 corrosion defect components necessitates extensive computational expenses. More specifically, assuming \(N_{\text{Sim}}\) is equal to \(10^5\), even a powerful computer system (core 17 \(\times\) 64 and 16 GB RAM) receives an “out of memory” error when performing such system reliability analysis in Matlab software. This reason illustrates the underlying merit of LHS method that could be easily applied in different cases of system reliability assessments.

Herein, there is another critical problem in case of utilizing either MCS or LHS for system reliability evaluation of corroded pipelines which has not been seriously addressed in the literature. In many cases for series systems, the target correlation matrices are not positive definite. Consequently, it is not possible to use Cholesky decomposition [21] and also the SA technique [22] is not applicable either, as the sample size of the correlation matrix is too large. As another solution, it is supposed to find the minimum perturbation to change the non-positive definite correlation matrix to a positive definite one. To this purpose, an efficient optimization algorithm called Iterative Spectral Method (ISM) [23] is implemented which is a simplified version of the Alternative Projection Method (APM) proposed by Higham [29]. Suppose that \(C = C_{d}\) is a non-positive definite correlation matrix and \(k = 1\). The sequential steps of ISM algorithm is as follows [23]:

1. Find \(L_{k}\) and \(S_{k}\) so that \(C_{k} = S_{k}L_{k}S_{k}^{T}\) by using spectral decomposition technique (spectral theorem). According to this theorem, any real symmetric \(n \times n\) matrix like \(C_{k}\) can be written as:

\[
C_{k} = S_{k}L_{k}S_{k}^{T}
\]

\(L_{k}\) is a diagonal matrix containing the \(n\) eigenvalues of the matrix \(C_{k}\) and the columns of \(S_{k}\) contain the eigenvectors of \(C_{k}\) in the same order as the corresponding eigenvalues in such a way that \(S_{k}\) is orthogonal.

2. If \(C_{k}\) is positive definite, the algorithm is terminated. Otherwise, go to the further step.

3. Let \(L_{k}^{+}\) be \(L_{k}\) with negative eigenvalues replaced by a small positive value \(a\).

4. Determine \(C_{k+1}\) by the expression \(C_{k+1} = S_{k}L_{k}^{+}S_{k}^{T}\).

5. Set the diagonal elements of \(C_{k+1}\) equal to unity.

6. Set \(k = k + 1\) and go back to step 1.

5. Results and discussion

5.1. Single corrosion defect

In the first step of this stochastic analysis, the results of conducted reliability evaluation for single corrosion defect is presented as:

5.1.1. Corrosion Perforation

For corrosion perforation, the Root Mean Square Error (PMSE) values between the failure probability based on MCS (\(N_{\text{Sim}} = 10^6\)) and LHS (\(N_{\text{Sim}} = [100 : 100 : 900] [1000 : 1000 : 10000]\)) methods are calculated by Eq. (10) considering different values of \(\text{Mean}(d)\) and \(\text{Mean}(t)\) as reported in Table 1.

\[
\text{RMSE} = \sqrt{\frac{1}{N} \left( \sum_{i=1}^{N} (\text{FP}_{\text{MCS}}(i) - \text{FP}_{\text{LHS}}(i))^2 \right)}
\]

In this equation, \(\text{FP}_{\text{MCS}}\) and \(\text{FP}_{\text{LHS}}\) are the failure probability obtained based on MCS and LHS methods. The series index \(i\) is associated with different values of \(\text{Mean}(d)\) (\(i = \text{Mean}(d) = 5.5 : 0.1 : 6.5\)) and \(N\) is equal to the size of \(\text{Mean}(d)\) (\(N = \frac{5.5 - 6.5}{0.1} + 1\)).

As expected, the RMSE values decrease as the number of LHS simulations increases. But, for number of simulations greater than 2000 the RMSE values reduce more gradually. Thus, \(N_{\text{Sim}} = 2000\) could be considered as an optimal benchmark for the number of LHS simulations in this study as its RMSE value is not noticeable and also it will decrease the computational expenses compared to MCS method.

Assuming \(N_{\text{Sim}} = 2000\), the failure probability based on both sampling methods has been computed for all the considered values of \(\text{Mean}(d)\) (horizontal axis) and \(\text{Mean}(t)\) (legend), and demonstrated in Fig. 2(b). As is clear, the results of both methods are very similar. The maximum discrepancy between the two methods might be related to the case with \(\text{Mean}(t) = 7.5\) and \(\text{Mean}(d) = 6\), which is not considerable, yet.

5.1.2. Local burst

In the next step, failure probability of local burst for different values of \(\text{Mean}(P_{b})\) have been demonstrated in Fig. 3. Herein, the Correlation Coefficient (CC) between the defect depth and defect length has changed from zero to unity with an increment of 0.1 using Cholesky decomposition technique as all these correlation matrices are positive definite.

Since the CC between the aforementioned variables do not considerably change the values of \(P_{b}\), the variation of \(P_{b}\) versus \(\text{Mean}(P_{b})\) based on both methods (\(N_{\text{Sim}} = 2000\) in LHS method) has been shown only for \(\mu_{dL} = CC = 0.5\). 1 to make the curves more distinguishable (see Fig. 3(a)).

As seen, all the curves based on either sampling method are very close to each other indicating that the correlation between the defect depth and length does not have noticeable impact on failure probability of single local burst defect. Analogous to Fig. 2, the RMSE between the defect based on MCS and LHS methods for different number of LHS simulations is illustrated in Fig. 3(b). It is clear that for \(N_{\text{Sim}} \geq 1000\), the RMSE values between the two sampling methods are not notable (\(\text{RMSE} \leq 0.005\)).

5.1.3. Rupture

In the last step of single corrosion defect reliability evaluation, failure probability of rupture mechanism has been plotted based on both approaches (\(N_{\text{Sim}} = 2000\) in LHS method) in Fig. 4(a). Moreover, the RMSE values between the two curves of Fig. 4(a), has been calculated for different number of LHS simulations (see Fig. 4(b)). It is again obvious that the results of both methods are quite similar.

5.2. Multiple corrosion defects

Due to the space limitation of the paper, local burst is the only failure mechanism which has been investigated for multiple corrosion defects considering two different classifications as:

5.2.1. The defect depth and defect length are uncorrelated in all the considered components

For uncorrelated variables in MCS method, it is not essential to generate the whole random realization matrix for all the components (a matrix with \(N_{\text{Cor}} \times N_{\text{Sim}}\) rows and \(N_{\text{Var}}\) columns), at once. In other words, it is possible to generate the random realization matrix for the first component (a matrix with \(N_{\text{Cor}} \times N_{\text{Sim}}\) rows and \(N_{\text{Var}}\) columns) which can be replaced by the second one after calculating its failure probability, and so forth. The reason is that for large number of simulations (\(10^6\)) the
randomly generated data will be uncorrelated unless a target correlation matrix is defined previously. But, in LHS method, since the correlation matrix of uncorrelated variables is equal to identity matrix (a positive definite matrix) Cholesky decomposition is employed to generate the random realizations. In this process, it is assumed that the number of defect components increases by the vector $N_{\text{Com}} = [10 : 100 : 90 : 100 : 1000]$. The failure probability of the system based on LHS method with different number of simulations versus number of defect components for 3 values of internal pressure ($p_a = 6, 8$ and $10$ Mpa) are compared by the ones based on MCS method in Fig. 5(a)–(c).

It is observed that the system failure probability increases with increase in number of defect components. In addition to this, it is readily deduced that the results of both methods are very close to each other. As is seen, the failure probability of the system for $p_a = 10$ has been depicted for the number of defect components less than 300 as the curves reach unity after this number. Also, the curves for greater internal pressure values are similar to Fig. 5(C), and this is the reason why they have not been presented in Fig. 5.

In order to quantitatively compare the efficiency of both sampling approaches the RMSE values between the system failure probability based on either method (in the whole range of $N_{\text{Sim}}$) are computed versus the number of LHS simulations ($N_{\text{Sim}}$) for different values of internal pressure (see Fig. 5(d)). As is evident, the RMSE values for the minimum and maximum internal pressure ($p_a = 6$ and $p_a = 14$), are very close to zero indicating that both approaches have quite similar results. Nevertheless, the RMSE values for intermediate internal pressure values are greater, and it seems that for $N_{\text{Sim}} \geq 2000$ the numerical values of RMSE are not much considerable.

5.2.2. The defect depth and defect length are correlated in different defect components

Herein, in order to consider the correlation of multiple components, the random simulations of all the components cannot be generated, separately. Due to this fact, it is not possible to employ MCS method and as a consequence, LHS method is implemented, instead. It should be stated that either for uncorrelated or correlated multiple components, the multivariate target correlation matrix should be defined by the user.
prior to generating the random simulations. In this investigation, the correlation between the defect depth or defect length of multiple components is mathematically modeled by two different scenarios as [30]:

1) In the first scenario, only neighboring segments (components) are correlated with each other, and the other ones are uncorrelated. Assuming $N_{var} = 3$ ($x$, $y$, and $z$), Fig. 6 shows a schematic correlation...
matrix for multiple components (a square matrix with $N_{\text{var}} \times N_{\text{com}}$ rows and $N_{\text{var}} \times N_{\text{com}}$ columns). In fact, each row or column of this symmetric matrix consists of $N_{\text{com}}$ correlation matrices which are associated with each single component (a symmetric matrix with $N_{\text{var}}$ rows and columns). In this scenario, there are three non-zero single component matrices in each row or column except the first and the last ones which have only two non-zero single component matrices. In this matrix, $\rho(x_i, y_j)$ denotes the correlation coefficient between the random variable $x$ in $i$th component and the random variable $y$ in $j$th component.

As it was observed that the effect of correlation between the defect depth and the defect length on failure probability of single component is negligible, such correlation is not considered in system reliability of correlated components for simplicity ($\rho(x_i, y_j) = \rho(x_i, z_j) = \rho(y_i, z_j) = 0$ and $i = j = 1, 2, 3, \ldots, N_{\text{com}}$) [13]. Further, to investigate the effects of correlated segments, the correlation between the defect depth of multiple components ($CC_d$) as well as the correlation between the defect length ($CC_l$) of the series components increase from zero to unity by an increment of 0.1 ($\rho(x_i, y_j) = 0.1 \cdot j - 0.1 \cdot i$ for $i \neq j$).

As described in section 4, there are a lot of non-positive definite correlation matrices in this assessment which should be corrected by ISM method prior to utilizing Cholesky decomposition approach. Assuming $N_{\text{com}} = 2000$ and $N_{\text{var}} = 100$, the $UB_{t_{1/2}}$ values of system failure probability versus $CC_d$ are shown in Fig. 7, considering different values of $CC_l$.

As is evident by the figure (first scenario), the $LB_{t_{1/2}}$ and $UB_{t_{1/2}}$ values of the system failure probability equal zero in case $p_a \leq 6$ Mpa and the $UB_{t_{1/2}}$ values are equal to unity if $p_a \geq 12$ Mpa. Relative variation in $UB_{t_{1/2}}$ values of the system failure probability caused by the change in either $CC_d$ or $CC_l$ is seen for the case $p_a = 8$. In such conditions, the $LB_{t_{1/2}}$ and $UB_{t_{1/2}}$ values increase when $CC_d$ lies in the interval $(0.5, 0.8)$ (except for $CC_l = 0$). Due to the artificial effect of statistical sampling variability, no uniform or monotonic trend is observed for either $UB_{t_{1/2}}$ or $LB_{t_{1/2}}$ values caused by the variation in $CC_d$ [15]. In addition to the mentioned points, the variation in $UB_{t_{1/2}}$ values are considerably greater than the variation in $LB_{t_{1/2}}$ values.

2) In the second scenario, the neighboring segments are correlated with each other with a correlation coefficient equal to $\rho$. The components located one segment away are still correlated with a correlation coefficient equal to 0.5$\rho$, and the other components are uncorrelated ($\rho = 0$). To comprehensively investigate the effect of correlation coefficient ($\rho$), $CC_d$ and $CC_l$ increase from zero to unity by an increment of 0.1. In this scenario, each row or column of the correlation matrix contains five non-zero single component matrices except the first two and the last two ones which have three (the first and last ones) and four (the second and the penultimate ones) single component matrices. Again, ISM method has been employed to correct the correlation matrix if it is not positive definite. Assuming $N_{\text{com}} = 2000$ and $N_{\text{var}} = 100$, the $UB_{t_{1/2}}$ and the $LB_{t_{1/2}}$ values of the system failure probability versus $CC_l$ are shown in Fig. 8, considering different values of $CC_d$ and $p_a$.

As seen, the results of Fig. 8 are almost similar to the previous one (Fig. 7). However, in this figure, the $LB_{t_{1/2}}$ values do not change by the variation in either $CC_d$ or $CC_l$, if $CC_l < 0.6$. Also, both $LB_{t_{1/2}}$ and $UB_{t_{1/2}}$ values slightly increase if $CC_l$ lies in the interval $(0.6, 0.8)$ and they decrease when $0.8 \leq CC_l \leq 1$. Supposing $CC_l = 0.8$, Fig. 8(b) illustrates

![Fig. 6. A schematic correlation matrix for a series system with 3 random variables and multiple components.](image-url)
that the $UB_{P_f}$ value for the case $CC_d = 0$ is maximum in contrast to Fig. 7(b), which was minimum. It is emphasized again that these differences might be related to the artificial effect of statistical sampling variability [15]. But, there is no certain illustrative reason which could justify such observations.

It is acknowledged that there are some other highlighted parameters such as the distance between the corrosion components and their positions (whether they are near any weld discontinuity and connection, buried, or in sea water experiencing hydrostatic pressure), which can considerably alter the system reliability of the corroded pipelines. Also, there are many strong references in the literature that have considered the aforementioned items [31–33]. However, these issues fall out of the scope of this article.

6. Summary and conclusions

This investigation compares the reliability analysis of pipelines against internal corrosion based on MCS and LHS methods, considering single and multiple corrosion components. For single corrosion defect, three failure mechanisms as: pitting perforation, local burst and rupture have been considered, although local burst is the only failure mechanism which has been studied for multiple corrosion defects when: 1) there is no correlation between the existing random variables 2) the defect depth and defect length of neighboring segments are correlated. In this process, a comprehensive attempt has been made to investigate the effect of number of LHS simulations on the accuracy of reliability analysis by comparing the results with MCS method. Additionally, the effect of correlation between the random variables and neighboring segments on failure probability of the pipeline has been inspected, separately (in both single and multiple corrosion defects). Since in many cases for multiple corrosion defects, the correlation matrix is not positive definite, the ISM method has been used to correct those matrices. The justifications of this study are drawn as follows:

1) Regarding single corrosion defect for the three considered failure mechanisms (pitting perforation, local burst and rupture), the results of MCS and LHS methods are very similar if the number of LHS simulations is greater than 1000 which can lead to RMSE values less than 0.01. Considering number of LHS simulations greater than 2000, the RMSE values between the LHS and MCS methods become less than 0.005, which is truly negligible in such reliability assessments. This remark has been investigated in different conditions considering various mean values of defect depth or pipeline wall thickness for pitting perforation, different mean values of internal pressure for both local burst and rupture and different correlation values between the defect depth and length for local burst mechanism.

2) The correlation between the defect depth and length of single corrosion defect has no considerable impact on failure probability of pipeline against local burst caused by internal pressure. This remark is not dependent on the mean values of internal pressure or any other random variable.

3) Regarding uncorrelated multiple defects, the RMSE values between the system failure probability based on MCS and LHS methods are very close to zero for low and high values of internal pressure if the number of LHS simulations is greater than 500. Meanwhile, the
RMSE values for intermediate pressure values are less than 0.005 if the number of LHS simulations is greater than 2000.

4) For correlated multiple defect components, it is not possible to perform MCS analysis with a number of simulation equal to $10^5$. Therefore, LHS method can be employed instead, to calculate the upper bound and lower bound of the system reliability. Both the considered scenarios reveal that the correlation between the defect depth or length of the neighboring segments can change the system failure probability despite the fact that there is no monotonic trend in the values of system failure probability by the variation in either $CC_d$ or $CC_L$. For instance, the upper bound of the system failure probability (for the case $p_a = 8$) increases when $CC_L$ lies in the interval $(0.6, 0.8)$ and decrease when $0.8 < CC_L < 1$.

5) The findings of this study indicates that LHS method might be preferable to MCS method when along acceptable accuracy, low computational expenses is of interest, as well.

Credit author statement

Mohsen Abyani: Conceptualization, Methodology, Software, Investigation, Writing - original draft, Writing - review & editing. Mohamad Reza Bahaari: Conceptualization, Methodology, Project administration, Supervision, Writing- Reviewing and Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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