Effect of radial injection on heat transfer of a Taylor–Couette–Poiseuille flow

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Mahdi Farsi, Sina Karbalaei M., Farshad Kowsary, and Pedram Hanafizadeh

School of Mechanical Engineering, College of Engineering, University of Tehran, Iran

Abstract

The effect of radial flow injection on the heat transfer characteristics of a Taylor–Couette–Poiseuille flow in an annulus is numerically investigated using the SST k-ω turbulence model. The ranges of the axial Reynolds number (Reₐ) and the rotational Reynolds number (Re₀) are 6.58 × 10⁴ – 1.37 × 10⁵ and 7.19 × 10⁴–1.8 × 10⁵, respectively. For every combination of axial and rotational Reynolds number, flow is injected radially into the air gap through a rectangular duct located on the inner cylinder with the injection rate \( m_{inj} = m_{axial} \) which varies from 0.02 to 0.14. Airflow in the air gap, before the injection, is in a fully developed condition and the effect of radial injection on the heat transfer of inner and outer cylinder is investigated in terms of the Nusselt number. In the studied range, radial injection increases the averaged Nusselt number up to 24% on the inner cylinder and up to 27% on the outer cylinder. In order to compare the heat transfer increase due to the radial injection, \( \frac{Nu}{Nu_{axial}} \) is defined as the difference between local Nusselt and Nusselt number of the fully developed flow happening at the same configuration without injection, and it is shown that though the trend of local Nusselt on the inner and outer cylinder is different, the averaged \( \frac{Nu}{Nu_{axial}} \) on both surfaces follows the same trend.

1. Introduction

This study concerns the effects of radial flow injection through the inner cylinder on the heat transfer of Taylor–Couette–Poiseuille flow, as shown in Figure 1. This flow occurs in the rotor-stator gap at the air-cooled turbogenerators where for cooling purposes, due to the rotation of the rotor, air from rotor sub-slot enters the radial slots in rotor conductors and then mixes with the air gap flow and affects the heat transfer characteristics of the air gap flow. Heat transfer in rotating machines is a major concern to designers, and in an effort to maintain the local temperature below allowable values, studying the heat transfer in different air passages in electrical machines is essential.

The cooling airflow in the rotor-stator gap is similar to the fluid flow between a concentric annulus with a rotating inner cylinder and stationary outer one with a superimposed axial flow. The combined axial and rotational flow in the absence of radial injection, known as Taylor-Couette-Poiseuille flow, has been studied extensively due to its many applications in the turbomachinery and electrical machines. Taylor [1] was the first to investigate the stability of the flow between two concentric rotating cylinders (Taylor-Couette flow). He determined once a critical rotation speed determined by Taylor number defined as Eq. (1) is exceeded, instabilities in shape of pairs of counter-rotating vortices arise in the cylindrical gap.
One of the earliest investigations of heat transfer of Taylor-Couette flows was carried out by Gazley [2]. He presented a correlation for the Nusselt number for the axial and rotational Reynolds numbers up to $10^4$ and $10^5$, respectively. Several other authors [3–7] have also developed correlations concerning the Nusselt number in different ranges of Taylor number and cylindrical gap aspect ratio in the form of $Nu = A(Ta)^a(Pr)^b$.

Goldstein [8] investigated the effect of superimposed axial flow on the stability of the Taylor-Couette flow and showed that the addition of axial flow would increase the stability. Kaye and Elgar [9] studied the effect of superimposed axial flow on a Taylor-Couette system experimentally. Based on the rotational speed, axial flow velocity, and radial gap size, their result showed four main flow regimes, namely laminar flow, laminar flow with Taylor vortices, turbulent flow, and turbulent flow with Taylor vortices. Becker and Kaye [3] performed temperature measurements for a broad range of rotational speed and superimposed axial flow rates and presented radial temperature profiles for different flow regimes in the diabatic flow. Kuzay and Scott [10] experimentally studied the turbulent heat transfer in an annulus between an insulated inner cylinder and heated stationary outer cylinder in the presence of axial flow. They obtained a correlation for the Nusselt number consisting of the Prandtl number, the axial Reynolds number, and a $\beta$ parameter combining both the rotational and axial flow speed. Based on experimental works for different ranges of Taylor number, axial Reynolds number, and cylindrical gap aspect ratio, correlations have been suggested for the Nusselt number [4,11–13]. Pfitzer and Beer [14] studied the effect of the rotation of inner cylinder of up to 2000 rpm and axial flow Reynolds of up to 3000 on the velocity profile and the heat transfer in a concentric annulus, both experimentally and theoretically. In the theoretical approach, it has been assumed that the flow is fully developed and the velocity profile and the Nusselt number is calculated at $z/D_h \rightarrow \infty$. They measured the Nusselt number in the experimental study at the axial position $z/D_h \approx 60$ so that the flow would be nearly fully developed. The theoretical approach mostly underpredicts the experimental results; however, they are in close agreement.

In recent years, thanks to the development of numerical methods and turbulence models, many authors have investigated this problem using computational fluid dynamics (CFD). Torri and Yang [15] compared the results of several different $k-e$ turbulence models for the entrance
Naser [16] also studied the flow through concentric annuli with inner cylinder rotation using the $k$-$\varepsilon$ turbulence model. He compared his results with the experimental data of Escudier and Gouldson [17]. His results showed discrepancy for the tangential velocities, and he attributed this discrepancy to the deficiency in the $k$-$\varepsilon$ turbulence model, which is not compatible with the simulated flow conditions. Char and Hsu [18] conducted a numerical computation for turbulent convection of airflow in a horizontal concentric annulus with a cooled outer cylinder and a heated rotating inner one. They adopted the $k$-$\varepsilon$ turbulence model with Kato-Launder modification, and comparing to existing experimental studies, they concluded this modification is an essential ingredient. Chung and Sung [19] performed a Large-Eddy Simulation (LES) of turbulent flow in a concentric annulus with inner wall rotation. They compared their results with the experiment one of Nouri and Whitelaw [20] and found a good agreement between mean velocity and second-order statistics with experimental data. Lee et al. [21] also performed a large-eddy simulation study on the heat and mass transfer of an annulus with rotating inner wall for several different Taylor and Reynolds numbers. They concluded that the Nusselt would increase with an increase in the Taylor number; however, at large Reynolds, the effect of the inner cylinder rotation would decrease significantly on the Nusselt number. Kuosa et al. [22] carried out modeling of gas flow and heat transfer in the air gap of an electrical machine. They compared the results of two-equation models ($k$-$\varepsilon$ and $k$-$\omega$ SST) with experimental data and concluded that in high rotation, turbulence models underestimate the heat transfer coefficient along both cylinders. Poncet et al. [23] considered turbulent flows in a differentially heated Taylor–Couette system with a superimposed axial Poiseuille flow using the Reynolds Stress Modeling (RSM). Their results were favorable compared to the velocity measurements of Escudier and Gouldson [17] and then extending their work to real operating conditions, they predicted the turbulent flow and the heat transfer in the gap of an electrical machine for a wide range of operating conditions. Poncet et al. [24] also studied turbulent Taylor-Couette-Poiseuille flows in a narrow-gap cavity using the LES model. They carried out the simulation for six different combinations of rotational and axial Reynolds numbers and they presented a correlation for the averaged Nusselt number along the rotor i.e. $Nu = ARe_q^{\alpha}Ta^{\beta}Pr^c$ with $A = 3.2 \times 10^{-5}$, $\alpha = 1.3$, $\beta = 0.145$, and $\gamma = 0.3$.

Several authors have studied the effect of air injection or ingestion on a Taylor-Couette-Poiseuille flow. Mayle [25] conducted an experimental analysis of changes in velocity ratio between rotating inner and stationary outer cylinder as a result of cooling flow injection. He assumed the flow in the gap must be considered undeveloped since the injection and ingestion continuously interrupt the flow in the air gap of a turbogenerator. Wilkinson and Dutcher [26] investigated the effect of radial fluid injection on the stability of the turbulent Taylor-Couette flow with the continuous axial flow and showed that only the highest injection rate modifies the

![Figure 1. Schematic of the general flow configuration showing a 20° sector of two coaxial cylinders with inner cylinder rotation and flow injected radially into the gap.](image-url)
turbulent Taylor vortex structure for a sustained period. Han et al. [27] considered the effect of multiple rotor and stator radial injection and ingestion on the air gap flow field and presented velocity contours and vectors demonstrating the complex flow field in air-gap. None of the above studies addressed the effect of radial injection on the heat transfer characteristic of the Taylor-Couette-Poiseuille flow. In terms of injection effect on heat transfer, Biswas et al. [28] numerically investigated the effect of injection through the cavity of a grooved channel. They analyzed the effect of the injection rate and position on the enhancement of heat transfer.

Numerical simulations were performed here to examine the effect of radial flow injection through rectangular ducts placed in the circumferential direction on the heat transfer of a Taylor-Couette-Poiseuille flow. As far as the authors know, the heat transfer characteristics of this configuration have not been studied. Twelve sets of axial and rotational Reynolds numbers are considered in this study, and for every combination, the injection influence is studied in term of injection rate \( F \) defined as:

\[
F = \frac{m_{\text{inj}}}{m_{\text{axial}}}
\]

that varies from 0.02 to 0.14.

In each simulation, the inner and outer cylinder is heated separately, and local Nusselt numbers are calculated along the axial direction on both surfaces, and the changes in Nusselt number values due to the injection is addressed.

2. Numerical method

The set-up consists of two concentric cylinders where the inner cylinder rotates with an angular velocity \( \Omega \) rad/s, and the outer cylinder is kept stationary. Cooling air is injected into the gap through rectangular ducts with a hydraulic diameter of 0.01. Eighteen ducts are arranged in a circumferential position along the inner cylinder with 20° spacing. Due to the repetition of radial inlet ducts in the circumferential direction, the calculations are confined to a 20° sector of full round and rotational periodic boundary conditions have been applied to the matching faces.

The analysis has been conducted by solving the Reynolds averaged continuity, momentum, energy, and turbulence model equations. Considering the importance of the flow behavior in the regions near the wall, as it dictates the heat transfer mechanism, and also according to the review of turbulence models given by Versteeg and Malalasekera [29] for modeling flow in a narrow annulus, low-Reynolds Shear Stress Transport \( k-\omega \) model is adopted.

The governing equations of continuity, momentum, and energy can be represented as:

Continuity:

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0
\]

Momentum:

\[
\rho \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + (\mu + \mu_t) \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j}
\]

Energy:

\[
\rho \frac{\partial}{\partial x_i} (\bar{T} \bar{u}_i) = \frac{\partial}{\partial x_i} \left[ \left( \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial}{\partial x_i} T \right]
\]

where \( Pr_t \) is turbulent Prandtl number and is set constant and equal to 0.85.

The transport equations of turbulent kinetic energy \( (k) \) and specific dissipation rate \( (\omega) \) for the model can be written as:
\[\frac{\partial}{\partial x_i}(k \overline{u}_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \frac{\tau_{ij}}{\nu_k} - \beta^r \rho \omega k \]  

(6)

\[\frac{\partial}{\partial x_i}(\omega \overline{u}_i) = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] + \frac{\tau_{ij}}{\nu_\omega} \frac{\partial \overline{u}_i}{\partial x_j} - \beta^r \rho \omega^2 - 2(1 - F_1) \rho \sigma_{\omega,2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \]  

(7)

where \(\sigma_k\) and \(\sigma_\omega\) are the turbulent Prandtl numbers for \(k\) and \(\omega\), and \(\mu_t\) is the turbulent viscosity.

\[\mu_t = \frac{\rho k}{\omega \max \left( \frac{1}{\sigma_k} \right)} \]  

(8)

\[\sigma_k = \frac{1}{\sigma_{k,1}} + \frac{1}{\sigma_{k,2}} \]  

(9)

\[\sigma_\omega = \frac{1}{\sigma_{\omega,1}} + \frac{1}{\sigma_{\omega,2}} \]  

(10)

\(S\) in Eq. (8) is equal to \(\sqrt{2S_{ij}S_{ij}}\) where \(S_{ij}\) is the mean rate of rotation tensor. The blending functions, \(F_1\) and \(F_2\) are computed using:

\[F_1 = \tanh \left[ \min \left( \max \left( \frac{\sqrt{k}}{0.09 \omega d} \cdot \frac{500 \theta}{\sigma_\omega CD_k d^2} \right), \frac{4 \rho k}{\sigma_{\omega,2} CD_k d^2} \right) \right]^4 \]  

(11)

where:

\[CD_k = \max \left( 2 \rho \frac{1}{\sigma_{\omega,2} \omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20} \right), \]  

(12)

\[F_2 = \tanh \left[ \max \left( \frac{2 \sqrt{k}}{0.09 \omega d} \cdot \frac{500 \theta}{\sigma_\omega d^2} \right) \right]^2 \]  

(13)

The full formulations along with equation constants of the SST \(k-\omega\) model has been given by Menter [30].

The present problem is solved using a commercial finite volume solver with SIMPLE algorithm for pressure-velocity coupling. The transport equations for momentum, \(k\), \(\omega\), and energy were discretized using the second-order upwind scheme. To take account of rotating components, the Multiple Reference Frame (MRF) method has been used to obtain time-averaged steady-state solutions. The MRF method does not rotate any part of the mesh, but instead provide the rotating regions in the domain with appropriate source terms. Fluid flow is assumed to be incompressible.
with constant fluid properties. The iterative solution is considered to be converged when the maximum of the normalized absolute residual across all nodes is less than $10^{-5}$.

### 2.1. Boundary conditions

As shown in Figure 2, a fully developed turbulent flow, both hydrodynamically and thermally, is set as axial flow inlet of the air gap. The inlet flow has a mass flow average temperature of 300 Kelvin. For modeling rotation of the inner cylinder and injection duct, these parts are considered as a single zone where rotating reference frame is applied to the region and rotates at the desired speed. Air is injected into the air gap through a rectangular duct placed from $Z^* = -0.4$ to $Z^* = 0.4$. The cooling air exits the annulus with a zero gauge pressure boundary condition prescribed on the exit flow passage. Periodic boundary conditions are applied in the circumferential directions. No-slip boundary conditions with constant heat flux are imposed on the inner and outer cylinder’s wall. It is noteworthy that the length of the computational domain in the axial direction is equal to $11\,\text{L}$.

### 3. Results and discussion

Test calculations of a concentric annulus with no injection were first carried out to confirm the validity of the numerical procedure. Then three-dimensional simulations were performed for the configuration explained in the previous part.

To take account of different axial velocity and rotational speed of flow in the annulus, the axial Reynolds and the rotational Reynolds numbers are defined respectively as Eqs. (14) and (15).

$$Re_A = \frac{V_a D_h}{\nu} \quad (14)$$
The axial Reynolds numbers and the rotational Reynolds number vary from $6.58 \times 10^4$ to $1.37 \times 10^5$ and from $7.19 \times 10^4$ to $1.80 \times 10^5$, respectively. The numerical simulation is carried out for twelve sets of axial and rotational Reynolds number combination, and for each configuration, the injection rate ($F$) varies from 0.02 to 0.14 with an increment of 0.02. The local Nusselt number is calculated as Eq. (16) and its variation on the inner and outer cylinder due to the flow injection is investigated.

$$\text{Nu} = \frac{q'' D_h}{\lambda (T_w, z - T_b, z)}$$  \hspace{1cm} (16)

### 3.1. Grid study

A mesh independence study was performed by increasing the number of mesh elements from 3.6 M to 14.6 M to ensure appropriate grid resolution. The local inner cylinder Nusselt number along the axial direction is compared in different grid sizes in Figure 3. With the increase in grid size, the difference between Nusselt values steadily declines until reaching a relative difference of 0.2% between the cases with element count of 8 M and 14.6 M. So, the mesh with 8 million hexahedral elements is chosen to conduct further simulations. Due to SST $k$-$\omega$ model requirements, the mesh refinement near the no-slip walls was set so that the near-wall meshes dimensionless distance $y^+$ value be close to 1.

### 3.2. Validation

Since turbulence models heavily depend on empirical constants for mathematical closure, it is essential to first establish the reliability and accuracy of the present model. To this end, the adopted SST $k$-$\omega$ model was validated by simulating a Taylor-Couette-Poiseuille flow in a concentric annulus of Pfitzer and Beer [14]. They performed an experimental and analytical study of the flow and heat transfer of a turbulent flow in a rotating annulus with a radius ratio of $\eta = 0.8575$. 

Figure 4. Nusselt number of a fully developed flow in a rotating annulus.
In the experimental study, the Nusselt is measured at $z/D_h = C_{25}/60$ while the theoretically determined values are valid for fully developed flow ($z/D_h \to \infty$). Both experimental and theoretical results shown to be in close agreement. The same configuration as Pfitzer and Beer [14] is modeled in the present study using the SST $k-\omega$ model, and by applying translational periodic boundary condition for annulus inlet and outlet, the flow is assured to be fully developed. Figure 4 shows the simulated results of the Nusselt number on the stator of an annulus with inner cylinder rotating compared with the theoretical result of Pfitzer and Beer [14] in the same configuration. The theoretical results of Pfitzer and Beer [14] are in close agreement with numerical results of the present study with an average deviation of 6.1% and the maximum deviation of 9%.

### 3.3. Flow field

Figure 5 shows the streamlines of the flow in the domain confined to $Z^* = -1.4 - 2.6$ for two injection rates of 0.02 and 0.14 in the case with axial and rotational Reynolds of $1.03 \times 10^5$ and

![Figure 6. Effect of flow injection on inner cylinder local Nu* for $Re_A = 6.58 \times 10^4$ and $Re_\omega = 1.08 \times 10^5$.](image)
The injection of fluid causes the formation of swirling structures emanating from the initial mixing of the two streams which flows downstream and affects flow field and heat transfer characteristics. The injection flow cools down the annulus walls downstream of the injection duct and increases the heat transfer coefficient. The inner cylinder wall cools down immediately downstream of the injection duct, though it can be seen that the swirling structures penetration into the annulus flow increases as it is convected downstream and increases the heat transfer coefficient on the outer cylinder as well. As the injection rate increases, these swirling structures become thicker and has more effect on the heat transfer coefficient increase.

3.4. Heat transfer

3.4.1. Heat transfer on the inner cylinder

Inner cylinder local Nusselt number varies in terms of $F$, $Re_a$, $Re_\omega$, and $z/D_h$. Figure 6 presents the inner cylinder local Nusselt number in the axial direction for four values of injection rate $1.08 \times 10^5$, respectively. The injection of fluid causes the formation of swirling structures emanating from the initial mixing of the two streams which flows downstream and affects flow field and heat transfer characteristics. The injection flow cools down the annulus walls downstream of the injection duct and increases the heat transfer coefficient. The inner cylinder wall cools down immediately downstream of the injection duct, though it can be seen that the swirling structures penetration into the annulus flow increases as it is convected downstream and increases the heat transfer coefficient on the outer cylinder as well. As the injection rate increases, these swirling structures become thicker and has more effect on the heat transfer coefficient increase.

![Figure 7](image-url)

**Figure 7.** Effect of rotation on the inner cylinder local $Nu^*$ for the injection rate of 0.06 and for (a) $Re_a = 6.58 \times 10^4$, (b) $Re_a = 1.03 \times 10^5$, and (c) $Re_a = 1.37 \times 10^5$.\n
NUMERICAL HEAT TRANSFER, PART B: FUNDAMENTALS
where \( \text{Re}_{in} = 1.08 \times 10^5 \) and \( \text{Re}_A = 6.57 \times 10^4 \). In order to show the plots vividly, the results of other injection rates are not displayed. The local Nusselt number on the inner cylinder is constant before the injection which is due to the fact that axial inlet flow is thermally and hydrodynamically fully developed. After the injection, a sudden rise occurs in the local Nusselt number following with a steep decline which finally stabilizes at a new value, more than the value before the injection as the mass flow has increased and so does the heat transfer. Nusselt number Peak happens at \( Z^* = 0.4-0.8 \) depending on the injection rate, and axial and rotational Reynolds number and at its maximum, its peak is 1.56 times more than the fully developed flow Nusselt number (where \( F = 0.14, \text{Re}_x = 1.8 \times 10^5, \) and \( \text{Re}_A = 6.57 \times 10^4 \)). The increase in Nusselt number is proportional to the injection flow rate, and as the injection rate increase, both the Nu peak and the Nu final value increase, resulting in a higher averaged Nusselt number. For the case shown in Figure 6, the Nu peak and the Nu final value for the injection rate of 0.14 are respectively 1.41 and 1.12 times more than the fully developed Nusslet number; while for the injection rate of 0.02 those values are 1.13 and 1.001 respectively.
In this study, in order to compare the increase in Nusselt number as a result of flow injection, the local Nusselt number is subtracted by the Nusselt number of a fully developed flow in the same configuration with identical rotational and axial Reynolds number but with no injection. With this regard, \( \frac{Nu}{C3} \) is defined as:

\[
\frac{Nu}{C3} = \frac{Nu}{C0} - Nu_{FD}
\]  

The effect of inner zone rotation on \( \frac{Nu}{C3} \) is shown in Figure 7. The plots are depicted for the injection rate of 0.06 and the trend is the same for all injection rates. Before the injection, the local Nusselt number is equal to the fully developed Nusselt number resulting in \( \frac{Nu}{C3} = 0 \). As was explained previously, that injection of flow causes a sudden rise in Nusselt number following by a decline reaching a new value higher than the value before the injection. As the rotational Reynolds increase, the \( \frac{Nu}{C3} \) peak increases as well. However, at higher rotational speed, the \( \frac{Nu}{C3} \) declines at a steeper slope and stabilize at a lower value. In \( Re_x = 6.58 \times 10^4 \) the \( \frac{Nu}{C3} \) peak caused by the injection rate of \( F = 0.06 \) in \( Re_\Omega = 1.8 \times 10^5 \) is 2.22 times more than the peak.

Figure 9. Inner cylinder averaged \( \frac{Nu}{C3} \) versus injection rate for different rotational Reynolds for (a) \( Re_\omega = 7.19 \times 10^4 \), (b) \( Re_\omega = 1.08 \times 10^5 \), (c) \( Re_\omega = 1.44 \times 10^5 \), and (d) \( Re_\omega = 1.8 \times 10^5 \).
happens in $Re_\Omega = 7.19 \times 10^4$ . This value is 1.95 and 1.89 for $Re_A = 1.03 \times 10^4$ and $Re_A = 1.37 \times 10^4$, respectively.

Figure 8 shows the effect of axial inlet velocity on the $Nu^*$ for the injection rate of 0.06. The trend of the plots for other injection rates is the same. The axial flow inlet velocity does not change the maximum rise tangibly but changes the slope at which the local $Nu$ decreases. In the same injection rate, local Nusselt number in cases with higher axial velocity, stabilize at a higher value, which is due to the higher mass flow injected into the gap. In $Re_\Omega = 7.19 \times 10^4$, the $Nu^*$ after the injection is 2.55 times more for the case with $Re_A = 1.37 \times 10^5$ than the case with $Re_A = 6.58 \times 10^4$. This value is 3.14 for $Re_\Omega = 1.08 \times 10^5$, 4.37 for $Re_\Omega = 1.44 \times 10^5$, and 5.94 for $Re_\Omega = 1.80 \times 10^5$.

To investigate the effect of injection on average Nusselt number, the local $Nu$ values have been averaged from $Z^* = -1.6$ to $Z^* = 5.4$. Depending on rotational and axial Reynolds number values, air injection increased the averaged heat transfer coefficient from 1% to 25%. Figure 9 plots the averaged $Nu^*$ as a function of the injection rate. Increase in the injection rate normally increases the averaged $Nu^*$. The results indicate that averaged $Nu^*$ increases with $Re_\Omega$ and $Re_A$, but as the rotational speed increases, the axial Reynolds number imposes less influence on heat transfer. In $Re_\Omega = 1.8 \times 10^5$ and injection rate of $F = 0.14$ in the case with $Re_A = 1.37 \times 10^5$, the averaged $Nu^*$ is 1.04 times greater than the case with $Re_A = 6.58 \times 10^4$. This value is 1.17, 1.73, and 2.19 for rotational Reynolds of $1.44 \times 10^5$, $1.08 \times 10^5$, and $7.19 \times 10^4$, respectively. This shows that as rotational speed decreases, axial flow velocity effect on averaged $Nu^*$ becomes more significant.

### 3.4.2. Heat transfer on the outer cylinder

Rotor radial injection has a substantial effect on the outer cylinder heat transfer as well as the inner cylinder. Once again, the Nusselt number varies with $F$, $Re_A$, $Re_\Omega$, and $z/D_h$. In Figure 10, the outer cylinder Nusselt value is depicted along the axial direction for four injection rate where the axial and rotational Reynolds are $1.03 \times 10^5$ and $1.08 \times 10^5$, respectively. Before the injection,
the Nusselt number is constant, which is due to the fact that at the inlet, a fully developed flow enters the computational domain. The flow injection causes an increase in the outer cylinder Nusselt number, which unlike the inner cylinder, is gradual and does not decline at a steep slope. Nusselt number peak occurs at a relatively long distance from the injection duct, and the lower the injection rate, the further the peak is. At its maximum in the case with $F = 1.4$, $Re_x = 7.19 \times 10^4$, $Re_\Omega = 1.08 \times 10^5$, $Re_A = 1.44 \times 10^5$, and $Re_\omega = 1.8 \times 10^5$, the outer cylinder Nusselt number peak is 1.71 times greater than the fully developed flow outer cylinder Nusselt number. The increase in stator Nusselt number is proportional to the injection flow rate and as the injection rate increase, both the Nu peak and the Nu final value increase, resulting in a higher averaged heat transfer coefficient. For the case shown in Figure 6, the Nu peak for injection rates of 0.14, 0.1, 0.06, and 0.02 are respectively 1.39, 1.28, 1.11, and 1.02 times more than the Nu value before the injection.

Similar to the previous part, the Nu values on the outer cylinder have been averaged from $Z^* = -1.6$ to $Z^* = 5.4$. The results indicate that flow injection increases the averaged heat transfer from 1% to 27% depending on the injection rate, the axial Reynolds number, and the rotational

Figure 11. Outer cylinder averaged Nu* versus injection rate for different rotational Reynolds for (a) $Re_\omega = 7.19 \times 10^4$, (b) $Re_\omega = 1.08 \times 10^5$, (c) $Re_\omega = 1.44 \times 10^5$, and (d) $Re_\omega = 1.8 \times 10^5$. 

NUMERICAL HEAT TRANSFER, PART B: FUNDAMENTALS
Reynolds number. The averaged $Nu^*$ versus the injection rate for different axial and rotational Reynolds number is shown in Figure 11. Though the trend of the local $Nu^*$ on the outer and inner cylinder is different, the averaged $Nu^*$ trend on both cylinders is almost the same, and averaged $Nu^*$ increases with $Re_X$ and $Re_A$. In $F=0.14$ and $Re_Q = 1.8 \times 10^5$, the averaged $Nu^*$ in the case with $Re_A = 1.37 \times 10^5$ is 2.55 times more than the case with $Re_A = 6.58 \times 10^4$. This value for rotational Reynolds of $1.44 \times 10^5$, $1.08 \times 10^5$, and $7.19 \times 10^4$ is 1.99, 1.31, and 1.13 respectively, meaning that at a high rotational speed, the axial flow has little influence on averaged $Nu^*$.

4. Conclusion

This work numerically investigates the effects of rotor radial injection on the convective heat transfer of a Taylor–Couette–Poiseuille flow in an annulus with axial Reynolds numbers of $6.58 \times 10^4$ to $1.37 \times 10^5$ and rotational Reynolds number of $7.19 \times 10^4$ to $1.8 \times 10^5$. Local Nusselt number on both inner and outer cylinders is calculated in the axial direction, and its variation due to the flow injection is studied. $Nu^*$ has been defined to compare injection effect.

1. Radially injected flow causes emergence of swirling structures that flows downstream and results in a sudden rise in the local Nusselt number of inner cylinder, following with a steep decline. The rise in the outer cylinder local Nusselt number is gradual and decreases gently after reaching the peak.

2. The inner cylinder $Nu^*$ peak increases with the $Re_Q$, while it does not change tangibly with the $Re_A$. In addition, as $Re_Q$ increases, the inner cylinder $Nu^*$ declines at a steeper slope and stabilize at a lower value. And as the $Re_A$ increases, the inner cylinder $Nu^*$ stabilizes at a higher value.

3. Radial injection of air in the studied range increases the averaged heat transfer 1 to 24 percent on the inner cylinder and 1 to 27 percent on the outer cylinder. Averaged $Nu^*$ follows the same trend on both inner and outer cylinder; in the way that averaged $Nu^*$ increases rapidly with $Re_A$ at low $Re_Q$, but as the $Re_Q$ increases, the axial Reynolds number imposes less influence on the averaged $Nu^*$.

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