Research Note: A unifying framework for the widely used stabilization of potential field inverse problems

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ABSTRACT

We present a brief review of the widely used and well-known stabilizers in the inversion of potential field data. These include stabilizers that use $L_2$, $L_1$ and $L_0$ norms of the model parameters and the gradients of the model parameters. These stabilizers may all be realized in a common setting using two general forms with different weighting functions. Moreover, we show that this unifying framework encompasses the use of additional stabilizations which are not common for potential field inversion.

Key words: Potential fields, Inverse theory, $L_p$-norm stabilizers, sparsity regularization.

INTRODUCTION

Many forms of stabilizers are used for the inversion of the ill-posed potential field data problem. The aim is the reconstruction of subsurface models that provide relevant physical interpretation. In order to reduce the possibility of over-interpretation of the data, it may be desirable to reconstruct a simple model with as little structure as possible, and to eliminate arbitrary discontinuities in the solution (Constable, Parker and Constable 1987). Such models can be expected to present only the important and large-scale features of the subsurface under the survey area. They can be obtained using a minimum roughness, or equivalently maximum smoothness, stabilizer that employs an $L_2$-norm of the gradient of the model parameters in the inversion algorithm (Constable et al. 1987; Li and Oldenburg 1996; Pilkington 1997; Li and Oldenburg 1998). A subsurface model exhibiting discontinuities may also be appropriate and physically relevant (Farquharson and Oldenburg 1998). These models are achieved by applying an $L_1$-norm total variation (TV) regularization, or an $L_0$-norm minimum gradient support (MGS) stabilizer for the gradient of the model parameters (Vatankhah, Renaut and Ardestani 2018a; Portniaguine and Zhdanov 1999; Bertete-Aguirre, Cherkaev and Oristaglio 2002). The $L_0$-norm, which counts the number of nonzero entries in the vector, does not meet the mathematical requirement to be regarded as a norm but does yield sparsity in the vector. Applying the $L_p$-norm, for $p = 0$ or 1, to the gradient of the model parameters minimizes the number of discontinuous transitions of the reconstructed model. Alternatively, when applied directly to the model parameters, these $L_p$-norms, with $p = 0$ or 1, provide sparsity in the solution, and are relevant when it can be assumed that the sources of interest are localized and compact (Last and Kubik 1983; Guillen and Menichetti 1984; Barbosa and Silva 1994; Portniaguine and Zhdanov 1999; Zhdanov and Tolstaya 2004; Ajo-Franklin, Minsley and Daley 2007; Vatankhah, Ardestani and Renault 2014a; Vatankhah, Renault and Ardestani 2014b; Vatankhah, Ardestani and Renault 2015; Vatankhah, Renault and Ardestani 2017). Inversion algorithms that employ such sparsity constraints yield sparse and focused images of the subsurface structures. Within the potential field literature, the use of the $L_0$-norm applied for the model parameters is generally referred to as a compactness, or minimum support, constraint (Last and Kubik 1983; Portniaguine and Zhdanov). Previous investigations have demonstrated that the $L_0$-norm stabilizer can yield a sparser solution than that

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