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Effect of Different Frequency Functions on Ferrofluid FHD Flow

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Abstract

A finite volume method is used to investigate the unsteady thermomagnetic convection of ferrofluid flow in a miniature channel under the effect of constant and oscillating magnetic fields. A magnetic line dipole as a magnetic source is placed under the lower wall of the two-dimensional channel whose lower and upper walls are subjected to a constant heat flux. To generate the oscillating magnetic field, four different frequency functions including rectangle, sine, triangle and sawtooth functions are implemented in the constant magnetic field. The study parameters include four frequency functions, magnetic field intensity as a magnetic number (\(Mn= 3.833\times10^8, 8.624\times10^8, 1.533\times10^9\)), frequencies (\(f=0.5\) to 5 Hz) and Reynolds numbers (Re=20 to 40). Results revealed that applying the constant magnetic field enhances heat transfer and as the Reynolds number increases, the effectiveness of magnetic field on heat transfer rate

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diminishes. The oscillating magnetic field, regardless of the type of the applied frequency function, improves heat transfer rate due to thermal boundary layer suppression and increase in velocity field. For all applied functions, results show that there is an optimum frequency where maximum heat transfer happens and this frequency increases for higher Reynolds numbers. For rectangle and sawtooth functions with instant change in the magnetic field, two simultaneous vortexes emerges which leads to better heat transfer than sine and triangle functions. The alteration of the pressure drop in one period is proportionate with the shape of the applied frequency even though the range of the alterations reduces with the increase of the Reynolds number.

**Keywords:** Ferrofluid; Heat Transfer; Magneto Convection; Frequency Functions; Oscillating Magnetic Field, Constant Magnetic Field

**Nomenclature**

- \( b \) Distance between pair of electric wires [m]
- \( B \) Vector magnetic density field [T]
- \( B \) Scalar magnetic density field [T]
- \( \text{BFD} \) Biomagnetic fluid dynamic [-]
- \( C_p \) Specific heat [J/kg·K]
- \( \text{CMF} \) Constant Magnetic Field
- \( \text{DI water} \) Deionized water [-]
- \( \text{Ec} \) Eckert number [-]
- \( e_r \) Radial unit vector [-]
- \( e_\phi \) Angular unit vector [-]
- \( f \) Frequency [Hz]
- \( F \) Body force [N/m³]
- \( \text{FHD} \) Ferrohydrodynamic [-]
- \( h \) Height of channel [m]
- \( H \) Vector magnetic field [A/m]
- \( \text{H} \) Scalar magnetic field [A/m]
\( H_r \)  
Magnetic strength reference [A/m]

\( I \)  
Electrical current [A]

\( k \)  
Thermal conductivity [W/m·K]

\( l \)  
Channel length [m]

\( m \)  
Magnetization moment [A·m]

\( M \)  
Magnetization vector [A/m]

Max. \( \text{Nu}_{\text{avg}}^{\text{osc}} \)  
Maximum value of average of Nusselt number for oscillating magnetic field [-]

Min. \( \text{Nu}_{\text{avg}}^{\text{osc}} \)  
Minimum value of average of Nusselt number for oscillating magnetic field [-]

MDT  
Magnetic drug targeting [-]

MHD  
Magnetohydrodynamic [-]

\( \text{Mn} \)  
Magnetic number [-]

\( \text{Nu} \)  
Nusselt number [-]

\( \text{Nu}_{\text{avg}}^{\text{cte}} \)  
Average Nusselt number for constant magnetic field

OMF  
Oscillating Magnetic Field

\( p \)  
Pressure [Pa]

\( \text{Pn} \)  
Pressure number [-]

\( \text{Pr} \)  
Prandtl number [-]

\( q'' \)  
Heat flux [W/m²]

\( r \)  
Radial coordinate [m]

\( \text{Re} \)  
Reynolds number [-], \( \frac{mD}{\mu C_p u} \)

\( t \)  
Time [s]

\( T \)  
Temperature [K]

TBL  
Thermal boundary layer

\( u \)  
Longitudinal velocity [m/s]

\( U_{in} \)  
Inlet velocity [m/s]

\( v \)  
Transverse velocity [m/s]

\( \nu \)  
Velocity vector [m/s]

\( x \)  
Longitudinal component [m]

\( y \)  
Latitudinal component [m]

**Greek**

\( \nabla \)  
Gradient operator [-]

\( \nabla^2 \)  
Laplacian operator [-]

\( \mu \)  
Dynamic viscosity [kg/m·s]

\( \mu_0 \)  
Magnetic permeability in vacuum \( (4\pi \times 10^{-7}[\text{N/m}^2]) \)

\( \alpha \)  
Thermal diffusivity [m²/s]
\( \beta \)  
Thermal expansion coefficient of ferrofluid \( (\beta = 5.6 \times 10^{-4}[1/K]) \)

\( \theta \)  
Normalized temperature [-]

\( \rho \)  
Density [kg/m\(^3\)]

\( \tau \)  
Period [-]

\( \varphi \)  
Angular coordinate [rad]

\( \Phi_{FHD} \)  
Magnetocaloric term [W/m\(^3\)]

\( \Phi_{vis} \)  
Viscos dissipation [W/m\(^3\)]

\( \chi_0 \)  
Magnetic susceptibility at reference temperature [-]

\( \chi_m \)  
Total magnetic susceptibility [-]

\( \phi \)  
Volume concentration [-]

Subscripts

avg  
Average

ff  
Ferrofluid

FHD  
Ferrohydrodynamic

in  
Inlet

max  
Maximum

p  
Nanoparticles

vis  
Viscosity

w  
DI Water

1 INTRODUCTION

Peer review of different cooling methods shows they are used abundantly in the wide range of systems, i.e. from small dimensions such as Micro and Nano Electromechanical Systems (MEMS and NEMS), microfluidics and nanofluidics to large ones such as power plants, petrochemical industries and oil refineries. The requirement of increasing the efficiency function always matters as a crucial subject in these systems because of energy saving and decreasing the cost of maintenance. The miniaturization of the implemented cooling systems and minimizing the number of their equipment is a prevalent solution in the optimization design. But the minimizing dimensions results in the heat transfer area reduction and causing the problem of high heat flux and local overheating areas due to the creation of hot spots. To assist the cooling
of the heat exchanging systems, active and passive methods are used. Active methods which demand an external power supply, such as a mechanical stirrer, vibration and applying magnetic or electrostatic fields. Passive methods, unlike the active methods do not require any external power supply, include adding particles and extending area heat transfer (fins and porous media) [1,2].

Increasing thermal conductivity of working fluids is one solution to compensate for the reduction of the heat transfer area. Using nanofluid as one solution to increase heat transfer by adding nano-sized particles with high thermal conductivity either metallic particles (Fe, Cu, and Ag) or non-metallic (Al₂O₃-TiO₂) into the working fluids (Water- Ethylene Glycol-Ethanol-Oil machine) has attracted many attentions in recent years [3-8]. Among these nanoparticles, the magnetic particles specifically magnetite (Fe₃O₄) not only have properties similar to the other non-metallic nanoparticles but also they respond and react toward an applied magnetic field. Ferrofluid is a stable mixture of nonmagnetic base fluid (DI-water, oil) and dispersed magnetized particle (magnetite, ferric oxide and iron-nickel oxide). The control of ferrofluid movement with applying magnetic field has led to increase the attraction of cooling applications of ferrofluid [9-11].

Because of numerous experimental and numerical studies have been done about ferrofluid heat transfer and its thermophysical properties variations under the effect of the magnetic field, the introduction tries to focus only on the numerical studies. Relate to the numerical studies of ferrofluid heat transfer, a majority of these studies have carried out about the free convection of ferrofluid in an enclosure under the effect of the magnetic field. Different geometries including rectangle enclosures with different aspect ratio [12-18], cubic [19-21], cylindrical [22, 23], annular [24-27] and triangular [28-30] cavities with different thermal boundary conditions and
mediums were considered. On the other hand, fewer of these numerical studies were devoted to
the thermomagnetic force convection of ferrofluid and could be categorized into rotating disk,
rectangular or circular ducts with a single or two-phase approach [31-40].

Krakov and Nikiforov [13] studied the effect of a uniform magnetic field on natural convection. Different orientations between the magnetic field and temperature gradient were studied and resulted that the different angles influenced the convection structure specifically for the case of zero gravity. Ganguly et al. [14] numerically studied heat transfer of ferrofluid under the influence of magnetic line dipole. The ferrofluid with lower temperature has a higher magnetic susceptibility and move toward the region of an applied magnetic field because of a high density of magnetic strength and displaces the warmer ferrofluid with lower magnetic susceptibility. By considering different variable such as length scale, temperature difference, magnetic field property and strength they obtained a dimensionless magnetic Rayleigh number which characterizes ferrofluid heat transfer. They also referred that applying magnetic field is an appropriate way to increase heat transfer in microsystems which natural convection induced by buoyancy motion is ineffective. Kefayati [17] studied the MHD free convection and entropy generation in a cavity for Cu-water nanofluid with non-Newtonian behavior. Different parameters including Rayleigh number, power-law index, Hartmann number and volume fraction of nanofluid were considered. Augmentation of power-law index caused to decrease heat transfer in the absence of magnetic field, while by applying the magnetic field, heat transfer enhanced even power-law index increased which showed the great effect of the presence of magnetic field. Applying magnetic field and increasing its intensity changed the power-law index and volume fraction entropy generation and also caused to weaken the total entropy generation. A kerosene-based ferrofluid in two cylinders with different dimensions was investigated as a two-phase
mixture of magnetic particles in a carrier phase by Jafari et al. [23]. Different temperature gradient and uniform magnetic field were applied over the geometry. Results showed applying magnetic field increased transport processes and also perpendicular orientation between a magnetic field and temperature gradient had better influence on heat transfer than the parallel orientation.

Ganguly et al. [31] studied force convection of ferrofluid under influence of magnetic line dipole. An induced generated vortex due to applying a magnetic field and dependent of ferrofluid magnetic susceptibility to temperature gradient changed the thermal boundary layer (TBL) and increased momentum and energy advection which caused to heat transfer enhancement. Adding extra dipoles enhanced heat transfer and also changing the relative position of the dipoles could alter the fluid flow and heat transfer of ferrofluid. Overall heat transfer enhancement predominantly depended on the net dipole strength and to some extent on the geometrical features of the magnetic field including size and shape. Strek and Jopek [32] simulated the ferrofluid transient heat transfer under the effect of magnetic line dipole. They showed that colder ferrofluid moved toward magnetic field gradient and displaced the warmer ferrofluid. The Kelvin body force could promote or inhibit the convection process due to the dependency of magnetic susceptibility to the temperature gradient. Xuan et al. [33] simulated the ferrofluid and thermal processes by developing a mesoscopic model by using the Lattice-Boltzmann method. The orientation and magnitude of magnetic field alterations resulted to control of flow and thermal processes. If the magnetic field gradient is parallel with the ferrofluid flow, best result relates to heat transfer enhancement acquired. If the magnetic field gradient opposes with the mainstream, heat transfer suppresses between ferrofluid and the walls of the channel. Aminfar et al. [36] have studied a subcooled ferrofluid boiling in an annular pipe under
the effect of a non-uniform longitudinal magnetic field with negative and positive gradients. Results showed critical heat flux increased under the effect of a negative magnetic gradient due to the decrease in evaporation rate in the wall surface.

Also, some numerical works on blood motion in the presence of an external magnetic field have been studied [41-48] which is known as biomagnetic fluid dynamic (BFD). Blood as the most important biofluid can be treated as a non-electrical fluid which results the governing equations similar to FHD flow. In BFD studies, magnetization vector is assumed to vary with temperature and magnetic field, linearly. The results showed temperature, heat flux and skin friction of blood enhance with the magnetic field.

In the literature review, to represent the applied magnetic field is changing, either spatially or temporally, the terms like “variable, oscillating, non-uniform and alternating” have been used vastly [49-58]. This non-uniformity includes as a spatial change in longitudinal and latitudinal directions with positive and negative signs, magnetic field variation as a function of distance from the source of a magnet, and more recently, a variation of magnetic field with time. Goharkhah and Ashajee [54] studied the ferrofluid laminar flow in the presence of an alternating magnetic field. Reynolds numbers, magnetic field intensities and frequencies were the variables of the study. Results showed that there is an optimum frequency for specific Reynolds number which increases with Reynolds number enhancement. Both heat transfer and pressure drop increase applying the magnetic field and maximum 13.9% heat transfer enhancement was observed for Re=2000 and f=20 Hz.

By considering the miniaturized configurations including microscale heat exchangers in MEMS devices and hypo or zero gravity conditions in Space, using a method to cool these
miniaturized systems demand novel methods besides to the previously presented applicable solutions (using ferrofluid with an external magnetic field). More recently, implementation of the OMF as a novel way is introduced to improve thermal behavior in the heat exchanging systems [59-63]. However, in the previous studies the influence of the CMF and OMF on heat transfer is investigated and numerical studies of time varying magnetic field are very scarce. To the best knowledge of the authors, no extensive works, either numerically or experimentally, has been done related to the effect of different frequency functions on thermomagnetic convection of ferrofluid. Therefore, this comprehensive paper tries to present a better and deeper understanding of this area.

Unsteady ferrofluid FHD flow in a 2D miniature channel subject to constant and oscillating magnetic fields have been investigated numerically. Symmetric constant heat flux imposed to the lower and upper walls while the cold ferrofluid with fully developed velocity profile flows in the channel. The main purpose of this paper is the study of variation of thermal and hydrodynamic behavior of ferrofluid along the channel due to the different frequency functions applied to the line dipole that generates the magnetic field. Four different frequency functions include rectangle, sine, triangle and sawtooth functions were applied to the line dipole to generate the OMF.

In the following, the outline of the paper is presented. Information about the geometry of the channel, implemented thermal and hydrodynamic boundary conditions and magnetic field position and its intensity is found in section 2. Set of equations including fluid flow and heat transfer for ferrofluid, Maxwell equations to compute magnetic body force, thermophysical equations for describing ferrofluid properties and dimensionless form of governing equations are presented in section 3. The numerical procedure, its verification and grid dependency are brought
in Section 4. The results corresponding to the heat transfer and pressure drop of the constant and oscillating magnetic fields in the form of contours, plots of velocity, temperature, Nusselt number and Pressure number and overall performance ratio are brought and discussed in section 5 and finally summarized in section 6.

2 PROBLEM DESCRIPTION

The moving ferrofluid inside a channel formed of 4% of magnetized particles (Fe$_3$O$_4$) and DI-water as a base fluid. The 2D miniature channel has a 2mm height ($h=2$mm) and length of 20mm ($l=20$mm) while the third dimension has much greater value. Hydrothermal conditions of inlet ferrofluid including $T_{in}=300$K to cool the channel with maintained symmetry heat flux conditions on lower and upper walls and to pretermit the hydrodynamic entrance region, the inlet velocity profile considered fully developed. A line dipole as a magnetic source is placed near and below a lower wall ($y/h=-0.5$) at the mid-length of the channel ($x/h=5.0$) which origin of the Cartesian coordinate system is placed at the left lower corner of the channel. A diagram giving the main information of the study is brought as Fig. 1.

![Figure 1. Schematic of 2D channel and position of magnetic dipole.](image-url)
For the case of constant magnetic field (CMF) study, 3 different magnetic strengths with corresponding magnetic number (Mn) \(Mn = 3.833 \times 10^8, 8.624 \times 10^8, 1.533 \times 10^9\) and considered for assumed Reynolds numbers. To generate OMF, four different common frequency functions, including rectangle, sine, triangle and sawtooth, were chosen and implemented to the line dipole. Frequencies vary in the range of \(f=0.5\) to \(5\)Hz. As seen in Fig. 2, the implemented functions cause the magnetic field varies in the range of \(-1 \leq B/B_{\text{max}} \leq +1\).

**Figure 2.** Distribution of different frequency functions to the magnetic dipole (a) rectangle, (b) sine, (c) triangle, and (d) sawtooth functions.
3 SET OF EQUATIONS

In this part, a comprehensive explanation including governing equations of fluid flow and magnetic field, thermophysical properties of ferrofluid and form of dimensionless of governing equations are introduced.

3.1 Fluid Flow and Heat Transfer

Thermofluid governing equations of ferrofluid which is subjected to the applied magnetic field contain conservation of mass, momentum equation of ferrofluid and energy equation for temperature. For more simplification of the governing equations include fluid flow and energy equations, some assumptions are taken as below: (a) the ferrofluid is Newtonian, incompressible and laminar; (b) the thermophysical properties are constant; (c) the work because of the pressure is neglected; (d) the flow field is two dimensional and transient; (e) ferrofluid as a nanofluid has a nanoparticles with uniform size and shape and they are well and uniformly distributed in the DI water; (f) the DI water as a base fluid and magnetic particles are in thermal equilibrium and the particles are so small in size which makes the particles have the same velocity as the base fluid and results in no slip condition between them. According to these assumptions, for modeling of the nanofluid, the single phase approach was chosen. Two methods including single phase and two phase approaches are commonly used by many researchers for modeling of convection heat transfer of nanofluid. Numerous researchers implemented the single phase approach for simulation of the nanofluid in a variety of geometries and applications [64-71].

Mass conservation for an incompressible fluid is:

\[ \nabla \cdot \mathbf{v} = 0 \]  \hspace{1cm} (1)
The fluid flow equation of magnetoconvective ferrofluid is a modified version of well-known Navier-Stokes relation which the Magnetization body force is added. The last term in Eq. 2 is known as Magnetization or Kelvin body force.

\[ \rho_{ff} \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu_{ff} \left( \nabla^2 \mathbf{v} \right) + (\mathbf{M} \cdot \nabla) \mathbf{B} \]  \hspace{1cm} (2)

A modified Fourier’s law, adding both terms viscous dissipation and magnetocaloric, presents the energy equation as below:

\[ \rho_{ff} C_{p,ff} \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k_{ff} \nabla^2 T + \Phi_{vis} + \Phi_{FHD} \]  \hspace{1cm} (3)

The basic parameters in the above equations: \( \mathbf{v}, p, T, \rho_{ff}, \mu_{ff}, C_{p,ff}, k_{ff} \) are velocity vector, pressure, temperature, density, dynamic viscosity, specific heat and thermal conductivity of moving ferrofluid. Viscous dissipation for a Newtonian fluid \( \Phi_{vis} \) is computed from:

\[ \Phi_{vis} = \mu_{ff} \left\{ 2 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \]  \hspace{1cm} (4)

Magnetocaloric term \( \Phi_{FHD} \) appeared in the energy equation can be assumed as a thermal power generated because of interaction between moving ferrofluid and applied magnetic field \[32\].

\[ \Phi_{FHD} = -\mu_0 T \frac{\partial \mathbf{M}}{\partial T} \cdot (\mathbf{v} \cdot \nabla) \mathbf{H} \]  \hspace{1cm} (5)

### 3.2 Kelvin Body Force

Ferrofluid in this study is assumed as a non-conducting electrical fluid and results to the static state form of Maxwell equations and the induced electromagnetic current can be negligible. Moreover, the effects of the electric field and the variation of the magnetic field because of the
temperature gradients in the ferrofluid can be negligible [28, 31]. Also for neglecting the electrokinetic effects on the fluid field, the dimensions of the channel in the order of millimeters is relatively large compared to the dimensions of the MEMS devices [31, 72].

\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{H} = 0 \]

Where \( \mathbf{B} \) is magnetic density field and \( \mathbf{H} \) is the magnetic field. From a constitutive relation, the Magnetization vector \( \mathbf{M} \) is related to the \( \mathbf{B} \) and \( \mathbf{H} \):

\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \]

In the above equation, \( \mu_0 \) is magnetic permeability in the vacuum and has a constant value \( (\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2) \). In this study, line dipole assumed as a magnetic source and based [31] the below equation can describe such a magnetic field and it verifies the static form of Maxwell equations.

\[ \mathbf{B} = \mu_0 (1 + \chi_m) m \left[ \frac{\sin \phi}{r^2} \hat{e}_r - \frac{\cos \phi}{r} \hat{e}_\phi \right] \]

As seen in Eq. 9, the presented magnetic field formulation is in the Cylindrical coordinate system which its virtual origin places at the exact position of line dipole \((x/h=5.0, y/h=-0.5)\) also \( \hat{e}_r \), \( \hat{e}_\phi \) are the radial and angular unit vectors. The moment magnet of the coil per length \( (m=I \cdot b/2\pi) \) describes the magnetic strength of the line dipole.

The appearance of \((\mathbf{M} \cdot \nabla) \mathbf{B}\) in the Eq. 2, which is known as the Magnetization force or the Kelvin body force, is created whenever a magnetic field applies to a magnetic fluid. Magnetization vector \( \mathbf{M} \) and magnetic field \( \mathbf{H} \) can linearly be related, for a limited range of temperature variation.
\[ M = \chi_m H \]  

Total magnetic susceptibility of the ferrofluid is a function of temperature [31]:

\[ \chi_m = \chi_m(T) = \frac{\chi_0}{1 + \beta_{ff}(T - T_m)} \]  

In above equation \( \chi_0 \) is magnetic susceptibility of ferrofluid at a reference temperature, \( \beta_{ff} \) is thermal expansion of ferrofluid. By substituting (10) in constitutive relation (8) we drive:

\[ B = \mu_0 (1 + \chi_m) H \]  

By substituting (10) and (12) on Kelvin body force statement we have:

\[ f_{Kelvin} = \mu_0 \left( \chi_m H \cdot \nabla \right) + \left( (1 + \chi_m) H \right) \]  

It should be noted that the temperature is the function of \((x, y)\). The last term of Kelvin body force is derived through manipulating the Eq. 13 as below:

\[ f_{Kelvin} = \frac{1}{2} \mu_0 (1 + \chi_m) \nabla \left( H \cdot H \right) + \mu_0 \chi_m H \left( (H \cdot \nabla) \chi_m \right) \]  

### 3.3 Ferrofluid Thermophysical Properties

To compute the thermophysical properties of ferrofluid which magnetite particles with 4% volume fraction \((\phi = 4\%)\) dispersed in the DI-water, the below formulas are used to determine density \( \rho_{ff} \), specific heat \( C_{p,ff} \), dynamic viscosity \( \mu_{ff} \) of ferrofluid:

\[ \rho_{ff} = (1 - \phi) \rho_w + \phi \rho_p \]  

\[ (\rho C_p)_{ff} = (1 - \phi)(\rho C_p)_w + \phi(\rho C_p)_p \]
\[ \mu_{ff} = \frac{\mu_w}{(1-\phi)^{2.5}} \]  

(17)

To calculate the dynamic viscosity due to a high volume fraction of magnetite particles in the water (\(\phi=4\%\)), the Brinkman equation was used to predict a more precise value for the dynamic viscosity of ferrofluid [73]. To calculate thermal conductivity \(k\) of ferrofluid below relation is used from [24]:

\[ k_{ff} = \frac{2k_w + k_p - 2\phi(k_w - k_p)}{k_w + k_p + \phi(k_w - k_p)} \]

(18)

In the above equations subscripts \text{ff}, \text{w} and \text{p} denote the ferrofluid, DI-water and magnetite particles, respectively. Table 1 represents the thermophysical of magnetite particles, water and ferrofluid.

**Table 1. Thermophysical properties of magnetite particles (Fe\(_3\)O\(_4\)), Base fluid (Water) and ferrofluid.**

<table>
<thead>
<tr>
<th></th>
<th>(\rho) [kg/m(^3)]</th>
<th>(\mu) [Pa.s]</th>
<th>(C_p) [j/(kg.K)]</th>
<th>(k) [W/(m.K)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magentite Particles(Fe3O4)</td>
<td>5100</td>
<td>-</td>
<td>660</td>
<td>7</td>
</tr>
<tr>
<td>Water</td>
<td>998</td>
<td>0.001003</td>
<td>4180</td>
<td>0.585</td>
</tr>
<tr>
<td>Ferrofluid((i=0.04))</td>
<td>1203.1</td>
<td>0.00114</td>
<td>3434</td>
<td>0.656716</td>
</tr>
</tbody>
</table>

**3.4 Dimensionless Equations**

In this chapter, the dimensionless form of governing equations to have a simpler frame of reference, are presented. At first, the dimensionless form of used basic parameters in the governing equations introduces:
It should be noted that the subscript "ff" denotes the implemented thermophysical properties are derived from equations 15-18 and $H_r$ is magnetic strength at the center of the lower wall ($H_r = H(x=a, y=0)$). By substituting basic variables in conservation equations (mass, momentum and energy), the governing equations become:

$$\frac{\alpha_{ff}}{h^2}(\nabla' \cdot \mathbf{v}') = 0$$  \hspace{1cm} (20)

$$\frac{\rho_{ff} \alpha_{ff}^2}{h^3} \left( \frac{\partial \mathbf{v}'}{\partial t'} + \mathbf{v}' \cdot \nabla \mathbf{v}' \right) = -\rho_{ff} \frac{\alpha_{ff}^2}{h^3} \mathbf{v}' + \frac{\mu_{ff} \alpha_{ff}}{h^3} \left( \nabla'^2 \mathbf{v}' \right) + \frac{\mu_0 H_0^2}{h} f_{K_{vel}}$$  \hspace{1cm} (21)

$$\frac{\rho_{ff} C_{p,ff} \alpha_{ff}}{h k_{ff}} \left( \frac{\partial \varphi'}{\partial t'} + \mathbf{v}' \cdot \nabla \varphi' \right) = \frac{q''}{h} \nabla'^2 \varphi' + \frac{\mu_0 H_0^2 \alpha_{ff}}{h^2} \left( \theta + \varepsilon \right) \frac{\partial (\chi_m' H')}{\partial \theta} \left( (\mathbf{v} ' \cdot \nabla ') H' \right)$$  \hspace{1cm} (22)

The appeared parameter $\varepsilon$ in Eq. 22 is defined as $\varepsilon = T_{in}/\Delta T$. The dimensionless form of Kelvin body force ($f'_{K_{vel}}$) can be written as follow:

$$f'_{K_{vel}} = \frac{1}{2} \mu_0 (1 + \chi''_m) \nabla' (H' \cdot H') + \mu_0 \chi''_m H' \left( (H' \cdot \nabla ') \chi'_m \right)$$  \hspace{1cm} (23)

For equation 23, the dimensionless form of magnetic susceptibility ($\chi'_m$) is as below:

$$\chi'_m = \chi''_m (\theta) = \frac{\chi_0}{1 + \beta \Delta T \theta}$$  \hspace{1cm} (24)

By multiplying equations 20, 21 and 22 in $h^2/\alpha_{ff}$, $h^3/\rho_{ff} \alpha_{ff}^2$ and $hk_{ff}/\rho_{ff} C_{p,ff} \alpha_{ff} q''$, respectively, and manipulating the resulted equations, the final terms become:
\[(\nabla' \cdot \nu') = 0 \quad (25)\]

\[
\left( \frac{\partial \nu'}{\partial t'} + \nu' \nabla' \nu' \right) = -\nabla' p' + \frac{\mu_f}{\rho_0 \alpha_f} (\nabla'^2 \nu') + \frac{\mu_0 H_r^2 h^2}{\rho_f \alpha_f^2} f_{\text{Kelvin}}' \quad (26)
\]

\[
\left( \frac{\partial \theta}{\partial t'} + \nu' \nabla' \theta \right) = \nabla'^2 \theta + \frac{\mu_f}{\rho_0 \alpha_f} \frac{a_k^2}{h^2} C_{p,f} q'' \Phi' + \frac{\mu_0 H_r^2 h^2}{\rho_f \alpha_f^2} \frac{a_k^2}{h^3 C_{p,f} q''} (\theta + \varepsilon) \frac{\partial \left( x_m H' \right)}{\partial \theta} \left( (\nu' \nu') H' \right) \quad (27)
\]

Some ratios appeared in the equations 26 and 27 can be replaced by well-known dimensionless numbers:

\[
Pr = \frac{\mu_f}{\rho_0 \alpha_f}, \quad Mn = \frac{\mu_0 H_r^2 h^2}{\rho_f \alpha_f^2}, \quad Ec = \frac{a_k^2}{h^3 C_{p,f} q''} \quad (28)
\]

Therefore, the final form of dimensionless governing equations obtains as below:

\[(\nabla' \cdot \nu') = 0 \quad (29)\]

\[
\frac{\partial \nu'}{\partial t'} + \nu' \nabla' \nu' = -\nabla' p' + Pr \nabla'^2 \nu' + Mn f_{\text{Kelvin}}' \quad (30)
\]

\[
\frac{\partial \theta}{\partial t'} + \nu' \nabla' \theta = \nabla'^2 \theta + Pr \Phi' + Mn Ec (\theta + \varepsilon) \frac{\partial \left( x_m H' \right)}{\partial \theta} \left( (\nu' \nu') H' \right) \quad (31)
\]

Based on the properties of Table 1, the Prandtl and Eckert numbers are \(Pr=5.9621\) and \(Ec=3.0202 \times 10^{-14}\). To characterize the behavior of ferrofluid in the heated channel, the below parameters were defined:

\[
\tau = tf = \frac{h^2 t'}{a_f} \quad (32)
\]

\[
Re = \frac{\rho_f U_m h}{\mu_f} \quad (33)
\]
\[ P_n = \frac{\Delta p}{\frac{1}{2} \rho U_0^2} \]  

(34)

To indicate a hydrodynamic behavior in the study, “Pn” as Pressure number is introduced which relates the pressure drop along the channel to the inlet velocity.

\[ \eta = \frac{\frac{Nu}{Nu_0}}{\frac{Pn}{Pn_0}} \]  

(35)

Total performance ratio \( \eta \) is defined to take into account the simultaneous variations of average Nu number and pressure drop along the channel in two periods as symbols of thermal and hydrodynamics behaviors, respectively. In the definition of \( \eta \) in the formula, the subscript “0” refers to CMF with intensity \( Mn=1.533 \times 10^9 \).

4 NUMERICAL METHOD

Set of governing equations are solved with the finite volume method (FVM). For solving the issue of coupling between pressure and velocity field SIMPLE C scheme is used and second-order upwind method used to discretize the momentum and energy equation. A symmetric heat flux and no slip condition were assigned for the upper and lower walls. A non-uniform and structured grid was employed to mesh the channel. It should be noted that the density of the produced mesh in the inlet, near the walls and the place of the implementation of line dipole are high because of the high amount of gradient of variables (velocity, pressure and temperature) specifically near the magnetic field. Root mean square (RMS) error for all physical quantities is assumed as \( 10^{-5} \) for convergence criteria.

In order to study the quality of the generated mesh (grid dependency) and the accuracy of computations, some meshes with different size were produced and studied. To ensure capturing
the most severe fluctuations of the parameters, the places of the channel which are subjected to
the near the lower wall and middle of the channel are chosen to study the longitudinal and
latitudinal fluctuations. For the assumed different mesh sizes, velocity and temperature profiles
for Re=20 and Mn= 1.533×10^9 are depicted in Fig. 3. Because of the presence of the magnetic
field and its drastic effect on the velocity and temperature fields in the vertical direction, Fig. 3
tries to present better and more accurate fluctuations in this direction.

Figure 3. Mesh dependency study for Re=20 and Mn=1.533×10^9 for variation of (a) longitudinal velocity, (b) latitudinal
velocity and (c) temperature.
Besides, to mesh dependency plots in Fig. 3, we analyzed a couple of grid sensitivity for the channel under the strongest magnetic intensity at Re=20. In the Table. 2, total 9 grids with different elements were tested for mesh dependency have been brought. To characterize the sensitivity of the number of elements, the average dimensionless temperature in the middle of the channel where the most severe variations occur, is brought. The relative change regard to the previous case is computed and reported in Table. 2. It should be noted that for the first case the relative change is not considered. The more number of elements increases, the more relative change of $\theta$ decreases. The highest number of elements ($n=6.0\times10^4$) had the best accuracy although the CPU time usage enhances considerably. Based on the variations of variables in Fig. 3, the relative errors and amount of CPU time usage in Table 2, the mesh size of ($100\times300$) is chosen for the remainder computations. Based on Table 2, the relative error of the grid with 100×300 division, is less than 1.0%, since this grid was selected as an accurate and grid independent mesh.
Figure 4. A graph of convergence of the variation of the dimensionless temperature at the middle of the channel based on the number of elements

In Fig. 4, for better illustration, the variation of the average of $\theta$ at the middle of the channel, where the influence of the magnetic is the most severe on the governing parameters, is introduced based on the information of the Table. 2. As seen from Fig. 4, the more number of elements of the grids increases the less the range of variations of the $\theta$ decreases. For the number of elements more than 30000, this variation is converged to a constant value. To check the verification and correctness of the implemented numerical solution and its results, in Fig. 5, the comparison between local Nusselt number from the numerical solution of this study and Ganguly et al. [31] is presented for different magnetic field strengths. The results show, the applied numerical method can predict the magnetic field effect result in a good agreement with [31]. The slight difference between the obtained results in the present study and the [31] can be related to the different implemented solvers, methods of the discretization also the number of elements and the criteria of convergence did not report in the [31].
5 RESULTS AND DISCUSSION

5.1 Applied Magnetic Field

The position of line dipole, as shown in Fig. 1, is near and below the lower wall (y/h=−0.5) at the mid length of the channel (x/h=5.0). The distribution of the magnetic field (B_x, B_y) and its magnitude is illustrated in Fig. 6. Moving from left or right corner of the lower wall to the middle of the channel, the magnetic field strength increases and its maximum value occurs exactly at the mid-length of the lower wall. In other words, by distance from the position of the line dipole, the magnetic field strength diminishes significantly with proportionate to the second order of distance (∝1/r^2) as Eq. 9 implies. The orientation of magnetic field using the vector field and its magnitude distribution contours are displayed in Fig. 7 and 8, respectively. Both the size of the vectors and the specific color pattern in contours show that the maximum strength of the magnetic field occurs at the middle of the channel and reduces to zero by moving toward the corner of the channel.
Figure 6. Distribution of normalized magnetic field on the lower wall.

Figure 7. Vectors of normalized magnetic field due to lower magnetic line dipole

Figure 8. Contours of normalized magnetic field distribution in the channel
5.2 Effect of Constant Magnetic Field

To have a better insight into the FHD thermomagnetic convection, in this part, a brief study of the influence of the CMF on force convection is presented. In Fig. 9, temperature distribution and streamlines for different magnetic numbers (Mn) at Reynolds number 20 (R=20) is indicated. According to Eq. 9, the magnetic susceptibility of ferrofluid is a temperature dependent property. Considering this dependency along with asymmetric temperature gradient through maintaining heat flux, rotational magnetic forces are induced. Recirculation zone or vortex area is emerged due to ferrofluid passing by the vicinity of magnetic dipole position and inevitable rotational magnetic forces. As it may be seen in Fig. 9, the size of vortex grows with magnetic field strength augmentation. The shape of the TBL alters due to the creation of the recirculation zone. Thickening and thinning of the TBL in the vortex area are the function of the streamlines direction. At the beginning of vortex area where the orientation of streamlines in vortex region and main flow are in the same direction, TBL becomes thicker and at the end of vortex area where the orientation of streamlines of vortex region and main flow are in opposite direction, TBL becomes thinner.
In Fig. 10, velocity and temperature distribution at the middle of the channel for different magnetic field strength are depicted. The negative value of longitudinal velocity ($u$) and appearance of latitudinal velocity ($v$) are interpreted as a sign of vortex formation where the variation of both velocities intensifies as magnetic field builds up. Heat transfer rate is enhanced by applying magnetic field due to the increase of velocity gradient near the wall leads to a decrease in temperature profile (Figure 10 (c)). Creation of vortex area and its growth by strengthening magnetic field, pushes (conducts) the ferrofluid to the upper wall and makes TBL thinner and increases the velocity in the middle of the channel. Eventually, this leads to increase in energy and momentum transfer and enhancement in heat transfer rate.
Figure 10. Effect of magnetic field strength on (a) longitudinal velocity (b) latitudinal velocity and (c) temperature profiles at Re=20.

To evaluate the heat transfer rate development under the effect of the magnetic field, local Nusselt number distribution for upper and lower walls are indicted in Fig. 11. For the case of the lower wall (Figure 11 (a)), the rise and fall of plot curves is pertinent to thickening and thinning of TBL, respectively. The irregular distribution of local Nu number (the rise and fall) is due to the emergence of the vortex region in the vicinity of the line dipole. In the vortex region where the orientation of the ferrofluid is according to the main flow stream in the channel, the TBL becomes thicker. Whereas, in the area of the vortex where the ferrofluid direction opposes with the direction of the main flow, the TBL thinning occurs. This thinning and thickening in TBL explain the rise and falls on the local Nu of the lower wall [31, 54 and 55]. For the case of the upper wall (Figure 11 (b)), the rise of the plot curve at the middle of the channel is affected by the movement of ferrofluid from the lower wall to the upper wall and suppression of the TBL in that region. In both plots, the variation of Nu corresponds to the size of the recirculation zone.
Temperature distribution and streamlines for different Reynolds numbers for the strongest magnetic field ($Mn=1.533\times10^9$) to investigate the effect of Reynolds number on ferrofluid magneto convection, is depicted in Fig. 12. When the magnetic field and fluid flow interact, the influence of inertia forces overtakes the magnetic forces with the increase of Reynolds number which is observed as the vortex area shrinks. Also, as shown in Fig. 12, temperature variation weakens with the increase of Reynolds number and causes a decrease in variation of the total magnetic susceptibility distribution near the magnetic dipole position.
Velocity and temperature distributions for different Reynolds numbers at Mn=1.533×10⁹ is illustrated in Fig. 13. The latitudinal velocity reduction (Figure 13 (b)) for higher Reynolds numbers indicates the shrinking of the vortex region. Also as Reynolds number increases at constant magnetic intensity, the corresponding velocity and temperature distributions tends to the non-magnetic profiles that show the weakening portion of magnetic forces in magnetic field and fluid flow interaction.
It is clear that by increasing Reynolds number, heat transfer rate improves as higher local Nu number profiles for both the lower and upper walls (Fig. 14). At the middle of the channel, Nu profiles of the lower Reynolds numbers reach to that of higher Reynolds numbers which expresses that heat transfer rate development in presence of magnetic field is more tangible at lower Reynolds numbers. Variations of Nu profiles at the middle of the channel is dependent to the size of the recirculation zone, as Reynolds number raises, local Nu profile experiences smaller variations which confirms vortex region shrinking.
Figure 14. Distribution of local Nu number for different Reynolds numbers along the (a) lower wall and (b) upper wall.

5.3 Effect of Oscillating Magnetic Field

In this part, the influences of generated OMF by different functions, as presented in Fig. 2, on thermo-fluid of ferrofluid is discussed. In Figures 15 to 18, the temperature contours and streamlines for Re=20 and f=1Hz are demonstrated. For each specific function, the time steps in one period are chosen in a way to cover the fluctuation of the ferrofluid movement. The conversion of CMF to oscillating one changes the pattern of moving fluid through relocating of the vortex and reducing or increasing of its size due to the variation of magnetic fields ($-1 \leq B/B_{max} \leq +1$). Applying a frequency function, regardless of its shape, causes the attraction or repulsion of the moving cold ferrofluid depending on the value of inducted magnetization force. The TBL near the position of the magnetic dipole, for example in Fig. 15, becomes thicker or thinner in one period resulted by the magnetic fields variation and yields in change of the total magnetic susceptibility of the ferrofluid. The attraction and repulsion causes the deviation of ferrofluid movement from the main migration path of magnetized particles and results in better mixing, suppression the growth of TBL and development of heat transfer rate. Moreover, the employed frequency functions with instant variation characteristic in the magnetic field
(rectangle and sawtooth) create two vortexes and results in better mixing of the fluid. This is mainly because near the step time boundaries, magnetic field experiences a sudden change from the negative value to the positive one or vice versa and therefore a drastic change occurs in the magnetic susceptibility of moving ferrofluid in the vicinity of the place of the magnetic field.

![Figure 15. Temperature and streamline contours for OMF at Re=20 for rectangle function.](image-url)
Figure 16. Temperature and streamline contours for OMF at Re=20 for sine function.

Figure 17. Temperature and streamline contours for OMF at Re=20 for triangle function.
Figure 18. Temperature and streamline contours for OMF at Re=20 for sawtooth function.

Velocity and temperature profiles for the rectangle, sine, triangle and sawtooth functions at the implemented time steps in Figures 15 to 18, are depicted in Figures 19 to 21. Effect of conversion of the CMF to the oscillating one can be seen from the variations of velocity and temperature distributions \((u, v, T)\) by time. In Figure 19 (a) and (d), for all functions, the longitudinal velocity \((u)\) varies with time due to the TBL thickening and thinning, attraction and repulsion of ferrofluid based on the sign of magnetic field. At time steps \(\tau=0.9\) and \(\tau=0.1\), as observed in Figures 15 and 18 respectively, for rectangle and sawtooth functions the intensity of mixing of the fluid is much higher and longitudinal velocity for both functions varies dramatically. Furthermore, the existence of latitudinal velocity results in emergence of vortex. In Figures 10 (b) and 13 (b) latitudinal velocity was constant but in this case the positive and
negative values of latitudinal velocities for OMF indicates the relocation of vortexes over time. For instance, in Figure 20 (a) and (d) at time steps when two vortexes are emerged, the latitudinal velocity increases compared to the sine and triangle velocities. In Fig. 21, temperature profiles shows that ferrofluid heat transfer in one specified period varies significantly due to continuous change in TBL. Appearance of two vortexes simultaneously for rectangle and sawtooth function (respectively at $\tau=0.9$ and $\tau=0.1$) results in better mixing of ferrofluid and lower temperature variations which can be observed in Figure 21 (a) and (d).

![Figure 19](image_url)

**Figure 19.** The alteration of longitudinal velocity profile in a period for (a) rectangle, (b) sine, (c) triangle and (d) sawtooth functions.
Figure 20. The alteration of latitudinal velocity profile in a period for (a) rectangle, (b) sine, (c) triangle and (d) sawtooth functions.
Figure 21. The alteration of temperature profile in a period for (a) rectangle, (b) sine, (c) triangle and (d) sawtooth functions.

In Fig. 22, variations of average Nu number ($\text{Nu}_{\text{avg}}$) and Pressure number (Pn) for different Reynolds numbers are demonstrated. For each frequency, the general shape of Pn plots conforms to the shape of its corresponding frequency function. By increasing the Reynolds number in all functions, the range of the variations of Pn number decreases because of the attenuating influence of magnetic forces against viscous forces. The alteration of the Pn plots corresponds to the transversal movement which is deviated from the main flow of ferrofluid in the channel. In one period, the variation of the magnetic forces which engendered from the
applied frequency functions leads to the temporal behavior of the vortex region including the relocation of the vortex and increase or reduction of its size. The sudden fluctuations in the orientation of applied magnetic field which originated from the applied rectangle and sawtooth frequencies (Fig. 2), displays drastic changes in both \( \text{Nu}_{\text{avg}} \) and \( \text{Pn} \) plots for aforementioned functions (Fig. 22 (a, a') and (d, d')). These alterations in \( \text{Nu}_{\text{avg}} \) and \( \text{Pn} \) plots in one period, are proportional to variations of the shape of frequency functions. This is because of the instantaneous change of thermal boundary thickness in the vicinity of the applied magnetic field.

\[ \begin{array}{c}
\text{(a)} \\
\text{Re=20} \\
\text{Re=30} \\
\text{Re=40} \\
\end{array} \]

\[ \begin{array}{c}
\text{(a')} \\
\text{Re=25} \\
\text{Re=30} \\
\end{array} \]

\[ \begin{array}{c}
\text{(b)} \\
\text{Re=20} \\
\text{Re=30} \\
\text{Re=40} \\
\end{array} \]

\[ \begin{array}{c}
\text{(b')} \\
\text{Re=20} \\
\text{Re=30} \\
\text{Re=40} \\
\end{array} \]
Figure 22. Thermal and hydrodynamic behaviors of ferrofluid movement at f=1 Hz for (a,a') rectangle (b,b') sine (c,c') triangle and (d,d') sawtooth frequency functions.

To have a comprehensive insight into the ferrofluid heat transfer rate under the effect of applied functions, the average Nu number parameter at the lower wall in each frequency for its corresponded time interval is averaged and plotted in Fig. 23. The weakening effect of magnetic forces at a constant Mn against increasing inertia forces result as an increasing trend with Reynolds number. There is an optimum frequency for all functions where maximum heat transfer rate happens and this frequency increases for higher Reynolds numbers. However, the achieved
Heat transfer enhancement weakens by the attenuated effect of the magnetic field as the Reynolds number increases.
According to Fig. 23, maximum and minimum average Nusselt number for OMF and for each Reynolds number is presented in Table 3. Furthermore, the average Nusselt number for CMF at each Reynolds number are also indicated in Table 3 for comparison and a better understanding of the influence of the OMF on heat transfer rate. The OMF develops thermal behavior of the channel significantly. Also, by increasing Reynolds number, the range of variation between maximum and minimum average Nusselt number for the OMF decreases which shows the reduction of the influence of magnetic forces compared to the viscous forces.

Table 3. Maximum and minimum variations of average Nusselt number for OMF for different Reynolds number

<table>
<thead>
<tr>
<th>Re</th>
<th>Nu_{av}^{cte}</th>
<th>Max. Nu_{av}^{osc}</th>
<th>Min. Nu_{av}^{osc}</th>
<th>(Max. Nu_{av}^{osc} - Nu_{av}^{cte}) \times 100/Nu_{av}^{cte}</th>
<th>(Min. Nu_{av}^{osc} - Nu_{av}^{cte}) \times 100/Nu_{av}^{cte}</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.888397</td>
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<td>5.750093</td>
<td>62.52457</td>
<td>17.62739</td>
</tr>
<tr>
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<td>4.932157</td>
<td>7.650787</td>
<td>5.830231</td>
<td>55.12051</td>
<td>18.20855</td>
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<tr>
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<td>7.51492</td>
<td>5.944396</td>
<td>49.82266</td>
<td>18.51162</td>
</tr>
<tr>
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<td>6.083568</td>
<td>44.37994</td>
<td>18.85985</td>
</tr>
<tr>
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<td>6.2322</td>
<td>39.94706</td>
<td>19.19069</td>
</tr>
</tbody>
</table>
The variation of total performance ratio ($\eta$) which quantifies the thermal and hydrodynamic behaviors of ferrofluid for different functions and Reynolds numbers are illustrated in Fig. 24. For rectangle and sawtooth plots (Figure 24 (a) and (d)), a sudden change on the applied magnetic forces has a tremendous effect on TBL due as two vortexes appears in the flow. Higher values of $\eta$ for all type of functions at lower frequencies are related to higher efficiency of the magnetic field. Similar to Fig. 23, there is an optimum frequency for each Reynolds number in which the maximum value of $\eta$ occurs.
CONCLUSION

This paper presents a comprehensive study about the effectiveness of constant and oscillating magnetic fields on the heat transfer of ferrofluid FHD flow. The studied parameters includes magnetic intensity (Mn number), Reynolds numbers, magnetic frequencies (f), frequency functions and pressure drop (Pn number). Highlighted results can be pointed as follow:

- Implementation of CMF results in suppression of the TBL growth, the creation of the recirculation zone, increasing velocity field near the magnetic dipole position and consequently, heat transfer enhancement.

- The more magnetic intensity increases, the better heat transfer is enhanced. But this enhancement will decrease for higher Reynolds number due to the weakening effect of magnetic forces in magnetic field and fluid flow interaction.

- Conversion of the CMF to oscillating one alters the TBL growth and vortex positions over time and provides a better mixing of fluid flow. This also enhances the level of

Figure 24. Total performance ratio (η) variations for (a) Re=20, (b) Re=25, (c) Re=30, (d) Re=35 and (e) Re=40.
momentum and energy transfer which resulted in heat transfer enhancement and variation in pressure drop along the channel.

- Between the implemented different frequency functions, the rectangle and sawtooth functions with sudden change in the magnetic field results in more heat transfer enhancement.

- Rectangle and sawtooth frequencies at time steps where a sudden change in the magnetic field occurs, two simultaneous vortexes emerges which leads to better mixing in flowing fluid and higher heat transfer enhancement.

- The results show there is an optimum frequency with maximum heat transfer enhancement. This optimum frequency, regardless of the type of the frequency function, increases in higher Reynolds numbers.

- Effect of frequency functions with sudden change in magnetic field on thermal and hydrodynamic parameters are more tangible than functions with smoother variations. For instance, variations of $P_n$ number for the sine and triangle functions pertains to the shape of functions.

- Thermofluid parameters including pressure drop along the channel and average Nusselt number in one period varies in accordance with the shape of the frequency function.

- Analysis of the total performance ratio ($\eta$) which takes into account both thermal and hydrodynamic variations shows that applying the OMF can enhance heat transfer depending on frequency and Reynolds number.

REFERENCES


Conventional cancer therapy methods encounter with serious drawbacks.

- A new trend is being emerged in clinical oncology to shift from monotherapy toward combination therapy methods.

- The effect of various cancer treatment modalities can be enhanced when combined with nanomaterials.

- Gold nanoparticle (AuNP) is widely used in combinatorial cancer therapy methods.

- Here, recent progress in the application of AuNPs in single-, double-, and triple-modality cancer therapies is reviewed.

- Also, the opportunities and challenges of the entry of AuNPs into the clinics are discussed.