Introducing unsteady and nonuniform source terms in entropic lattice kinetic models using Fourier series

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(Received 22 April 2018; revised manuscript received 5 October 2018; published 6 November 2018)

In this work, for the constant speed kinetic model (CSKM) of Zadehgol and Ashrafizaadeh [J. Comput. Phys. 274, 803 (2014)], a different nonuniform and unsteady mass and force source term is proposed. Employing a Fourier series expansion of the source term and applying the standard Chapman-Enskog analysis, we show that the desired source term can be determined by evaluating the expansion coefficients of the Fourier series in terms of the mass and force sources and the known macroscopic properties of the fluid. To rectify the pressure and speed of sound, rest particles are introduced. Moreover, it is shown that the present results can be applied to three-dimensional (3D) cases by employing four-dimensional (4D) Fourier series in the 4D hypercomplex space and projecting the results into the 3D space. The accuracy and thermodynamic consistency of the present method are numerically verified by simulating the following 2D and 3D benchmark flows: (i) Womersley flow, (ii) unsteady Taylor-Green vortex flow, (iii) flow in a concentric annular pipe, and (iv) the force-driven flow of a fluid inside a 3D channel. The present results are in excellent agreement with the exact and analytical solutions, and it is shown that the implementation of the present formulation of source term does not affect the thermodynamic consistency of the CSKM.

DOI: 10.1103/PhysRevE.98.053303

I. INTRODUCTION

The lattice Boltzmann method (LBM) [1–4] has widely been used as a viable alternative to conventional computational fluid dynamics (CFD). The standard CFD is an Eulerian model and it relies on the macroscopic governing equations. The LBM, on the other hand, is a Lagrangian model and it relies on simplified (reduced) mesoscopic kinetic equations. The LBM has attracted the attention of researchers in the past two decades and has been used in several different fields [5–7], because of its (i) relative simplicity, (ii) computational efficiency, (iii) high accuracy, and (iv) ease of parallelization.

The collision integral of the Boltzmann equation has a complicated structure, and it is usually replaced by a simpler expression which is referred to as a collision model. The simplified Boltzmann equation is called a kinetic model [1]. A widely used collision model which ensures the recovery of the weakly compressible Navier-Stokes equations is the so-called Bhatnagar-Gross-Krook (BGK) model, where the probability distribution function is relaxed toward a local equilibrium distribution function (EDF) at each time step, using a single relaxation time scheme. Discretization of the velocity space using the discrete-velocity models (DVMs) [8] leads to further simplification of the kinetic models.

The most important task, in the collision stage of the kinetic models, is to define an appropriate EDF. The Maxwellian distribution of the classical kinetic theory is compatible with the H-theorem, while the discrete form of the Maxwellian does not yield a thermodynamically consistent kinetic model. In fact, it has been shown [9] that, for the conventional LBM with polynomial EDF, the H-theorem does not exist, where an attractor plays the role of a true equilibrium.

In spite of noticeable achievements of the LBM, it is still in the development stage. Numerical instability at low viscosity is one of the most important issues of the present-day LBM. To overcome such shortcomings, modifications have been proposed. For example, while the regularized and multiple relaxation times (MRT) methods provide improvements over the standard LBM at a higher computational cost, the instability still remains an issue in these models [4]. Recently, the cumulant [10–13] and cascaded [14,15] LB approaches have been introduced to improve the MRT models. While these complicated and expensive modifications have significantly improved the LBM, the choice of the relaxation rates of different moments can seriously affects the simulation results. Furthermore, the thermodynamic consistency is not satisfied in these models.

It has been noted [16–18] that the lack of the H-theorem is at least partly responsible for the numerical instability of the conventional LBM. Hence, kinetic models in which the H-theorem is respected have been proposed [16–22], in the past two decades, and they have been referred to as the entropic kinetic models. Two different types of entropic kinetic models are found in the literature [4]. The first type, the so-called