Generalizing the Boltzmann equation in complex phase space

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In this work, a generalized form of the BGK-Boltzmann equation is proposed, where the velocity, position, and time can be represented by real or complex variables. The real representation leads to the conventional BGK-Boltzmann equation, which can recover the continuity and Navier-Stokes equations. We show that the complex representation yields a different set of equations, and it can also recover the conservation and Navier-Stokes equations, at low Mach numbers, provided that the imaginary component of the macroscopic mass can be neglected. We briefly review the Constant Speed Kinetic Model (CSKM), which was introduced in Zadehgozl and Ashrafizaadeh [J. Comp. Phys. 274, 803 (2014)] and Zadehgozl [Phys. Rev. E 91, 063311 (2015)]. The CSKM is then used as a basis to show that the complex-valued equilibrium distribution function of the present model can be identified with a simple singularity in the complex phase space. The virtual particles, in the present work, are concentrated on virtual “branes” which surround the computational nodes. Employing the Cauchy integral formula, it is shown that certain variations of the “branes,” in the complex phase space, do not affect the local kinetic states. This property of the new model, which is referred to as the “apparent jumps” in the present work, is used to construct new models. The theoretical findings have been tested by simulating three benchmark flows. The results of the present simulations are in excellent agreement with the previous results reported by others.

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I. INTRODUCTION

The kinetics-based models have widely been accepted as useful tools in computational fluid dynamics (CFD) and as alternatives to the continuum-based models. The kinetics and continuum-based models, which rely on the Lagrangian and Eulerian points of view, respectively, can be used for fluid flow simulations, using the Boltzmann and Navier-Stokes equations.

The Boltzmann equation (BE) provides an accurate statistical description of the rarefied gas, under certain conditions. For example, all collisions should be binary and the number of molecules per volume should be high enough to allow for an accurate statistical description of the gas. Therefore, the BE is accurate only at a certain range of Knudsen numbers. Using the Chapman-Enskog (CE) analysis, it can be shown that the first- and second-order approximations of the BE result in the Euler and Navier-Stokes equations, respectively [1].

Solving the Boltzmann equation is difficult, mainly due to the complicated structure of the collision kernel. To overcome this difficulty, several simplifications have been proposed. For example, in the lattice Boltzmann model (LBM), the virtual particles can collide and stream on a regular lattice. These simplifications, can introduce new invariants with no real counterparts, as a result of which spurious (nonphysical) effects may occur. For example, the conventional LBM with polynomial equilibrium distribution function cannot allow for an H-theorem [2]. Hence, they are not entropic and they cannot satisfy the second law of thermodynamics.

The LBM was originally introduced [3] as an extension of the lattice gas cellular automata (LGCA) [4]. Later, it was shown [5] that it can directly be derived from the continuous Boltzmann equation (CBE). The pioneer works on the LGCA and LBM can be found in Refs. [3–7].

Constant speed kinetic models have been proposed and studied (e.g., in Refs. [5,8]) in the past. Qu et al. [9] proposed a fixed speed lattice Boltzmann model where the equilibrium probability density function is referred to as the “circular function” by the authors and it has a simple fractional form. The virtual particles, in their model, are concentrated on the collision circles centered at \((\bar{u}_x, \bar{u}_y)\) with reference to the computational nodes, where \(u_x\) and \(u_y\) are the components of the macroscopic velocity of the fluid. For the streaming stage, in the “circular model,” the Lagrange multiplier technique is used to discretize the circular function. Yang et al. [10] showed that the circular function can be derived from the Maxwellian. Wang et al. [11] extended the model of Qu et al. [9] by replacing the “circular function” with a new equilibrium distribution function which they referred to as the “polynomial kernel function.” They showed that their model can be used to simulate compressible and viscous flows with flexible specific heat ratios and Prandtl numbers. Zadehgozl and Ashrafizaadeh [12] and Zadehgozl [13] proposed an entropic constant speed kinetic model (CSKM), in which the virtual particles are concentrated on collision circles centered at the computational nodes. Moreover, the equilibrium distribution functions, in the CSKM, can be identified with the Poisson kernel of the Poisson integral formula [13], and the model is based on the unconventional entropies of Burg (for two dimensions (2D)) and Tsallis (for nD with \(n \geq 3\)).

Wagner [2] proved that the conventional LBM with polynomial equilibrium distribution function (EDF) cannot allow for an H-theorem. To overcome this deficiency, the entropic lattice Boltzmann model (ELBM) [14–20] has been proposed, in which the discrete entropy is given by \(H = \sum f_i \log (f_i/w_i)\), while the classical Boltzmann-Gibbs entropy is given by \(H = \sum f_i \log f_i\). Boghosian et al. [21] used the Burg entropy of \(H = \sum \log f_i\) (for 2D cases) and Tsallis entropy of \(H = \sum f_i^{1-\frac{1}{\alpha}}\) (for nD cases with \(n \geq 3\)) to construct a new class of lattice Boltzmann models. Zadehgozl and Ashrafizaadeh [12] and Zadehgozl [13] extended the work of Boghosian et al. [21], in the limit of fix speed continuous velocities, and showed that the equilibrium distribution function, in the extended model,