Three-dimensional numerical investigation of a single bubble behavior against non-linear forced vibration in a microgravity environment

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ABSTRACT

A single gas bubble behavior under non-linear oscillating excitation of the liquid container is studied numerically. The volume of fluid (VOF) method is used for the interface tracking between gas and liquid, implemented in the open source OpenFOAM CFD toolbox. The surface tension force at the interface is evaluated using the continuum surface force (CSF) model. Bubble behavior under forced vibration is examined by considering translational motion and shape deformation of the bubble, while the volume oscillations are ignored regarding the applied frequency range (f<250Hz). Bubble response to the forced vibration is studied in three types of regular oscillation, chaotic oscillation, and bubble breakup. It is shown that the inertia of the surrounding liquid and the bubble shape at the beginning of each period have the main effects on the bubble breakup. In order to survey the chaotic response, bubble shape is decomposed into a linear sum of the second to ninth shape modes using Legendre polynomials. The results obtained from the modal analysis of the bubble shape oscillation reveal that the bubble response is regular at frequencies below the second mode natural frequency of the bubble. On the other hand, bubble shape oscillation is chaotic at frequencies close to the third mode natural frequency, which in turn causes the erratic motion of the bubble in the liquid. Finally, the bubble responses to a wide range of frequencies and amplitudes are presented in a diagram based on the Bond number (the ratio of the vibration force to the surface tension force, Bo) and the ratio of the vibration amplitude to the bubble diameter (A/D).

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1. Introduction

Mechanism of bubble/droplet oscillation and its effects on the flow field features have always been of interest to researchers as a challenging subject among different aspects of gas-liquid two-phase flows. Several studies have been carried out in this area, such as bubble/droplet oscillation impact on heat and mass transfer (Rose and Kintner, 1966; Nigmatulin et al., 1981; Kim et al., 2004; Hayashi et al., 2014; Xie et al., 2015), bubble column reactor performance (Ellenberger et al., 2003; Waghmare et al., 2009; Budzyński et al., 2017), bubble condensation (Tang et al., 2016), and droplet evaporation rate (Sanyal et al., 2014; Rahimzadeh and Eslamian, 2017). Applications of bubble oscillation in therapy and drug delivery are the more recent topics that have been studied in the past decade (Ferrara et al., 2007; Wu and Nyborg, 2008; Wiedemair et al., 2012; Kooiman et al., 2014).

Bubble oscillations may be free or forced. In free oscillations, bubble deviation from its equilibrium shape acts as the exciting operator, while in forced oscillations there is always an external force that can be caused by an acoustic field or applied vibrations. Since the experimental investigation of the bubble dynamic under forced vibration requires a reduced gravity environment, most of the studies belong to the application of acoustic fields. The acoustic wave applies an oscillatory pressure distribution to the flow field which results in a force known as primary Bjerknes force (Crum, 1975). Based on the pressure oscillation amplitude, acoustic levitation can lead to an instability in bubble shape and also translational motion, which is called the erratic dancing motion of the bubble (Eller and Crum, 1970). Theoretical studies of the bubble center of mass velocity and shape deformation revealed that the erratic motion occurs as a result of excitement and interaction of two adjacent bubble shape modes (Benjamin and Ellis, 1990; Mei and Zhou, 1991; Feng and Leal, 1995).

Without any restriction on oscillation modes and natural frequency of the bubble, Doinikov (2004) presented a set of coupled equations governing all three types of bubble oscillations, i.e., volume oscillation, shape oscillation, and translational motion, using
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(V)</td>
<td>bubble volume, m³</td>
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<td>(A)</td>
<td>vibration amplitude, m</td>
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<tr>
<td>(a_c)</td>
<td>container acceleration, m/s²</td>
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<td>(a_n)</td>
<td>nth mode shape amplitude, m</td>
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<td>(B_0)</td>
<td>Bond number, dimensionless</td>
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<td>(C_0)</td>
<td>Courant number, dimensionless</td>
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<td>(f)</td>
<td>vibration frequency, Hz</td>
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<td>surface tension force per unit volume, N/m³</td>
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<tr>
<td>(x_c)</td>
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<td>(Y)</td>
<td>bubble center position in vertical direction, m</td>
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<td>(Z)</td>
<td>global coordinate</td>
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Greek symbols

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<td>(\alpha)</td>
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<td>(\tau)</td>
<td>shear stress, N/m²</td>
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<tr>
<td>(\Delta t_{\text{max}})</td>
<td>maximum allowed time step, s</td>
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Subscripts

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<tr>
<td>(b)</td>
<td>bubble</td>
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<td>(c)</td>
<td>container</td>
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<tr>
<td>(g)</td>
<td>gas phase</td>
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<td>(l)</td>
<td>liquid phase</td>
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A perturbation analysis. The results obtained from numerical solution of the presented equations showed that the odd modes of the bubble shape oscillation can excite all other oscillation modes, including the translational motion and the volume mode. While the even modes do not have this ability and can only excite the other even shape modes and the volume oscillation of the bubble.

If the imposed force is strong enough, the shape deformation increases and eventually, the bubble may collapse. Experimental observations and numerical predictions based on the boundary integral method (BIM) revealed that the key factor in the collapse of a non-spherical bubble is the liquid jet development that penetrates into the bubble core and results in a toroidal shape (Crum, 1979; Blake et al., 1999; Calvisi et al., 2007).

As mentioned earlier, in order to study the bubble dynamics under forced vibration of the container, it is necessary to have a reduced gravity environment without buoyancy effects. In this regard, a series of experiments known as interfacial stability and capillary wave (ISCAP) were performed on the microgravity vibration isolation mount (MIM) during STS-85 mission aboard the Space Shuttle in 1997; in order to investigate the translational motion of an air bubble under low-amplitude vibrations (Kawaiji et al., 1999). Results showed that exertion of a low-frequency vibration (\(\sim 2.5\) Hz) to the liquid container induces oscillatory motion of the bubble at the same frequency as the forced vibration (Kawaiji et al., 1999; Hong and Kawaiji, 2003). Using another series of experiments in microgravity, Farris et al. (2004) examined the motion of a spherical bubble under forced vibrations at frequencies less than \(10\) Hz. Bubble frequency response from experimental data showed that the ratio of the bubble oscillation amplitude to the container displacement does not depend on the Reynolds number and amplitude of the container, while it is a strong function of the forced vibration frequency.

Bubble deformation under forced vibration was not considered in the studies mentioned so far. Imposed acceleration due to the forced vibration has a significant role in the bubble deformation. The shape deformation increases with rising the imposed acceleration on the container. In a large enough acceleration, bubble shape changes into a toroidal shape, which may lead to bubble breakup (Zoueshtagh et al., 2006). Experimental observations of Yoshikawa et al. (2010) in normal and reduced gravity environments revealed that the bubble breakup occurs at a constant acceleration of the container, which is known as critical acceleration. The critical acceleration in the normal gravity was larger than in the reduced gravity condition. Besides, high inertia of the surrounding liquid was determined as the dominant mechanism of the breakup process. The authors noted that the bubble breakup does not occur at a constant acceleration in high-frequency vibrations (\(> 20\) Hz), and the bubble behavior was not examined in this frequency range.

Due to the limitations of experimental techniques, a number of computational fluid dynamics (CFD) methods have been provided to study multiphase flows. The most commonly used techniques for investigation of two-phase flows are Volume of Fluid (VOF) methods (Hirt and Nichols, 1981; Rudman, 1997; Rider and Kothe, 1998; Bussmann et al., 1999), front tracking techniques (Unverdi and Tryggvason, 1992; Tryggvason et al., 2001), Level Set methods (Sussman et al., 1994, 1999) and the Lattice Boltzmann Method (LBM) (Sankaranarayanan et al., 2002; Cheng et al., 2010).

In a numerical research aiming at understanding the bubble dynamic under forced vibration, Friesen et al. (2002) examined the results of ISCAP experiments. They investigated low-frequency bubble oscillations using Level Set and VOF methods and clarified that both numerical results are consistent with the experimental data. Using the VOF method, Movassat et al. (2012) stated that by increasing the forced vibration amplitude and subsequently the bubble shape deformation, the bubble response becomes nonlinear as a result of the interaction between translational motion and shape deformation. Another numerical study was recently carried out by Movassat et al. (2015) using a Level Set method. The authors presented a map for dynamic response of a \(4\) mm air bubble in water under different vibrational conditions, and it was claimed that the chaotic motion of the bubble is due to its large shape deformation. However, there was no comment on how the shape oscillation becomes chaotic, and how it affects the translational motion of the bubble.

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In the present work, a single bubble behavior under forced vibration is investigated numerically, focusing on the shape deformation and translational motion of the bubble. Given that the frequency range used in this study (<250Hz) is much smaller than the Minnaert resonant frequency (Martin et al., 2008) of a 5 mm air bubble immersed in water (about 1.3 kHz), which is surveyed here, the volume oscillations are not considered. All simulations are carried out in three dimensions, while an improved VOF method is implemented to track the interface. Different types of the bubble response to forced vibrations are described. To study the coupling between translational motion and shape deformation of the bubble, which was inadequately studied in previous works, the bubble shape during oscillations is decomposed into a linear sum of shape modes. At the end, a parametric study is also performed to determine the bubble response to a wide range of oscillatory conditions.

2. Numerical method

2.1. Mathematical model

Gas and liquid are considered as two immiscible, incompressible, and isothermal fluids with constant physical properties. The Volume of Fluid (VOF) method is used as the interface tracking approach to advance the interface in a fixed Eulerian mesh. In the VOF method, which was first introduced by Hirt and Nichols (1981) for incompressible fluids, a scalar function $\alpha$ is defined indicating the volume fraction of each phase in computational cells:

$$\alpha(x,t) = \begin{cases} 
1 & x \text{ is liquid} \\
0 < \alpha < 1 & x \text{ is interface} \\
0 & x \text{ is gas} 
\end{cases}$$ (1)

In the current VOF method, the transport equation of the scalar $\alpha$ is expressed as follows:

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) = 0,$$ (2)

where $\mathbf{U}$ is the velocity vector and $t$ is time. Two different fluids are treated as one effective fluid which its physical properties are calculated as a weighted average with respect to the volume fraction of each phase in computational cells:

$$\rho = \rho_{\alpha} \alpha + \rho_{g} (1 - \alpha).$$ (3)

$$\mu = \mu_{\alpha} \alpha + \mu_{g} (1 - \alpha).$$ (4)

where $\rho$ is the fluid density, $\mu$ is the fluid dynamic viscosity, and the subscripts $l$ and $g$ denote the liquid and gas phases, respectively. Accordingly, an inaccurate calculation of the volume fraction changes the computed physical properties of the fluid throughout the domain. In addition, calculations of the interface and surface tension force between two phases are performed based on the volume fraction. Since the interface curvature significantly affects the shape deformation calculation, an improved model is used for the volume fraction solution (Rusche, 2002). In this method, the effective fluid velocity is calculated as the weighted average of each phase velocity, and an additional term is introduced into the volume fraction transport equation,

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) + \nabla \cdot [\mathbf{U} \alpha (1 - \alpha)]= 0,$$ (5)

where $\mathbf{U}_{r} = \mathbf{U}_{l} - \mathbf{U}_{g}$ is the relative velocity between the phases, which is also called the compression velocity. The last term on the left hand side of Eq. (5) is the added expression called the artificial compression term, which acts only in the interface region and vanishes at upper and lower limits of $\alpha$, i.e. in the liquid and gas phases. The additional compression term results in a more sharpened interface area between the two phases.

In the VOF method, the $\alpha$ transport equation is solved simultaneously with governing equations, i.e. continuity and momentum:

$$\nabla \cdot \mathbf{U} = 0,$$ (6)

$$\rho \frac{\partial \mathbf{U}}{\partial t} = -\nabla p + \nabla \cdot \tau + \mathbf{F}_{\alpha} + \mathbf{F}_{g},$$ (7)

where $p$ is the pressure and $\tau$ is the shear stress tensor which is calculated as $\tau = \mu \left( \nabla \mathbf{U} + (\nabla \mathbf{U})^{T} \right)$ for Newtonian fluids, where the superscript $T$ stands for transpose. $\mathbf{F}_{\alpha}$ and $\mathbf{F}_{g}$ represent the surface tension force and body forces per unit volume. The surface tension force at the interface is evaluated per unit volume with a constant surface tension using the continuum surface force (CSF) method proposed by Brackbill et al. (1992):

$$\mathbf{F}_{\alpha} = \sigma \kappa \nabla \alpha,$$ (8)

where $\sigma$ is the surface tension and $\kappa = -\nabla \cdot (\nabla \alpha / |\nabla \alpha|)$ is the interface curvature.

By applying vibration, the whole system, including liquid and immersed bubble, moves along with the container. The momentum equation in the moving reference frame attached to the liquid container moving with $\mathbf{U}_{c}$ velocity is outlined as:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} - \mathbf{U}_{c}) \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \tau + \frac{1}{\rho} \mathbf{F}_{\alpha} + \frac{1}{\rho} \mathbf{F}_{g} + \frac{\partial \mathbf{U}_{c}}{\partial t},$$ (9)

Since the container velocity is constant in the entire domain ($\nabla \mathbf{U}_{c} = 0$), the momentum equation is expressed as follows using the relative velocity $\mathbf{U}' = \mathbf{U} - \mathbf{U}_{c}$.

$$\frac{\partial \mathbf{U}'}{\partial t} + \mathbf{U}' \cdot \nabla \mathbf{U}' = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \tau + \frac{1}{\rho} \mathbf{F}_{\alpha} + \frac{1}{\rho} \mathbf{F}_{g} + \frac{\partial \mathbf{U}_{c}}{\partial t}. $$ (10)

where the last term on the right hand side of Eq. (10) represents the imposed acceleration. By applying a non-linear cosine vibration, the displacement, velocity, and acceleration of the container are written as:

$$x_{c} = A \cos(2\pi ft).$$ (11)

$$u_{c} = -A(2\pi f)\sin(2\pi ft).$$ (12)

$$a_{c} = -A(2\pi f)^{2}\cos(2\pi ft).$$ (13)

where $A$ and $f$ represent the amplitude and frequency of the forced vibration, respectively. Using Eq. (13) as the imposed acceleration, and the fact that there is no body force without gravity ($\mathbf{F}_{g} = 0$), the final momentum equation is written as:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \tau + \frac{1}{\rho} \mathbf{F}_{\alpha} + A(2\pi f)^{2}\cos(2\pi ft).$$ (14)

Note that ' $t$ ' is omitted for simplicity, while $\mathbf{U}$ represents the velocity vector in the moving reference frame attached to the container. Since the forced vibration is applied in one direction, the amplitude vector has only one component which is depicted as $A$ in the rest of this paper.

2.1.1. Non-dimensional equations

Effective dimensionless quantities in the bubble dynamic under forced vibration can be identified using non-dimensional form of the governing equations. Using the imposed force period ($T$) as the time scale, bubble diameter ($D$) as the characteristic length and
physical properties of the surrounding liquid as the reference properties, dimensionless variables are defined as:

\[ t^* = \frac{t}{T}, \quad U^* = \frac{U}{U_0}, \quad p^* = \frac{Dp}{\sigma}, \quad \kappa^* = \kappa D. \]

Using these quantities, non-dimensional form of the conservation of mass and momentum equations are written as:

\[ \nabla \cdot U^* = 0, \quad (15) \]

\[ \frac{\partial U^*}{\partial t^*} + U^* \cdot \nabla U^* = - \frac{A}{D \beta_0} \nabla p^* + \frac{A}{D \alpha \beta_0} \nabla \cdot \tau^* + \frac{A}{D \beta_0} \mathbf{F} = 4\pi^2 \frac{A}{D} \cos(2\pi t^*). \quad (16) \]

As a result, the involved dimensionless numbers are \( A/D, \beta_0, \alpha \) and \( \beta_0 \). Non-dimensional amplitude, \( A/D \), is the ratio of the forced vibration amplitude to the bubble diameter. Bond number, \( \beta_0 = \rho(Af^2)d^2/\alpha \), is defined as the ratio of the induced oscillatory body force to the surface tension, which is an appropriate definition in the context of the present study. Reynolds number, \( Re = \rho(Af)/D\mu \), is the ratio of the inertia to the viscous force. Just as Movassat et al. (2015) reported, our simulation results also showed that viscous forces have little effects on the bubble behavior under forced vibration. Since the viscosity only appears in the Reynolds number in Eq. (16), all discussions in this study are presented based on \( \beta_0 \) and \( A/D \) non-dimensional numbers.

2.2. Computational details

Numerical computations are performed using open source CFD package OpenFOAM (Weller et al., 1998). The solution domain and governing equations terms are discretized based on the finite volume method (FVM) on a staggered grid. Gauss' theorem is used for converting volume integrals to surface integrals surrounding computational cells in discretization of the spatial derivatives. Since the values are stored on the cells centers, second order interpolation methods are used to compute the values on the cells faces, which are needed for calculation of the surface integrals. Gradient terms are interpolated using a central difference scheme. The vanLeer interpolation method (van Leer, 1974) is used for convection terms, with a flux limiter function as:

\[ \psi(r) = \frac{1 + |r|}{1 + |r|}. \quad (17) \]

where \( r \) is the ratio of consecutive gradients called the smoothness parameter (Darwish and Moukalled, 2003). The added compression term in Eq. (5) is discretized using the “interfaceCompression” method (Weller, 2008; Berberović et al., 2009).

Discretization of time derivatives is accomplished by an implicit Euler method. Moreover, an adaptive time step is used to ensure the stability of the numerical solution over the time. The Courant number of the solution is defined as:

\[ Co = \frac{|U| \delta t}{\delta x}, \quad (18) \]

where \( \delta t \) is the time step, \( |U| \) represents velocity magnitude and \( \delta x \) is the size of the computational cell along the velocity vector. The maximum local Courant number in the entire domain (Co\( ^\text{max} \)) is calculated by Eq. (18) and the new time step is determined using the expression:

\[ \Delta t^n = \min \left\{ \frac{Co_{\text{max}}}{Co}, \left( 1 + \frac{Co_{\text{max}}}{Co} \right) \Delta t^0, \lambda_2 \Delta t^0, \Delta t_{\text{max}} \right\}. \quad (19) \]

where \( Co_{\text{max}} \) and \( \Delta t_{\text{max}} \) are preset values for the maximum allowed Courant number and time step, respectively. A maximum Courant number of 0.2 is proposed by Berberović et al. (2009) for gas-liquid two-phase flows with a large deformation of the interface. In this study, the maximum Courant number is set to 0.1 to ensure the stability of the solution. Besides, the maximum allowed time step is assumed to be \( 10^{-3} \) s, while the appropriate time step based on Eq. (19) is much lower than this value (about \( 10^{-4} \) s) for all simulations. According to Eq. (19), if the calculated value of \( Co^\text{max} \) exceeds the maximum permissible value for Courant number (\( Co_{\text{max}} \)), the new time step will decrease and vice versa. The constants \( \lambda_1 = 0.1 \) and \( \lambda_2 = 1.2 \) are damping factors, which are used to prevent the time step fluctuations.

The solution procedure is treated based on the Pressure Implicit with Splitting of Operators (PISO) iterative segregated algorithm (Issa, 1986) for pressure-velocity coupling. A summary of the solution steps is as follows:

1. Initialize the problem variables including velocity, pressure and also volume fraction.
2. Solve the advection of volume fraction and update the physical properties throughout the domain.
3. Calculate the interface curvature using the current volume fraction from the previous step.
4. Start the PISO loop to achieve a specified tolerance for the pressure-velocity coupling.
5. Determine the new time step based on the Courant number of the solution, and return to the second step (advance in time).

3. Computational domain and grid resolution

The liquid container is considered as a 12 mm × 20 mm × 12 mm cell in \( x, y \) and \( z \) directions, respectively; while a \( D = 5 \) mm spherical bubble is located at the center of the container at the initial state. It is true that the domain size is relatively small compared to the bubble diameter and wall effects may interfere with bubble behavior, but simulation results revealed that we can still investigate phenomena such as bubble breakup with a reasonable accuracy using the mentioned container size. Water and air properties at 20°C are considered as the liquid and gas phases properties. Computational domain with vibration direction are shown in Fig. 1. The origin of the coordinate system is located at the center of the container, which coincides with the bubble center position. Boundary conditions are the same on all sides of the container. Viscous boundary condition is set to a fixed value of \( U = 0 \) on the walls, for satisfying no-slip criterion. A fixed flux condition is used...
Fig. 2. Uniform grid at middle plane of the domain.

Fig. 3. Comparison of bubble center of mass vertical position between the grids of 40, 60, and 80 CPD.

Fig. 4. Comparison of bubble surface area and breakup moment between the grids of 40, 60, and 80 CPD.

for pressure on the walls, known as “fixedFluxPressure” boundary condition in OpenFOAM, which adjusts pressure gradient according to the calculated velocity field. A zero normal gradient condition is also applied to the volume fraction function on the walls, which means a constant contact angle of 90°.

As is shown in Fig. 2 for the middle plane of the domain, a uniform grid is used for 3D simulations in this article. Due to the symmetry of the problem to xy and yz planes, one quarter of the domain is modeled using the symmetry plane boundary condition. Three different meshes with 40, 60 and 80 cells per bubble diameter (CPD) in each direction are considered for a grid dependency test. Bubble behavior is investigated by applying a vibration with frequency of \( f = 54 \) Hz and amplitude of \( A = 0.5 \) mm, which results in a case with \( B_0 = 0.5 \) and \( A/D = 0.1 \).

Variations of the bubble center position in vertical direction with time (non-dimensionalized by the bubble diameter and period of oscillation, respectively) for the grids of 40, 60 and 80 CPD reveal that there is a minor difference between the two lateral meshes (Fig. 3). Since the VOF technique is highly dependent on

the grid resolution and may result in an artificial bubble breakup (Hoang et al., 2013; Cano-Lozano et al., 2015), the breakup time is monitored in different grids to investigate to what extent the mesh size limitation may affect the bubble behavior, particularly the breakup phenomenon. Evolution of bubble surface area (S) for three different grid sizes of 40, 60 and 80 CPD is illustrated in Fig. 4. The point at which the bubble surface peaks represents the breakup moment. It can be observed from Fig. 4 that there is a negligible difference in the calculated breakup time between the cases of 60 CPD (\( t/T = 2.234 \)) and 80 CPD (\( t/T = 2.236 \)). Therefore, it seems that the bubble breakup can be modeled with a reasonable accuracy using 60 CPD grid. Furthermore, the bubble shape and velocity magnitudes for the grids of 60 and 80 CPD are compared in Fig. 5. It is perceived that the bubble shape and velocity field are the same for these two grids, while there is only an insignificant difference in the velocity magnitudes which is applicable for the purposes of the present work. Consequently, the grid with 60 cells per bubble diameter is used in this study, which results in a total of 5,400,000 cells for a \( 2.5D \times 4D \times 2.5D \) domain.

4. Results and discussion

4.1. Validation

Results of the interfacial stability and capillary wave (ISCAP) experiments (Kawaji et al., 1999) are used for validation of the present numerical method. A water-surfactant solution (\( \rho_f = 996 \) kg/m³, \( \mu_s = 8.54 \times 10^{-4} \) kg/ms) and air (\( \rho_g = 1.18 \) kg/m³, \( \mu_g = 1.86 \times 10^{-5} \) kg/ms) with a surface tension of \( \sigma = 0.022 \) N/m were used as the liquid and gas phases in the experiments. The schematic configuration of the experiments is illustrated in Fig. 6.

The experimental data revealed that the bubble oscillates at the same frequency as the imposed force for low-frequency (under 2.5 Hz) vibrations, which simulation results from the present study also confirm it very well. For instance, variation of the bubble center position in vertical direction for a vibration with \( A = 2 \) mm and \( f = 0.6 \) Hz is shown in Fig. 7. As it is observed, the bubble oscillates exactly at the same frequency as the container. Present simulation results for variation of the bubble translational amplitude (\( A_b \)) with container vibration amplitude (\( A \)) in a constant frequency of \( f = 0.6 \) Hz are compared with ISCAP experimental data.
Fig. 5. Comparison of bubble shape and velocity magnitude contours between the grids of 80 CPD (left) and 60 CPD (right).

Fig. 6. Schematic configuration of the ISCAP experiments. The bubble is sandwiched between front and rear walls of the container.

Fig. 7. Variation of bubble center position in vertical direction for an imposed vibration with $A = 2$ mm and $f = 0.6$ Hz. The bubble oscillates at the same frequency as the forced vibration.

Fig. 8. Bubble motion amplitude ($A_b$) based on the container vibration amplitude ($A$) in a constant frequency of $f = 0.6$ Hz. A comparison between present simulations, ISCAP experimental data (Kawaji et al., 1999), and numerical results of Movassat et al. (2012).

(Kawaji et al., 1999) and Movassat et al. (2012) numerical results in Fig. 8. Kawaji et al. (1999) showed that there is a linear relationship between $A_b$ and $A$, which the present numerical results confirm it well. In addition, simulation results for the amplitude of bubble motion reasonably agree with ISCAP experiments and predictions of Movassat et al. (2012). So, the current computational method models an oscillating bubble motion with a proper accuracy comparing to the past experimental and numerical researches.

Because of the small interface deformation in the ISCAP experiments, those data cannot be used for verification of simulations in high frequencies or amplitudes where the bubble shape deformation is quite large, especially the breakup phenomenon. For this reason, the experimental results provided by Yoshikawa et al. (2007) are implemented in order to check the validity of our numerical approach in modeling bubble breakup. Yoshikawa et al. (2007) studied a $V = 3$ cm$^3$ (V: spherical bubble volume) air bubble behavior immersed in a water rectangular container at 25°C, which is subjected to sinusoidal excitations.
In a set of experiments, the imposed frequency was increased gradually in a constant amplitude of the container in order to evaluate the critical frequency \( f_c \) where the bubble breakup is observed. Fig. 9 indicates the critical frequency of the bubble, normalized by its natural frequency \( f_n = \left(4\sigma/\pi\rho V\right)^{1/2} \), based on different container amplitudes, normalized using the bubble diameter. Present simulations results for the same cases, with an uncertainty of 0.05 Hz in calculated critical frequencies, are also shown in Fig. 9 along with the experimental data from Yoshikawa et al. (2007). As can be observed, the predicted values for critical frequencies are always lower than the measurements. This discrepancy may be caused by the fact that the microgravity quality in the experiments was in the order of 0.01 g \((g = 9.81 \text{ m/s}^2)\) (Yoshikawa et al., 2010), while we have used a free-gravity condition for our simulations. In this regard, Yoshikawa et al. (2007) stated that the frequency threshold for bubble breakup in normal gravity is always larger than in microgravity. Nevertheless, the maximum difference between calculated frequencies and experimental data is about 13\%, which is quite applicable and proves that the present numerical method performs well in the simulation of large interface deformations including bubble breakup.

4.2. Bubble responses to forced vibration

Bubble translational motion and shape deformation are examined as the two main parameters of studying bubble behavior under forced vibration. Variation of the bubble center of mass vertical position non-dimensionalized by the bubble initial diameter, \( Y/D \), is used to investigate the translational motion. In order to study the shape deformations, a dimensionless parameter known as sphericity is defined as \( S^* = S/\left(\pi D^2\right) \), where \( S \) represents the bubble surface area during oscillations. In addition to the bubble surface, sphericity is also a measure of bubble deformation from the spherical state. \( S^* = 1 \) belongs to a completely spherical shape, while deformation of the bubble results in an increase in \( S^* \). Bubble response to the forced vibrations is classified into three main categories of regular oscillation, chaotic response, and bubble breakup.

4.2.1. Regular response

In the regular response, the bubble oscillates repetitively after several early periods, with the same frequency of the container vibration. Bubble oscillation in a vibrating container with \( Bo = 0.2 \) and \( A/D = 0.1 \) is considered here for illustration of the regular response. Fig. 10(a) indicates variation of the bubble sphericity with time for the first 25 periods. The maximum surface area, which means the largest deviation from the spherical shape, occurs during the first period. The bubble experiences regular and stable shape oscillations after the first five periods. For studying the translational motion, variation of the bubble center position during the 25 periods is examined (Fig. 10(b)). Similar to the bubble shape, the bubble center also oscillates regularly after the fifth period. Besides, the bubble oscillation frequency is equal to the applied vibration frequency, which is in agreement with the definition of regular response in the literature (Kawaji et al., 1999; Friesen et al., 2002; Movassat et al., 2012).

4.2.2. Chaotic response

In a chaotic response, bubble oscillation period \( T_b \) is different from the applied vibration period \( T \). The chaotic oscillation period varies from twice the period of forced vibration to an infinite value, which leads to an aperiodic oscillation (Lauterborn and Kurz, 2010). Therefore, a non-repetitive pattern for bubble motion and shape deformation is observed during each period of vibration in a chaotic response. A case with vibration parameters of \( Bo = 0.4 \)
4.2.3. Bubble breakup

In order to analyze the bubble breakup, the behavior of a bubble under forced vibration with $Bo = 0.5$ and $A/D = 0.175$ is exam-

and $A/D = 0.05$ is considered here to study the bubble chaotic behavior.

Variation of the sphericity with time (Fig. 11(a)) demonstrates that the bubble experiences chaotic and non-repetitive shape oscillations. According to the bubble center position variation, as is shown in Fig. 11(b), the bubble translational motion also has an erratic pattern during the first 25 periods. Hence, it is concluded that the bubble response is chaotic for $Bo = 0.4$ and $A/D = 0.05$.

Fig. 11. Variations of (a) bubble sphericity and (b) bubble center position in vertical direction, for oscillation with $Bo = 0.4$ and $A/D = 0.05$.

Fig. 12. Bubble shapes, velocity magnitudes and vectors for $Bo = 0.5$ and $A/D = 0.175$. 

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Period, posed to period, which for Fig. 13. Bubble shape evolution during the first nine periods for $Bo = 0.4$ and $A/D = 0.1$.

Fig. 14. Variation of curvature radius for the bubble top surface during the first nine periods for $Bo = 0.4$ and $A/D = 0.1$.

The upward motion in the bottom part of the bubble and the liquid jet downward motion contribute to a reduction in core region thickness of the bubble. During the third quarter period, the liquid jet continues to penetrate into the bubble due to the high liquid inertia. Eventually, the liquid flow pierces the bubble and breakup occurs with detachment of tiny bubbles from the main bubble (Fig. 12(e)). The resulting toroidal bubble and separated tiny bubbles oscillate in the liquid by continuing forced vibrations (Fig. 12(f)). Regarding the above description, the high inertia of the surrounding liquid is one of the most important factors causing the bubble to break up. Another case with oscillation parameters of $Bo = 0.4$ and $A/D = 0.1$ is considered here to identify the other significant factors involved in the breakup process. The bubble shapes in the first nine periods are shown in Fig. 13. The amplitude of the container vibration has decreased compared to the previous case, which delays the breakup to the ninth period. According to Fig. 14, which illustrates the evolution of curvature radius for the bubble top surface, it can be perceived that the curvature radius peaks at the start of ninth period. On the other hand, a large curvature radius decreases the bubble resistance against the shape deformation caused by the liquid inertia force. Hence, the liquid jet formation and its progress into the bubble are performed with the beginning of the ninth period.

Looking into the two previous cases reveals that in addition to the liquid inertia, the bubble shape, and more precisely the curvature radius of the bubble top surface at the beginning of each period plays an essential role in the formation of liquid jet and bubble breakup. As discussed, the bubble shape in a chaotic response does not repeat in time and experiences significant changes during each period; therefore, the bubble may break up during any period due to the random shape deformation in a chaotic response. Given these results, a specific boundary could not be explained between the chaotic response and bubble breakup. It should be noted that the first 10 periods of oscillations is considered in this study as a measure for determination of the bubble response.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f$ (Hz)</th>
<th>$A$ (mm)</th>
<th>$Bo$</th>
<th>$A/D$</th>
<th>Bubble response</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>34.18</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td>Regular</td>
</tr>
<tr>
<td>B</td>
<td>34.18</td>
<td>0.25</td>
<td>0.1</td>
<td>0.05</td>
<td>Regular</td>
</tr>
<tr>
<td>C</td>
<td>68.36</td>
<td>0.25</td>
<td>0.4</td>
<td>0.05</td>
<td>Chaotic</td>
</tr>
<tr>
<td>D</td>
<td>68.36</td>
<td>0.125</td>
<td>0.2</td>
<td>0.025</td>
<td>Chaotic</td>
</tr>
</tbody>
</table>

Table 1 Data used for modal analysis of the bubble oscillation.

Fig. 15. Comparison of reconstructed bubble shapes (left) with those simulated (right) at different times for case A with $Bo = 0.2$ and $A/D = 0.1$. (Fig. 12(d)).

4.3. Modal analysis of bubble oscillation

Modal analysis of bubble oscillation is performed here in order to characterize the bubble behavior under forced vibrations. As previously mentioned, translational oscillation is investigated by monitoring the variation of bubble center of mass location with
time. For shape oscillations, Legendre polynomials can be used to decompose the resulting bubble shape into different shape modes. In this method, which was first proposed by Rayleigh (1879) in order to evaluate a drop oscillation around its spherical shape, bubble shape is decomposed into a linear sum of second to ninth
shape modes using the following expression:

\[ r(\theta, t) = \frac{D}{2} - \sum_{n=2}^{9} a_n(t)P_n(\cos \theta) = 0. \]  \hspace{1cm} (20)

where \( r(\theta, t) \) function is the distance from points on the bubble surface to the bubble center in polar coordinates, which is derived from simulations. \( D \) is the bubble initial diameter, \( a_n(t) \) represents the \( n \)th shape mode amplitude and \( P_n(\cos \theta) \) is the \( n \)th order Legendre polynomial.

Variations of \( a_2 \) to \( a_9 \) with time represent the oscillation of bubble shape amplitudes in each mode, which can be used to evaluate the shape oscillations. Natural frequencies of the bubble surface oscillations are calculated according to the following expression which was proposed by Lamb (1932) for a gas bubble immersed in the liquid:

\[ (2\pi f_n)^2 = (n + 1)(n - 1)(n + 2) \frac{\sigma}{\rho R^3}. \]  \hspace{1cm} (21)

where \( f_n \) represents the \( n \)th mode natural frequency of the bubble, and \( R \) is bubble radius. For a \( D = 5 \) mm bubble with water and air properties at 20°C, the second, third, and fourth mode natural frequencies are calculated as \( f_2 = 37.7 \text{ Hz} \), \( f_3 = 68.8 \text{ Hz} \), and \( f_4 = 103.2 \text{ Hz} \).

In this section, four cases (A–D) with different frequencies and amplitudes are surveyed in order to analyze bubble oscillation in terms of shape modes. A summary of the properties and results for cases A to D is given in Table 1.
The first case is considered here for modal analysis (case A) is the same as the investigated regular case, with $Bo = 0.2$ and $A/D = 0.1$. The imposed vibration amplitude is 0.5 mm and the frequency is 34.18 Hz, which is somewhat smaller than the second mode natural frequency of the bubble. As discussed in Section 4.2.1, both translational (Fig. 10(b)) and shape (Fig. 10(a)) oscillations of the bubble are regular. For modal analysis of the shape oscillations, amplitudes of the second to ninth shape modes are computed using Eq. (20) and the bubble shape is reconstructed from the linear sum of different modes. Slight differences between reconstructed and simulated bubble shapes, as can be seen from Fig. 15, confirm that the present approach performs with a reasonable accuracy in the bubble shape reconstruction.

Bubble oscillation with the same frequency of case A, $f = 34.18$ Hz, is studied as case B, while the amplitude of forced vibrations, $A = 0.25$ mm, is reduced by half. Using these values results in dimensionless numbers as $Bo = 0.1$ and $A/D = 0.05$. Variations of the sphericity (Fig. 16(a)) and bubble center position (Fig. 16(b)) indicate that the shape deformation and translational motion of the bubble reach to stable oscillations after several early periods. So, the bubble response is regular in this case.

The imposed frequency and amplitude of the case C are $f = 68.36$ Hz and $A = 0.25$ mm, with $Bo = 0.4$ and $A/D = 0.05$, which is the same as the reviewed chaotic case. The applied frequency is approximately equal to the third mode natural frequency of the bubble shape oscillations. As observed in Section 4.2.2, the bubble experiences chaotic oscillations in both translational motion (Fig. 11(b)) and shape deformations (Fig. 11(a)). The reconstructed bubble shapes with those simulated are shown together in Fig. 17. As can be seen, the bubble shape does not repeat in time during the oscillations.

Forced vibrations with $Bo = 0.2$ and $A/D = 0.025$ is the last case (D) for modal analysis of the bubble oscillations. Similar to case C, the imposed frequency is $f = 68.36$ Hz, which is close to the third mode natural frequency. The forced vibration amplitude for this case is $A = 0.125$ mm, which is lower than the previous studied cases, so the bubble shape deviation is much smaller compared to the other ones. The shape deformation and translational motion of the bubble for this case in the first 20 periods are chaotic as can be perceived from Fig. 18.

The imposed frequency of cases A and B is close to the second mode natural frequency of the bubble, while the vibration amplitude of case A is twice as large as case B. As discussed, both translational motion and shape oscillation of the bubble in these cases are regular. Calculated amplitudes of the second and third shape modes are shown in Figs. 19 and 20 for cases A and B, respectively. It should be noted that variations of higher modes amplitudes are negligible compared to the second and third ones, so they are not

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shown in the figures. The results reveal that the second and third shape modes amplitudes oscillate regularly after several early periods for both cases A and B. It can be understood that all shape amplitudes oscillate regularly when the forced vibration frequency is close to the second mode natural frequency of the bubble shape oscillation and the second mode acts as the dominant mode. As a result, the shape deformation is also regular which means that the bubble shape is repeated at the end of each period, causing the same pressure distribution around the bubble. Eventually, the bubble translational motion is also regular due to the repeated pressure distribution in the entire domain.

On the other hand, both translational motion and shape deformation of the bubble are chaotic in case C, which the imposed frequency is close to the third mode natural frequency of the bubble. With decreasing the amplitude of forced vibration in case D, while the frequency is kept constant, the bubble shape deviation from spherical state also decreases; however, the bubble oscillation remains chaotic. As is shown in Figs. 21 and 22, the second and third shape mode amplitudes of cases C and D experience random oscillations. The resulting chaotic shape deformation of the bubble causes an irregular pressure pattern in the flow field, which in turn causes the bubble erratic motion. Therefore, in cases C and D with an imposed frequency close to the third mode natural frequency of the bubble shape oscillations, the second shape mode is excited by the third one and the interaction between these two consecutive modes leads to the excitation of the first mode, which relates to the translational motion. While the second shape mode does not have this capability and cannot excite the other oscillation modes, as observed in cases A and B.

Regarding the given description, the amplitude of forced vibration does not play a significant role in the bubble chaotic response. In summary, chaotic shape deformations are observed when the imposed force frequency is close to the third mode natural frequency of the bubble surface oscillation, which in turn makes the translational motion chaotic. This conclusion is analogous to the results obtained by Doinikov (2004) for a bubble dynamic in an acoustic field, who stated that the odd surface modes play an important role in the behavior of an oscillating bubble. Doinikov (2004) showed that the third surface mode can excite the second one, and due to the non-linear second order interaction between two adjacent modes, the bubble translational motion is also excited, while the even surface modes, including the second mode, do not have this feature.

4.4. Bubble response diagram

In order to characterize the bubble response to different vibrational conditions, a parametric study is carried out on a wide range of amplitudes and frequencies. Considering three types of bubble dynamic response as regular oscillation, chaotic oscillation,
and bubble breakup, the results are presented in the diagram of Fig. 23 in terms of \( Bo \) and \( A/D \). The constant frequency lines in the diagram represent the second, third, and fourth mode natural frequencies of the bubble.

The green circles in Fig. 23, representing the regular response of the bubble, are observed in cases with a low \( Bo \) number (\( Bo < 0.3 \)) and in a broad range of amplitudes. As expected, in the region with frequencies below the second mode natural frequency, bubble oscillation is regular. While Movassat et al. (2015) reported some chaotic responses in this area by increasing container amplitude, it is observed that the amplitude does not have a noticeable influence on the chaos of bubble response. Rising the frequency in this region for high amplitude cases leads to the breakup, without any chaotic response, because of the large shape deformation. By increasing the frequency to the third mode natural frequency in low amplitude cases, the chaotic response is observed (the blue diamonds in Fig. 23). The bubble behavior at a fixed Bond number of 0.3 shows that the chaotic oscillation of the bubble changes to regular with an increase in the amplitude of vibrations. In fact, with rising the amplitude at a constant Bond number, the vibration frequency decreases according to the Bond number definition. Therefore, reducing the imposed frequency to below the second mode natural frequency causes the bubble response to change. Breakup and toroidal shape of the bubble is also indicated by the red triangles in Fig. 23. The breakup occurs in relatively high amplitudes and Bond numbers, in which the imposed force on the container is sufficient for large deformations of the bubble and liquid jet development.

5. Conclusions

Effects of non-linear forced vibrations on an air bubble behavior in a liquid container were studied numerically. All discussions were performed based on the dimensionless parameters of \( Bo \) and \( A/D \). Bubble behavior was investigated in terms of translational motion and shape oscillations. Bubble response to the forced vibration was studied in three categories of regular oscillation, chaotic oscillation, and bubble breakup. In the regular response, the bubble oscillates at the same frequency as the applied force. The bubble oscillations become stable after several initial periods so that the translational motion and shape deformation are repeated during each period. The regular response was observed in a wide range of amplitudes, but only at low frequencies of the imposed force.

In the chaotic response, the bubble experiences a completely erratic motion in the liquid, while the shape deformation is also unpredictable. Increasing the amplitude of the container motion causes a larger shape deformation. Due to the high inertia of the surrounding liquid, a high-speed liquid jet penetrates into the bubble core and makes the bubble to pierce. As a result, the breakup occurs by detachment of tiny bubbles from the main bubble. This phenomenon was observed in all applied amplitudes. The results showed that in addition to the liquid inertia, the bubble shape at the beginning of each period plays a key role in the breakup process.

Modal analysis of the bubble shape oscillation was performed in order to study the relationship between bubble response and forced vibration parameters. The analyses revealed that for the imposed frequencies below the second mode natural frequency of the bubble shape oscillation, the second and third shape mode amplitudes oscillate regularly. Hence, the translational motion and shape oscillation of the bubble are also regular. Raising the imposed force frequency to the third mode natural frequency makes the second and third shape mode amplitudes oscillate erratically. Due to the contribution of these two modes, the shape oscillation and subsequently the translational motion of the bubble also become chaotic. As a result, the increment of applied frequency to the third mode natural frequency is the main factor of the bubble chaotic response, while the forced vibration amplitude does not have a significant impact.

A parametric study was also conducted to determine the bubble response in various vibrational conditions. The results were presented in a diagram based on \( Bo \) and \( A/D \), taking into account the effects of bubble natural frequencies. The Bubble oscillates regularly at frequencies below the second mode natural frequency, while the breakup is also observed in large amplitude cases. The bubble response is chaotic at frequencies higher than the second mode natural frequency with a relatively small amplitude, while the bubble breakup occurs by increasing the vibration amplitude.

References


