A geostatistical investigation of 3D magnetic inversion results using multi-Gaussian kriging and sequential Gaussian co-simulation

Hemn Rahimi a, Omid Asghari b,*, Ahmad Afshar c

a Department of Mining Engineering, Faculty of Engineering, Urmia University, Urmia, Iran
b Simulation and Data Processing Laboratory, School of Mining Engineering, University College of Engineering, University of Tehran, Tehran, Iran
c School of Mining Engineering, University College of Engineering, University of Tehran, Tehran, Iran

A R T I C L E   I N F O
Article history:
Received 2 May 2017
Received in revised form 2 May 2018
Accepted 5 May 2018
Available online 7 May 2018

Keywords:
Inversion
Magnetic anomaly
Multi-Gaussian kriging (MGK)
Physical property distribution method (PPDM)
Sequential Gaussian co-simulation (SGCS)

A B S T R A C T
Inversion of magnetic data has a significant role in geophysical exploration and it can be utilized in the 3D (three-dimensional) modeling of subsurface bodies (e.g. geological structures, mineral deposits, etc.). In this study, Physical Property Distribution Method (PPDM) was utilized for the inversion of magnetic data and the reliability of the resultant model was later examined by means of the geostatistical methods. For magnetic anomalies, PPDM first divides the surface of interest into a large number of cells with constant dimensions and then calculates the susceptibility value for each cell. On the other hand, geostatistical methods can calculate the ore grade value at different depths of a mineral deposit. In this study, we have used Multi-Gaussian Kriging (MGK) method for estimation of iron grade (Fe %) in Darreh-Ziarat Iron Deposit. MGK is a powerful non-linear estimation method that first provides normal distribution of grade values using the plot variogram of these new values and then makes use of one of the linear estimation approaches to prepare the ore grade distribution of each block. Also, Sequential Gaussian Co-Simulation (SGCS), which is based on the normal distribution of the variables of interest, can simulate iron grade value of each block using a secondary value. Here, SGCS method was utilized as a simulation algorithm to determine the relationship between Fe and susceptibility variables. Considering the fact that magnetic susceptibility variables exist in all simulated blocks, SGCS results are highly dependent on inversion results. So, Iron grade values at different depths of Darreh-Ziarat Iron Ore deposit were calculated using MGK and SGCS methods in order to assess the correctness of inversion results. Finally, the results revealed a high degree of validity at shallower depths and a lower degree of confidence at deeper depths.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction
Subsurface geological structures, depending on their nature, are characterized by variations on the physical parameters of the Earth (e.g. magnetic susceptibility and density). Therefore, estimation of these parameters through geophysical data inversion (magnetic and gravity data) can reveal the existence of these subsurface structures. Inversion algorithms are classified into two groups: parametric inversion method and Physical Property Distribution Methods (PPDMs (Namaki et al., 2011)). In the parametric method, it is assumed that the geophysical anomalies consist of homogeneous bodies with constant physical properties; and through inversion of an overdetermined problem, the parameters related to the geometry, location and the physical property will be estimated. This method demands reasonable hypotheses about the origin of the anomaly and its shape, and is applicable for the bodies with simple geometric shape, e.g. dyke and sphere (Namaki et al., 2011; Oldenburg and Pratt, 2007).

In PPDMs, the goal is to find a three-dimensional function that can model the physical property distribution. In fact, the volume under exploration area is divided into a large number of rectangular cells and with assumption that the value of the physical parameter (magnetic susceptibility in this study) at each cell is constant and its value has been calculated by means of inversion (Aster et al., 2011; Ellis and Oldenburg, 1994; Last and Kubik, 1983; Lelièvre and Oldenburg, 2009). Li and Oldenburg (1996) have presented an algorithm for inversion of magnetic data and have developed it for magnetic studies that formulated the problem in a practical and flexible way and capable of dealing with real data.

Geostatistical methods are considered powerful techniques for quantitative determination of uncertainty in ore grade in the field of mining engineering (Benndorf and Dimitrakopoulos, 2013; Pyrzcz and Deutsch, 2014). For this purpose, geostatistical estimation and simulation methods present themselves as appropriate methods. Generally, geostatistical estimation methods, based on kriging estimator, are classified to linear and non-linear methods. In linear geostatistics, the
variable under study is estimated using a parametric function, and the shape of the function has been defined in the initial conditions of the problem (Chilès and Delfiner, 2012). In these methods, the results are unrealistic from the practical point of view due to the smoothing of properties. Both over- and under-estimation is likely (Costa, 2003; Cressie and Johannesson, 2001). Among the widely accepted non-linear methods, disjunctive kriging (DK) and multi-Gaussian kriging (MGK) are suitable and do not involve smoothing effect (Afzal et al., 2015). MGK method, that is based on preparation of normal distribution from the variable under study, regardless of whether mean is known or unknown, is categorized to two cases of Ordinary Multi-Gaussian (oMK) and Simple Multi-Gaussian (sMK) (Emery, 2006a, 2008). This method can estimate the variable studied by making a conditional distribution at each estimated block, after determination of grade behavior at it.

Algorithms of geostatistical simulation had been developed to quantify the uncertainty at un-sampled locations (Chilès and Delfiner, 2012). These approaches rely on the interpretation of regional variables such as realization in the random spatial field and making multiple realizations of these fields that can reproduce changes in space and constitute alternative results of real variables (Safikhani et al., 2017). Many algorithms are used for geostatistical simulations but the commonest type is the sequential simulation. In this approach, nodes are simulated consecutively and they depend on the distributed nature of data used and are divided into various methods. Sequential Gaussian Simulation (SGS) has the most applicability but it is necessary to transform data distribution into standard Gaussian and therefore the simulation at each estimated block, after determination of grade behavior at it.

In this method, it is assumed that there are no remnant magnetization and demagnetization effect between cells. So the relationship between observed magnetic data and susceptibility distribution (model) is linear and can be written as (Li and Oldenburg, 1996; 2003; Pilkington, 1997).

\[ d = Gm. \]

Here \( m = [m_1, m_2, \ldots, m_d]^T \) is magnetic susceptibility vector of cells, \( d = [\Delta T_1, \Delta T_2, \ldots, \Delta T_M]^T \) is the total intensity of magnetic field measured at stations 1 to N and \( G \) is the sensitivity matrix or magnetic kernel.

Data misfit function is the first criterion for solving the inverse problem. The meaning of misfit is that the difference of measurement between the observed and calculated data that must be kept at the minimum. This criterion is defined for inversion of magnetic data as follows in Eq. (2) (Aster et al., 2011).

\[ \varphi_d = \sum_{i=1}^{N} \left( \frac{d_{i}^{\text{obs}} - d_{i}^{\text{cal}}}{\sigma_i} \right)^2 = W_d \parallel d^{\text{obs}} - d^{\text{cal}} \parallel^2. \]

\( d^{\text{obs}} \) and \( d^{\text{cal}} \) denote the observed data; whereas \( d^{\text{cal}} \) is the calculated data and \( \sigma_i \) is the standard deviation of \( i_{th} \) data. Assuming uncorrelated data, the standard deviation is on the main diameter of \( W_d \) matrix and in fact, takes a weight proportional to the inverse of their error value (Li and Oldenburg, 1996); while \( \parallel d^{\text{obs}} - d^{\text{cal}} \parallel^2 \) is the two-Euclidean norm of \( d^{\text{obs}} - d^{\text{cal}} \) vector and is equal to the sum of the square of its components.

For controlling the inverse problem, regularization methods with imposing restrictions to the solution can remove instabilities in solving the problem. For this purpose, a Multi-term objective function such as Eq. (3) can be put into practice for subsurface model estimation of magnetic susceptibility distribution (Belge et al., 2002).

\[ \varphi(m) = \frac{1}{\sigma^2} \parallel d - Gm \parallel^2 + \sum_{i=1}^{M} \sum_{j=1}^{K} \beta_i f_j \left( (L, m)_{ij} \right). \]

where \( f_j \) are regularization functions to exert intended conditions; \( \beta_i \) denote regularization parameters that show the importance of each parameters concentrating nearest to the data location. A depth-weighting function is essential to compensate for the natural decay of the kernel amplitudes, which actually decrease with depth (Cella and Fedi, 2012).

The performing minimum structure inversion will produce susceptibilities concentrating nearest to the data location. A depth-weighting function is introduced this function as:

\[ w(z) = (z + z_0)^{-\beta} / \beta, \]

Here \( z \) is the depth to the center of a cell, \( z_0 \) is adjustable parameter which is obtained through the fitting function of \( w(z) \), with the field produced at an observation point by a column of cells. \( \beta \) measures the field decay of the magnetic source and plays a critical role in estimating the depth of the causal body. In the magnetic case, Li and Oldenburg (1996) suggested that \( \beta = 3 \), because cubic cells act as a dipole source and the magnetic field decays by a cubed inverse distance. Cella and Fedi (2012) pointed out that the inversion does not depend on the shape of the cell. To select the appropriate value of \( \beta \), they suggested that \( \beta \) could be assigned using the structural index \( N \), this last estimated by analyzing the behavior of the field with known techniques such as Euler deconvolution.
Finally, the inversion by the PPDM became a general objective function (Eq. (5)) by means of defining the model objective function (Li and Oldenburg, 1996):

\[
\psi(m) = \phi_d + \beta \psi_m(m) = \left( d^{x_0} - d^x \right)^T W_d^T W_d \left( d^{x_0} - d^x \right) + \beta (m - m_0)^T W_m^T W_m (m - m_0),
\]

where \( W_m \) is cumulative weight matrix of the model and \( W_d \) is the cumulative weight matrix of the observed data.

2.2. Geostatistical technique

2.2.1. Variogram analysis

In geostatistics, the regional variable variographic studies are performed to find spatial structure. Variogram creates a mathematical model for spatial analysis that quantifies the spatial variability of random variables between two points (Antunes and Albuquerque, 2013). A variogram is a tool for determination of spatial correlation of a regional variable and provides important information about the geological processes, anisotropy and different constraints of forming processes (Marinoni, 2003). Accordingly, experimental variogram is calculated among separated data point. By fitting the empirical variogram into a theoretical model (e.g. spherical, exponential, Gaussian, cubic, gamma and stable model) variogram modeling obtained (Deutsch and Journel, 1998). In order to obtain the best variogram modeling, a good physical knowledge of the phenomenon under perusal is required. Variogram modeling is based on range and sill and any anisotropy of the spatial variability can be detected via calculating variogram in various directions. If the range of variogram modeling changes, the anisotropic type is geometric and if sill changes, it is called zonal anisotropy (Eriksson and Siska, 2000). Generally, in order to find the direction and dip of anisotropy, two or three final directions are required, which are called major, minor and vertical directions.

2.2.2. MGK and oMK

Multi-Gaussian models have more applications in geostatistical concepts of non-linear estimation and simulation (Chilès and Delfiner, 2012; Emery, 2007; Verly, 1983). In the MGK estimation method, a region variable under study is considered as a non-linear realization of a random field transformation, \( Y_\alpha \times \mathbb{R}^3 \), with a univariate Gaussian distribution and the mean and variance of \( Y_\alpha \) are usually set to 0 and 1 for converting to a standard Gaussian variable (Emery, 2006a). Row variable is transformed to the normal distribution as Eq. (6) (Emery, 2008).

\[
\forall \alpha \in \{1, \ldots, n\}, \quad y_\alpha = C^{-1} \left( \sum_{i=1}^{n-1} p_i + 1/2 \rho \right).
\]

where \( y_\alpha \) is the transformed value into Gaussian distribution, \( C^{-1} \) is the inverse of the standard Gaussian cumulative distribution function and \( p_\alpha \) is the classified weight amount of \( \alpha \)th variable. After distribution transformation, MGK is classified to ordinary multi-Gaussian kriging (oMK) and the simple multi-Gaussian kriging (sMK) depending on the use of ordinary kriging (OK) or simple kriging (SK) estimation methods. Due to using the oMK in this study, the only the oMK is described here. The oMK is an unbiased estimation of recovered functions that does not use the mean value of normal distribution (Emery, 2006b).

For conducting estimation process after providing normal distribution, experimental variogram must be plotted on new data and according to kriging estimation routine, final variogram modeling must be determined and then the OK method estimation and estimation variance will be calculated. Estimation distributions of the oMK and the OK methods for participation in the oMK equation can be obtained through Eqs. (7)–(10) (Emery, 2006b, 2006c).

\[
\begin{align*}
\forall \alpha = 1, \ldots, n, \quad & \sum_{i=1}^{n} \omega_i \mathbb{C}(V_\alpha, V_\alpha) + \mu = \mathbb{C}(V, V_\alpha), \\
& \sum_{i=1}^{n} \omega_i = 1
\end{align*}
\]

\[
\sigma^2_{\varphi(Y)}(V) = \sum_{i=1}^{n} \omega_i \varsigma^2(Y(U_i)) + \mu^2 - \mu^2
\]

where \( \varphi(Y) \) is the back-transformed value, \( \varsigma^2(Y(U_i)) \) is the variation value of the \( Y(U_i) \) and \( \mu \) is the sampled value. Each probability intervals will be achieved through the system of MGK equations. The covariance function is obtained from the input variogram model, \( C(h) = C(0) - \gamma(h) \). Where \( \gamma(h) \) is the specific value of the variogram between the two samples that are spaced at distance \( h \).

Finally, the OK methods for participation in the oMK equation can be obtained through Eqs. (7)–(10) (Emery, 2006b, 2006c).

\[
\begin{align*}
\forall \alpha = 1, \ldots, n, \quad & \sum_{i=1}^{n} \omega_i \mathbb{C}(V_\alpha, V_\alpha) + \mu = \mathbb{C}(V, V_\alpha), \\
& \sum_{i=1}^{n} \omega_i = 1
\end{align*}
\]

\[
\sigma^2_{\varphi(Y)}(V) = \sum_{i=1}^{n} \omega_i \varsigma^2(Y(U_i)) + \mu^2 - \mu^2
\]

\[
\begin{align*}
\forall y \in [y_{\text{min}}, y_{\text{max}}] \quad & \phi(y) = z_{\text{min}} + (z_0 - z_{\text{min}}) \exp(-\lambda_0y)\exp(\lambda y), \\
& \forall y \in [-y_{\text{max}}, -y_{\text{min}}] \quad \phi(y) = z_{\text{max}} + (z_0 - z_{\text{max}}) \exp(-\lambda_0y)\exp(-\lambda y)
\end{align*}
\]

where \( \phi(y) \) is the back-transformed value, \( z_{\text{min}} \) and \( z_{\text{max}} \) are the minimum and maximum quantities of variable studied; \( \lambda \) and \( \lambda \) are the respectively lower tail and upper tail of the value converted due to exponential distribution, and \( y \) is the initial value obtained by the oMK.

Given that the oMK results are just distribution, in addition to calculation of final estimated values: expected tonnage, expected metal content and expected mean grades above each cut-off can be calculated with different cut-off grades. Also, lower bounds and upper bounds of probability intervals will be achieved through the system of MGK equations. Eqs. (12) to (14) demonstrate the calculation method of the oMK local variance, expected tonnage and expected metal content above the cut-off of \( z \) (Emery, 2006a).
where \( \text{var}(\phi(Y_u)) = [\phi(Y_u)]^2 \) is local variance calculated with oMK method; \( \phi(Y)^T \) is the expected tonnage above cut-off \( z \); \( \phi(Y)^G \) is the expected metal content above cut-off \( z \) and \( \{\phi(Y(u_i)); z\} \) is an indicator function and is equal to 1 if \( Z > z \) and otherwise it is equal to 0.

2.2.3. SGCS

Stochastic simulation provides an approach for merging various types of uncertainty into a prediction of a complex system respond (Gotway and Rutherford, 1994). The SGCS is a sequential simulation method that makes use of Gaussian distribution space similar to the oMK and afterward utilize the back-transform process. For conducting the SGCS, datasets of the primary and secondary variables must be transformed to Gaussian distribution first (according to Eq. (6)). In the second stage, it is necessary to plot the variograms for both variables in Gaussian distribution and plot cross-variogram between variables. Subsequently, a random direction is selected in a manner that it passes all points of the estimated grid blocks, and using co-kriging estimation, a local distribution function is produced according to the quantity of estimated variance and average (estimated value (Emery and Peláez, 2011; GW Verly, 1993)). In the final stage, the quantity of simulated variable is back-transformed to main space. Upon completion of this process, the SGCS's realization is obtained. By avoiding drawing mistaken conclusions, SGCS process can be made on a set of realizations (Emery and Peláez, 2011).

In fact, in sequential simulation smoothing effect of kriging will be removed by adding random variable and effect it in another point. In Eqs. (15) and (16), \( Y(u) \) denotes the simulated value, \( Y \) is the expected metal content above cut-off \( z \) and \( \{Y(u_i); z\} \) is an indicator function and is equal to 1 if \( Z > z \) and otherwise it is equal to 0.

\[
Y_s(u) = Y(u) + R(u) \tag{15}
\]

\[
\text{Cov}[Y_s(u), Y(u_i)] = E[Y_s(u), Y(u_i)] - E[Y_s(u)]E[Y(u_i)] = E\left\{\sum_{i=1}^{n} \lambda_i \cdot Y(u_i) + R(u) \cdot Y(u_i)\right\} = \sum_{i=1}^{n} \lambda_i \cdot E\{Y(u_i) \cdot Y(u_i)\} + E\{R(u) \cdot Y(u_i)\} \tag{16}
\]

This expression is equal to zero according to the mean of \( R(u) \). Consequently, the estimation covariance with simulation is equal to the estimation covariance without simulation. SGCS allows for using a Gaussian variable as a secondary variable for simulation if its correlation with the primary variable is available (Karacan and Olea, 2013). In other words, we can use Markov models instead of full Co-kriging. In full co-kriging mode, cross-variogram plot is necessary, while Markov model does not demand it. Markov models are classified as Markov model #1 (MM1) and Markov model #2 (MM2).

MM2 has been developed for such cases that the volume support of the secondary variable is larger than primary variable (Journel, 1999); therefore, in the present study with regard to the magnetic susceptibility MM2 is applied for co-kriging estimation in SGCS since this secondary variable is wider than the primary variable (Fe variable).

The hypothesis in this Markov-type model is as following form:

\[
E[Z_1(u)Z_2(u + h)] = E[Z_1(u)|Z_2(u)] \tag{17}
\]

The dependence of the primary variable to the secondary is restricted to the co-located secondary variable. Therefore, the cross-variogram is proportional to the covariance of the secondary variable as can be seen in Eq. (18):

\[
C_{12}(h) = C_{12}(0) = C_{22}(h) \tag{18}
\]

where \( C_{12} \) is the cross-variogram between the two variables; \( C_{22} \) is the covariance of the secondary variable; \( C_{11} \) is the covariance of the primary variable and finally \( Z_1 \) and \( Z_2 \) are primary and secondary variables respectively.

Fig. 1. Geological map of Darreh-Ziarat iron deposit (Scale: 1:20000; Honarpazhouh, 2013).


139
3. Study scope

3.1. Geological setting of Darreh-Ziarat iron deposit

Darreh-Ziarat Iron Ore deposit is situated in the northern part of Sanandaj-Sirjan Structural Zone (SSZ), in the NW Iran and 30 Km away from Saqqez city. The northern part of SSZ is a metamorphic belt that is raised because of a continental clash between the Iranian microcontinent and Afro-Arabian continent and is divided into numerous small zones (Mohajjel et al., 2003). In general, the area consists of a set of metasedimentary rocks, such as sericite-quartz schist, a variety of carbonate units and interlayers of phyllite. These units are folded.

Fig. 2. (a) magnetic anomaly map, (b) 3D view of drilling boreholes in the study area.
with northeast-southwest direction, and subsequently, granite-gneiss masses are intruded in the axis of these folds (Honarpazhouh, 2013). Iron ore in this area is in the form of magnetite-hematite horizons that have taken place in sericite-quartz schist (Mt\textsuperscript{sch}1 unit) host rock (Fig. 1). Deposits related to the middle to lower Paleozoic (Ordovician to Carboniferous) have outcrops in the region as metamorphic

Fig. 3. 3D view of inversion results of magnetic data using PPDM.

Fig. 4. The observed (a) and predicted (b) magnetic anomalies which are plotted on the same scale for comparison.
units and they are covered as non-harmonic layers of with Permian limestone.

Mineralization in this deposit is occurred in two parallel veins with a transverse spacing of 10 m apart from each other in compliance with layered schist units in northern of Darreh-Ziarat village and protraction of iron horizons is N65E with a slope of 20 degrees toward the NW (Honarpazhouh, 2013).

3.2. Datasets

In the study area, the magnetic survey has been done at profiles with a distance of 20 m and station distance of 10 m. Altogether 2050 magnetic data were measured. On the other hand, 10 drilling boreholes were drilled along major magnetic anomaly around geophysical survey. A geophysical survey was employed for inversion of data and drillholes
data were used for the calculation of Fe grade values by means of oMK and SGCS. Fig. 2 demonstrates the geophysical observed data and total magnetic anomaly, after accounting for International Geomagnetic Reference Field (IGRF). It also depicts the 3D view of drilling boreholes with regard to Fe grade value.

4. Results and discussion

4.1. Geophysical analysis

For performing magnetic data inversion process, after checking and removing noise and finding regional IGRF, the total magnetic anomaly of each point was calculated. For conducting inversion by PPDM, cells dimensions of 15, 15 and 7.5 respectively for length, width and height were chosen. After that, inversion on magnetic data was done with the Mag3D software package. As can be seen in Fig. 3 inversion results based magnetic susceptibility, have a range of 0 to 1.33. High susceptibility values are an indicator of high magnetic property.

As shown in Fig. 4, The predicted magnetic anomaly for the model (Fig. 4b) and the observed magnetic anomaly (Fig. 4a) have a good consistent.

4.2. Geostatistical analysis

After obtaining the inversion results for each blocks (cells) of the model space, the estimation space was selected in the model space based on boreholes and surface geology data in order to attenuate the estimation error of the oMK. Therefore, cells that were deeper than 1630 m, and cells outside the study area were removed. The final estimation space is shown in Fig. 5.

After providing estimation space, descriptive statistical parameters on Fe variable were prepared. Because the height of estimation space was 7.5 m, borehole samples were also composited to 2.5-meter long. For calculating oMK estimation, in the first stage after compositing the samples must be transformed into a normal distribution. Fe grade variable histogram in the form of raw samples and normal score transformation is depicted in Fig. 6. As one can see in Fig. 6b, for Fe normal variable distribution, both mean and variance are close to zero and one. These are the main conditions of standard Gaussian distribution.

Similar to all geostatistical estimation, experimental variogram of normalized Fe variable must be plotted in the second stage of oMK. Afterward, variogram modeling is done in major direction and other directions are prepared. Final variogram modeling in major and vertical directions with scatter plot of their experimental variograms are demonstrated in Fig. 7. Because experimental variogram in minor direction was not able to find sample pairs for the fitting model, its variogram model and experimental variogram are not included here.

After the variogram modeling is done and final variogram parameter and search ellipsoid are determined, estimation with the oMK method is conducted on Fe variable by means MATLAB software. 3D view of estimated block in the study area is depicted in Fig. 8. Black points show the location of drill boreholes. One can consider the high range
of iron grade values and inclusion of all of them as the advantages of this estimation method, and smoothing does not have an adverse influence on the analysis as well.

In order to validate the results in geostatistical estimation methods sometimes cross-validation and jackknife techniques are used. Cross-validation normally uses the real data and in each step one of the actual data is set aside (removed) and it is re-calculated using the other data in the vicinity (Deutsch and Journel, 1998; Issaks and Srivatsa, 1989). The term jackknife applies to resampling without replacement which the dataset divided into two parts randomly (analysis and test) and then re-estimating the locations of one dataset by another (Efron, 1982). Both of these approaches are used for validation of variogram modeling and estimation method and since the jackknife does not use the non-overlapping data to establish the statistical parameters, it is a more precise approach (Pyrcz and Deutsch, 2014). In this study, the jackknife was performed three times. Once the data were randomly divided into two equal parts, this means that one part of the data is the test dataset and the other part is analysis dataset, while the test dataset is estimated by the analysis dataset with oMK estimation method. This process was also performed once in a way in which 30% of the data was test dataset and 70% was analysis dataset, and once in a way in which the test dataset was 70% of the whole data. The validation results are illustrated in the scatter plot, assigning predicted value from estimation to the horizontal axis, and real data to the vertical axis. (Fig. 9). As can be seen in all three cases investigated with the jackknife, correlation coefficient and goodness of fit have high values and the slope is close to 1 and also the standard error of estimation amount in all three cases are close to each other. By observing these curves and the parameters obtained from them, we can conclude that oMK results offer a good degree of accuracy and therefore they are deemed acceptable.

The next stage of the analysis process is to calculate Fe variable at each cell by means of the SGCS method. For doing so, this process must be performed on susceptibility variable in addition to the transformation of Fe variable normal score and variography study of the normalized variables. Initially, susceptibility variable must be transformed to the Gaussian distribution in a similar manner to Fe variable. One can observe the susceptibility variable histogram for real distribution in Fig. 10. It is apparent from the diagrams that the mean and variance of the Gaussian distribution are respectively 0 and 1. In the second stage, experimental variogram for this new variable is calculated and final variogram modeling is conducted and the model is recognized in major direction. Variogram modeling, as well as experimental variograms in major horizontal, minor horizontal and vertical directions, are shown accumulatively in Fig. 11.

![Fig. 9. Scatter plot: real data versus predicted value from oMK using the jackknife validation technique in cases of (a) 30% of the data, (b) 50% of the data and (c) 70% of the data.](image1)

![Fig. 10. Histogram of susceptibility variable in (a) raw value distribution, (b) normalized value distribution.](image2)
As mentioned earlier in the methodology section, in this research MM2 technique was employed as a co-kriging method for SGCS simulation. In addition to the variogram modeling for both variables used, the coefficient of correlation between the primary and secondary normalized variables must be determined. Because the spaces of the two variables are different, correlation coefficient obtained after assigning susceptibility variable values to the drilling boreholes is 0.55. Finally, Fe variable is simulated in each cell using SGeMS software package (Remy et al., 2009) in order to complement information for SGCS. In order to evaluate the simulation results of different realizations and gaining a higher depth of verification, SGCS process was done for 50 realizations and the average value of these realizations was considered as the final result of SGCS. Because of susceptibility values, as the secondary variable, exist for each simulated block results of these realizations were found to be close to each other. 3D view of the simulated block is shown in Fig. 12; in which the drillhole localities are marked with black points.

As scientifically expected, SGSC results are highly effective of susceptibility variable in the simulated block and actually, they are the reflection of inversion results.

4.2.1. Post-processing of oMK results

One of the advantages of the oMK method is its applicability in the assessment of the tonnage, metal content and the mean grade above the specified threshold (e.g., >20% “start of Fe mineralization”: Fig. 13). These maps are among the most effective mineral inventory information and this is very advantageous at different stages of ore body evaluation. They provide an instinctive evaluation of the general size of the resource (quantity of metal or tons of ore) based on exploratory data in the first steps of assessment with regard to different cut-off grades. In this initial step, the estimations cannot be considered either resources or reserves in the strict sense. During mine planning stage, the tonnage of ore and the content of metal mined is incorporated with these maps according to the different depths of the orebody and the changes in ore recovery and grade (Sinclair and Blackwell, 2002). As can be seen in Fig. 13, the most prominent quantities of tonnage, metal content and mean-grade (above the desired cut-off) is located approximately in the center of the area under study (elevation 1723.75 m, X = 615,050, Y = 3,984,300) and that indicates the commencement of the first population derived from the oMK.

4.3. Integration of results

The final step of this research was to compare the results from oMK and SGCS at different elevations in order to evaluate the veracity of inversion results. In order to accomplish this, the following parameters were calculated: The slope and coefficient of correlation between results of oMK and SGCS. The plots of relationship for changes in correlation coefficient and slope between oMK and SGCS results, in 10 different realizations and final value, are demonstrated in Fig. 14. It is observable that in both higher and lower elevation depths, correlation coefficient and slope show smaller values; and it means that in both

---

**Fig. 11.** Experimental variogram and variogram modeling of normalized susceptibility variable at directions of major horizontal, minor horizontal and vertical axes.

**Fig. 12.** 3D view of simulated blocks using SGCS.
methods Fe quantities calculated for extreme elevations are very different. According to the dependence of SGCS results with inversion, one can deduce that the inversion results in these depths do not enjoy a high degree of accuracy and hence the results are not considerably reliable. Unlike extreme elevations in the middle of the studied area elevation-wise, correlation coefficient and slope both have higher quantities and this confirms that the results of both methods for this region are close to each other; therefore, the inversion results enjoy more accuracy and shall be considered more dependable. From another point of view, one can deduct that inversion results in higher
Elevations are generally more accurate compared to those of the lower depths. According to correlation coefficient and the slope of the linear regression curve, the highest and the lowest quantities of these values demonstrate the nearest and furthest calculated Fe value through both methods of oMK and SGCS. Therefore, the calculated Fe variable using oMK and SGCS as well as the difference variable (the difference of calculated quantities with various methods), and also the inversion results are shown in three different elevation depths, namely at depths with the highest, the lowest and near zero quantity of correlation coefficient. As can be seen in Fig. 15 for the best correlation, results of both methods are close to each other in the high-grade zone and far apart in the low-grade zone. For other cases, as shown in Figs. 16 and 17, the significant difference between the results of oMK and SGCS methods.
Fig. 16. Plan view at an elevation with the lowest quantity of correlation coefficient.

Fig. 17. Plan view at an elevation with near zero quantity of correlation coefficient.
can be related to the estimation of blocks with medium susceptibility in middle part of the mineral body; and it can be considered as an error of inversion process. As a result of this medium susceptibility, Fe variable calculated with SGCS method is increased and therefore have a significant difference with estimated value through oMK especially for in the lowest correlation. According to the figures, one can say that the difference in Fe variable values between oMK and SGCS methods is more pronounced for low-grade zones; and oMK results shall be treated with more acceptability than SGCS results.

5. Conclusions

Results of this research indicate that oMK technique is a suitable method in order to reduce uncertainty in mineral resource estimation; this method considers all grade values and also it does not involve the adverse effect of smoothing. Validation of OMK outcome corroborates that this method is able to yield a high depth of accuracy in grade estimation and the results are generally trustworthy. As an additional bonus, this method is capable of delineating the distribution of valuable mineral and estimating the ore tonnage, average grade, and metal content, which are very useful information for the next stages of mine design and planning.

The depth of inverted model depends heavily on the chosen depth-weighting exponent. Using an exponent lower than 3, going from 3 to 1, we'd have got models at shallow (perhaps better) depth. Looking at the figure of inverted model (elongated horizontal shape) it seems that the source distribution has nothing to do with a spherical distribution, roughly speaking. 2 could have been a better choice.

The most significant conclusion of the present study is with regard to the inversion results. Application of geostatistical methods in addition to inversion proves to be very useful for assessing Fe grade value in the mineralized zone. Actually, these techniques improve the ability of inversion in recognizing mineralization zone. Comparing the results from geostatistical methods indicates that thus at the highest and lowest depths, the accuracy of the inverted model is not good but at middle and also relatively high depth inversion yields a model with a good degree of precision, and the results are close to those obtained from the geostatistical analysis.

Acknowledgment

The authors would like to appreciate the cooperation of Engineer Mohammad Golkar as the managing director of Zamin Kavan Gharb Iranian Company (ZKG) for authorizing us to use Darrehz-Izarat Iron Ore exploration datasets. The authors are also very grateful to Dr. Nasser Madani, the anonymous reviewers and Professor Maurizio Fedi for helping to improve the text.

References


