Simulation of a falling droplet in a vertical channel with rectangular obstacles

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ABSTRACT

Droplet microfluidic systems have attracted a large amount of research due to their numerous applications in biomedical micro-devices and drug discovery/delivery platforms. One of the most important problems in such systems is to investigate deformation, coalescence, and breakup of droplets within the channel. The present study demonstrates numerical simulation of a falling droplet subject to gravitational force in a channel with embedded rectangular obstacles. The lattice Boltzmann method incorporated using He–Chen–Zhang method for two phase flow is employed. Two rectangular obstacles with inverse aspect ratios are introduced to investigate the mechanism of breakup and deformation of the droplet. The influence of gravity magnitude, viscosity and surface tension on the deformation rate of droplet for two different aspect ratios of the obstacle is studied. It is observed that increasing the gravity force, decreasing the viscosity or surface tension increase droplet deformation rate resulting in more stretched filaments and so breakup occurs in a shorter time.

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1. Introduction

Multiphase flow is a common type of flow in a wide range of natural phenomena such as water penetration in soil, atmospheric rain drop formation, and oil flow inside petroleum wells. Using those phenomena as initial inspirations, several engineering applications are introduced in different industries like atomization [1], inkjet printers [2], and oil recovery technology [3]. More recently, microfluidic systems have attracted a large amount of research due to their numerous applications in biomedical micro-devices and drug discovery/delivery platforms [4,5]. One of the most important problems in such systems is to investigate the deformation, coalescence, and breakup of droplets within the channel.

Studying droplet is a challenging task due to its complex nature of interfacial fluid dynamics and topological changes. The large number of works is dedicated to address different aspects of this topic experimentally. One of the earliest works was done by Clift et al. [6] who presented a collection of experimental results in the field of drop deformation and bubble dynamics. Zhao et al. [7] presented an experimental study of liquid droplet impingement upon a substrate and visualized the droplet deformation process using a two-reference-beam pulse holography method. Sikalo et al. [8,9] investigated the impact of three different liquid drops on flat and inclined solid surfaces experimentally.

Numerical simulation of multiphase flows is a crucial part of studying engineering systems. The traditional way to model such flows is to discretize the Navier–Stokes equations and use a suitable interface capturing technique to track interface between different phases. Gerlach et al. [10], used three volume-of-fluid methods to simulate the Rayleigh–Taylor instability, an equilibrium rod and a capillary wave. Moshiri et al. [11] performed a numerical simulation of multiphase flows in porous media with gravitational effects using dominant wave method. Ray et al. [12], utilized numerical simulations applying the CLSVOF method to investigate the drop impact on deep and shallow liquid and discussed the coalescence and splashing regimes of a spherical water drop. Mashayek et al. [13] investigated the collision of two liquid drops using a Galerkin finite element method in conjunction with the spine-flux method for the free surface tracking. Kadivar et al. [14] employed a boundary element method to study the deformation of a single droplet in a microfluidic device with consideration of different parameters. A detailed knowledge of two-phase flows can be found in the book by Liu [15]. Another approach which has become popular in the recent years is lattice Boltzmann method (LBM) which is a reduced-order kinetic model and solves the mesoscopic kinetic equations in a lattice structure [16,17]. Some of the exclusive features of this scheme are parallelism of computations, capture of the complex geometries, and simple coding procedure.
Nomenclature

**Roman letters**
- \( A \) Atwood number
- \( C_0 \) Lattice speed of sound
- \( D \) Initial diameter of the droplet
- \( e_\alpha \) Lattice velocity in direction \( \alpha \)
- \( f \) Distribution function
- \( F \) Effective molecular interaction force
- \( g \) Pressure distribution function
- \( G \) Gravitational force
- \( k \) Surface tension force
- \( P \) Pressure
- \( R \) Universal gas constant
- \( r \) Droplet radius
- \( u \) Macroscopic fluid velocity
- \( V_c \) The centroid velocity of the droplet
- \( w_{\alpha} \) Weighting coefficient

**Greek letters**
- \( \delta, \delta_\alpha \) Lattice constant
- \( \mu \) Dynamic viscosity
- \( \nu \) Kinematic viscosity
- \( \rho \) Density
- \( \sigma \) Surface tension coefficient
- \( \lambda \) Relaxation time
- \( \xi \) Microscopic velocity
- \( \phi \) Interface function between different phases

**Subscripts**
- \( \alpha \) Lattice streaming vector direction
- \( d \) Initial diameter of the droplet
- \( h \) Maximum value of the index function
- \( l \) Minimum value of the index function

**Superscripts**
- * Quantifier denoting dimensionless variables
- \( \text{eq} \) Quantifier denoting equilibrium properties
- --- Quantifier denoting average values

In the recent decades, several lattice Boltzmann models have been proposed to simulate multiphase flows such as the pseudo-potential model [18], the free-energy-based approach [19], the chromodynamic model [20], and a multiphase LBM based on the kinetic theory for dense fluids [21]. He and Doolen [22] reviewed these methods and investigated the weak points of each method. He et al. [23], implemented the LBM based on kinetic theory for dense fluids and successfully employed the scheme to simulate incompressible multiphase flow and Rayleigh–Taylor instability. There are many other studies which adopted He–Chen–Zhang model [23] in two-phase flow problems such as effect of surface tension in 2D Rayleigh–Taylor instability [24], interactions of two deformable drops [25], and Kelvin–Helmholtz instability [26]. Recently, the problem of a falling droplet in confined channels has attracted many researchers’ attention. Bosse et al. [27], numerically investigated the settling process of initially spherical suspension drops under gravity for low and moderate flow Reynolds numbers. Ni et al. [28], performed a direct simulation of falling droplet in closed channel with density ratios of 1.05 and 1.125 using a projection method coupled with a level set method. Fakhrari and Rahimian [29], simulated the falling droplet under gravity using lattice Boltzmann method and found that at low values of Eotvos number and high Ohnesorge values, the drop deforms slowly without break up. In another paper, Fakhrari and Rahimian [30] studied various breakup modes of an axisymmetric falling droplet under buoyancy force using multiple-relaxation-time lattice Boltzmann method. Bararnia and Ganji [31], studied the breakup and deformation of a falling droplet under high voltage electric field. They addressed the effect of permittivity and conductivity ratio on droplet shape. Shen et al. [32], used lattice Boltzmann with implementation of pseudo-potential model to simulate the deformation of a droplet after impact on a 2D curved surface.

The review of the previous works indicates that a large number of investigations were conducted on the two phase flow of a falling droplet in confined channels. However, according to the author’s knowledge, there exists few research about the effect of obstacles and their shapes on the droplet deformation and breakup regime [33–36]. Furthermore, little attention is allocated to combined effect of the obstacles and fluid parameters to control deformation rate of a falling droplet. In the present work, we employ lattice Boltzmann method using He–Chen–Zhang model [23] to investigate the effect of different parameters on the deformation of a falling droplet due to gravity in a vertical channel with two aspect ratios of rectangular obstacle. Then, two concepts including wetting of droplet on a vertical obstacle and fluid thickness on a horizontal obstacle are also discussed.

2. Model and computational details

2.1. Governing equations; LBM for multiphase flow

The primary equation governing the flow is based on the Boltzmann equation as:

\[
\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \xi \cdot \nabla f = \frac{f - f^{\text{eq}}}{\lambda} + \left( \xi - u \right) \left( F + G \right) f^{\text{eq}}
\]

(1)

where \( f \) is the single particle distribution function, \( t \) denotes time, \( f^{\text{eq}} \) is the equilibrium distribution function for incompressible fluids which mimics the Maxwellian distribution in phase space, \( \lambda \) is the relaxation time, \( \xi \) and \( u \) are the microscopic and macroscopic velocities, respectively, \( \rho \) is density, \( R \) is the universal gas constant, \( T \) is the temperature, \( G \) is the gravitational force and \( F \) is the effective molecular interaction force calculated as,

\[
F = -\nabla \psi + F_r.
\]

(2)

The Carnahan–Starling equation of state gives the pressure \( P \) as [37],

\[
P = \rho RT \frac{1 + \eta + \eta^2 - \eta^3}{(1 - \eta)^4} - a \rho^2; \quad \eta = \frac{b \rho}{4}.
\]

(3)

Both two parameters of “a” and “b” are constant which correct the attractive of molecules and the volume respectively. The function \( \psi \) in Eq. (2) shows the non-ideal part of Carnahan–Starling equation of state and written as,

\[
\psi(\rho) = p - \rho RT.
\]

(4)

The surface tension force can be written as,

\[
F_s = k \rho \nabla \nabla^2 \rho.
\]

(5)

Here, parameter \( k \) denotes surface tension’s strength.

2.2. Numerical procedure

Solving Eq. (1) directly leads to numerical instability due to large values of the gradient term of the intermolecular force near the interface [23]. Therefore, a suitable transformation is introduced as,

\[
g = RT + \psi(\rho)\Gamma(0)
\]

(6)
where \( g \) is called the pressure distribution function. In order to make our notations shorter we define the \( \Gamma \) function which relates to equilibrium distribution function by,

\[
\Gamma = \frac{f_{eq}(\rho)}{\rho}.
\]  

(7)

Substituting Eq. (6) into Eq. (1), and employing the definition given in Eq. (7), we obtain the following set of Eqs. [38]

\[
\begin{align*}
\frac{DF}{Dt} &= f - f_{eq} - \frac{\rho}{2\lambda} \nabla \psi(\phi) \Gamma(\rho) \\
\frac{D\rho}{Dt} &= \frac{\rho - \rho_{eq}}{\rho} + (\xi - u) \cdot \nabla \psi(\phi) \frac{\Gamma(\rho)}{\rho}
\end{align*}
\]

(8)

where \( \phi \), which is known as the index function, tracks the interface between the phases. Considering \( \bar{a} = 4 \) in Carnahan–Starling equation of state, one can write \( \psi(\phi) \) as:

\[
\psi(\phi) = \phi \sqrt{\rho} \left( 4 - 2\phi \left( 1 - \phi^2 \right) - 3\phi^2 \right)
\]

(9)

In the above equation, to ensure phase segregation the value of \( \alpha \) should be opted to be more than 10.601 \( \rho T \). In this research it is assumed that \( \alpha = 12 \) the same as the previous works [24,26,39]. D2Q9 lattice structure shown in Fig. 1 was used to discretize Eq. (8). Looking into Fig. 1, the discrete microscopic velocities are given as,

\[
e_{\bar{a}} = \begin{cases} 
0\alpha = 0 \\
\sqrt{2}(\cos \theta, \sin \theta), \theta = (\pi - 1)\pi/2, \alpha = 1, 2, 3, 4 \\
\sqrt{2}(\cos \theta, \sin \theta), \theta = (\pi - 5)\pi/2 + \pi/4, \alpha = 5, 6, 7, 8.
\end{cases}
\]

(10)

Corresponding weight coefficients [40],

\[
w_{\bar{a}} = \begin{cases} 
\frac{4}{9}, \alpha = 0 \\
\frac{1}{9}, \alpha = 1, 2, 3, 4 \\
\frac{1}{36}, \alpha = 5, 6, 7, 8.
\end{cases}
\]

(11)

Space and time steps are selected to be \( \delta_s = \delta_t = 1 \) which leads to lattice sound speed of \( c_s^2 = \rho T = 1/3 \). Introducing the following transformation variables, an explicit scheme can be adopted as [23],

\[
\tilde{f}_{\bar{a}} = f_{\bar{a}} + \frac{(e_{\bar{a}} - u) \cdot \nabla \psi(\phi)}{2\rho T} \Gamma(\rho) \delta_t
\]

(12)

\[
\tilde{g}_{\bar{a}} = g_{\bar{a}} - \frac{1}{2}(e_{\bar{a}} - u) \cdot \nabla \psi(\phi) \frac{\Gamma(\rho)}{\rho} \delta_t.
\]

(13)

After substitution of Eqs. (12) and (13) into Eq. (8), we obtain the ultimate discretized Boltzmann equations,

\[
\tilde{f}_{\bar{a}}(x + e_{\bar{a}}, t + \delta_t) = \tilde{f}_{\bar{a}}(x, t) - \frac{\Gamma(\rho)}{\rho T} \frac{(e_{\bar{a}} - u) \cdot \nabla \psi(\phi)}{2\rho T} \Gamma(\rho) \delta_t
\]

(14)

\[
\tilde{g}_{\bar{a}}(x + e_{\bar{a}}, t + \delta_t) = \tilde{g}_{\bar{a}}(x, t) - \frac{\Gamma(\rho)}{\rho T} \frac{(e_{\bar{a}} - u) \cdot \nabla \psi(\phi)}{2} \Gamma(\rho) \delta_t
\]

(15)

where

\[
\Gamma(\rho) = \rho\left(1 + 3(e_{\bar{a}} \cdot u) + \frac{9}{2}(e_{\bar{a}} \cdot u)^2 - \frac{3}{2}u^2 \right)
\]

(16)

\[
\tilde{f}_{\bar{a}}(\rho \cdot \nabla \psi(\phi)) = \rho\left(1 + 3(e_{\bar{a}} \cdot u) + \frac{9}{2}(e_{\bar{a}} \cdot u)^2 - \frac{3}{2}u^2 \right)
\]

(17)

\[
\tilde{g}_{\bar{a}} = \rho\left(\rho - \rho_b\right) g.
\]

(19)

After the streaming step in space, the macroscopic variables are calculated as follows,

\[
\rho = \sum_{\bar{a}=0}^{\bar{a}=8} \tilde{f}_{\bar{a}}
\]

(20)

\[
p = \sum_{\bar{a}=0}^{\bar{a}=8} \tilde{g}_{\bar{a}} - \frac{\nu T}{2} \frac{(F_1 + G) \delta_t}{2}
\]

(21)

The kinematic viscosity is related to dimensionless relaxation time by,

\[
\nu = (\tau - 0.5)RT \delta_t.
\]

(23)

Also, the density of the fluid can be obtained by a simple interpolation,

\[
\rho(\phi) = \rho_l + \frac{\phi - \phi_l}{\phi_h - \phi_l} (\rho_h - \rho_l)
\]

(24)

where \( \rho_l \) and \( \rho_h \) denote the light and heavy fluids densities respectively, and \( \phi_l \) and \( \phi_h \) are the minimum and maximum values of the index function.

For an equilibrium state of contact angle on an ideal solid wall, the interfacial tensions are in balance at the contact point. The contact angle can be measured from a balance of forces at the contact line in which interfacial tensions and the contact angle satisfy Young’s equation. In conventional numerical methods, free energy approach [41] is implemented for the precise prediction of the contact line motion and the bounce-back rule with equilibrium condition for the mass conservation is used for implementation of wetting condition [42]. Recently, the geometric scheme has been employed for the wetting boundary condition in the framework of LBM to investigate the dynamics of droplets [43–46]. Here, we employed the geometric formulation proposed by Ding and Spelt [47] as follows:

\[
\frac{\partial \phi}{\partial n} = -\tan(\theta) |t_n \cdot \nabla \phi|
\]

(25)

where \( t_n \) are the unit tangential and normal vector on the obstacle, respectively. \( \theta \) is the microscale contact angle. When a contact angle is prescribed, the scheme of using geometric formulation.
has advantages over the surface-energy formulation [47–49]. In the design of geometric method, it is obvious that the tangential component of \( \nabla \phi \) cannot be modified during the simulation and the microscopic contact angle can only be enforced through the change of the normal component [47]. Owning to this, the geometric method performs better than other surface-energy-based boundary conditions in assuring that the microscopic contact angle matches the specified value. Here, the geometric formulation for a horizontal surface can be written [48]

\[
\phi_{i,0} = \phi_{i,2} + \tan\left(\frac{\pi}{2} - \theta_s\right) \left|\phi_{i+1,1} - \phi_{i-1,1}\right|.
\]  

(26)

Considering the horizontal obstacle as a wall we should use the Eq. (26) for applying wetting condition at solid nodes. To obtain the terms of \( \nabla \psi(\phi) \), it should be crucial to have the values of index function, \( \phi \), on the solid nodes. Thus, above equation respected as a geometric formulation for implementation of wetting condition can be imposed through the computation of \( \phi \) on the solid nodes. Here, \((i, 0)\) denotes boundary or solid node on the obstacle, the first and second subscripts, \((i, 1)\) and \((i, 2)\) describe the upper nodes of boundary node. In addition,\((i - 1, 1), (i + 1, 1)\) are left and right neighboring nodes of \((i, 1)\). Note that in our model the static contact angle has the value of 55.2.

### 3. Numerical validation

This section covers the results for verification of our two dimensional lattice Boltzmann code. We start with wetting of a droplet on a flat surface and then we consider Rayleigh–Taylor instability. Finally, we examine the grid independency of cross sectional centroid velocity on three different grids.

#### 3.1. Wetting condition of the droplet on a flat surface

Initially, we place a semi-circular droplet on a flat surface with initial contact angle of 90 and then the droplet starts to move toward its equilibrium state called static contact angle. Droplet starts increasing its area with the flat surface until the droplet reaches its equilibrium state. To verify our code, we compare our results with the following analytical relation in Fig. 2 for the maximum height of droplet versus the contact angle. Our results are in a good agreement with analytical approach [36].

\[
\frac{h_{\text{max}}}{r} = (1 - \cos \theta) \sqrt{\frac{\pi}{2\theta - \sin 2\theta}}
\]  

(27)

where, \( h_{\text{max}} \) is maximum height of droplet, \( r \) is the initial radius of semi-droplet and \( \theta \) is contact angle.

#### 3.2. Rayleigh–Taylor instability

Here, the simulation of a 2D Rayleigh–Taylor instability is presented in Fig. 3. We consider a heavy fluid which falls down and
its structures grows into a light fluid. The density ratio is chosen to be 3 while two fluids have the same kinematic viscosity. As can be seen in Fig. 3, at the $t^* = 0$, grow of the fluid interface is smooth and nearly flat. During the early stages ($t^* = 1$ and $t^* = 2$) the evolution of the roll-up of the heavy fluid exhibits the high deformation of interface until at $t^* = 3$ this roll-up looks like a mushroom. Then, at $t^* = 4$ this roll-up has been stretched into two layers folded upward. Dimensionless time is defined by $\frac{t}{\sqrt{Dg}}$.

In this case an oscillatory wave will be produced. Initially the amplitude of this perturbation is extremely smaller than the wave length and then it starts to grow. The result is that small amplitude perturbations grow exponentially with time, which can be given by:

$$h = h_0 e^{\alpha t}$$

(28)

where $h_0$ represents the initial amplitude, $h$ is the amplitude at time $t$ and $\alpha$ stands for the growth rate of perturbation. It is evident that the growth rate is a function of Atwood number $A = (\rho_l - \rho_h)/(\rho_l + \rho_h)$ and the wave number $k = 2\pi/W$ with $W$ as the channel width. In Fig. 4 for Atwood number equal to 0.5 the dependence of the rate of growth on the value of the wave number has been plotted and compared. As can be seen, our results are in a good agreement with those reported by He–Chen–Zhang model [23].

3.3. Droplet falling in the microchannel due to gravity

Here, we evaluate the accuracy of our computational code by placing a droplet in the channel under gravity. First, a circular droplet is initially at rest and then allowed to move down due to gravitational force. In Fig. 5, the dimensionless cross-sectional velocity passing through the centroid of falling droplet plotted against the transverse position of channel at the particular time where $g = 0.0004$, $\nu = 0.233$ and $\frac{\rho_l}{\rho_h} = 5$. The grid study was carried out to examine the sensitivity of the solution to the number of cells with three different number of grid points, namely $64 \times 128$, $128 \times 256$ and $256 \times 512$. Obviously, this study demonstrates that increasing the resolution from $128 \times 256$ to $256 \times 512$ has minimal effect on the dimensionless cross-sectional velocity, which indicates the independency of our numerical method from number of meshes.

In order to validate the numerical method, we have compared our result with another published numerical study. Fig. 6 shows the qualitative and quantitative comparison between our results and those reported by [50]. As can be seen, after a slight deformation at the early stages, the back of droplet pushes in and deforms into a crescent-like shape. With further progress in time, the back side of droplet becomes flat. However, it takes less time in another computational study [50] to undergo an elliptical shape with its flat back. Finally, it reaches a steady fall velocity. In another computational study [50] droplet is more stretched perpendicular to the flow in comparison with our presented simulation. In addition, the result corresponding to Fig. 6 is based on centroid velocity of droplet versus dimensionless time. Comparing our results with
numerical data obtained in the literature [50] reveals that in the reported numerical simulation since the droplet is more stretched in the normal direction of its motion, it slows down faster than that simulated in our study. As can be seen the dimensionless centroid velocity \( V_c \) starts to deviate from those obtained by J. Han and G. Triggeson [50] after \( t^* = 5 \). However, the difference remains relatively small. The discrepancies between our results and those in Ref. [50] may be due to the fact that our simulations are in 2D while those in Ref. [50] are axisymmetric.

4. Falling droplet under the gravity on the obstacle

In this section, we probe the main objective of our study, the simulation of the falling droplet under gravitational effects on the obstacle with different configurations. The circular droplet with diameter \( D = W/4 \) is centered at \((W/2, H - 0.375 \times W)\), where \( W \) and \( H \) as width and height of channel are 80 and 160, respectively (see Fig. 7). To take into account the impact of a droplet on an obstacle, we placed obstacle vertically or horizontally in the channel. Free slip boundary condition is applied at the upper boundary. Also, at the vertical walls, bounce-back boundary condition is used. The bottom border \( (y = 0) \) is treated as the open boundary and then \( \bar{f}_2, \bar{f}_5 \) and \( \bar{f}_6 \) are needed to be calculated. Therefore, the following extrapolation scheme is used to determine the values of the distribution functions:

For \( \alpha = 2, 5, 6 \rightarrow \begin{cases} \bar{f}_\alpha (x, 0) = 2\bar{f}_\alpha (x, 1) - \bar{f}_\alpha (x, 2) \\ \bar{g}_\alpha (x, 0) = 2\bar{g}_\alpha (x, 1) - \bar{g}_\alpha (x, 2) \end{cases} \) \hspace{1cm} (29)

In addition, the investigation of the falling droplet requires examination of the most significant dimensionless numbers described by parameters as follows:

\[ t^* = \frac{t}{\sqrt{D/g}} \] \hspace{1cm} (30)

\[ Oh = \frac{\mu_d}{\sqrt{\rho_d D \sigma}} \] \hspace{1cm} (31)

\[ Eo = \frac{g \Delta \rho D^2}{\sigma} \] \hspace{1cm} (32)

where \( D \) is the initial diameter of the droplet, \( g \) is gravitational acceleration, \( \rho_d \) and \( \mu_d \) are density and viscosity of the droplet, respectively, and \( \Delta \rho \) is the density difference. In our simulation, the density ratio is fixed at 5 and viscosity value of two fluids is equal.

4.1. The effect of the surface tension coefficient

Surface tension plays an important role in the deformation of the falling droplet and leads to different configurations. Here, while gravity and viscosity are fixed to \( \nu = 0.2333 \) and \( g = 0.0004 \) at the \( k = 0 \) (red color), \( k = 0.15 \) (black color) \( (Oh = \infty, Eo = \infty \) and \( Oh = 0.87, Eo = 32 \) \( k = 0 \) (red color), \( k = 0.15 \) (black color) at selected dimensionless times: \( 1.1t^* = 4.47, 2t^* = 5.37, 3t^* = 6.26, 4t^* = 7.16, 5t^* = 8.05, 6t^* = 8.95 \). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Fig. 9. Evolution of a falling droplet in channel with a horizontal obstacle with $\nu = 0.2333, g = 0.0004$ $(Oh = \infty, Eo = \infty$ and $Oh = 0.82, Eo = 32)$ $k = 0$ (red color), $k = 0.15$ (black color) at selected dimensionless times: (1) $t^* = 4.47$, (2) $t^* = 5.37$, (3) $t^* = 6.26$, (4) $t^* = 7.16$, (5) $t^* = 8.05$, (6) $t^* = 8.95$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 10. Evolution of a falling droplet in channel with a vertical obstacle, $k = 0.15$ and $g = 0.0004, (Oh = 0.24, Eo = 32$ and $Oh = 0.53, Eo = 32), \nu = 0.0067$ (red color), $\nu = 0.15$ (black color) at different dimensionless times: (1) $t^* = 4.47$, (2) $t^* = 5.37$, (3) $t^* = 6.26$, (4) $t^* = 6.71$, (5) $t^* = 7.16$, (6) $t^* = 7.60$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. The effect of viscosity

In addition to surface tension coefficient, viscosity of fluids can affect droplet behavior. Figs. 10 and 11 depict the deformation of the falling droplet in a channel with a vertical and horizontal obstacle at different dimensionless times for two viscosity values of $\nu = 0.0067$ and $\nu = 0.15$.

To evaluate the effect of viscosity in our research, we change viscosity from $\nu = 0.0067$ to $\nu = 0.15$, Fig. 10 shows the evolution of the droplet in the channel with vertical obstacle for two cases; $Oh = 0.24, Eo = 32$ (red color) and $Oh = 0.53, Eo = 32$ (black color). At $t^* = 4.47$, the impact between droplet and obstacle lead to a formation of a dent in the droplet. In the case of lower viscosity, $t^* = 5.37$ is the onset of droplet detachment from the obstacle while in the case of higher viscosity this phenomenon takes place in the later dimensionless time. Afterwards, in the case of lower viscosity, two daughter droplets are generated and completely detached from the obstacle at $t^* = 6.26$. Nonetheless, considering the dimensionless times from $t^* = 6.71$ to $t^* = 7.60$, we can observe that higher viscosity results in disintegration of droplet into four droplets. It is necessary to say that larger daughter droplets can easily turn into circular shape in the case of higher viscosity compared to lower one.

According to Fig. 11, the result was obtained by considering horizontal obstacle in a channel with increasing Oh number from 0.24 to 0.53 at a constant $Eo$ number. In this figure, lower viscosity causes the droplet to break up sooner compared to the case of higher viscosity. This phenomenon occurs because as the viscosity increases, the drops deform much more slowly and the droplet shows more resistance to the gravitational forces. Moreover, the maximum thickness of droplet on top of the obstacle is higher in
Fig. 11. Evolution of a falling droplet in channel with a horizontal obstacle, \( k = 0.15 \) and \( g = 0.0004, (Oh = 0.24, Eo = 32 \) and \( Oh = 0.53, Eo = 32) \), \( \nu = 0.0667 \) (red color), \( \nu = 0.15 \) (black color) at different dimensionless times: (1) \( t^* = 4.47 \), (2) \( t^* = 5.37 \), (3) \( t^* = 6.26 \), (4) \( t^* = 6.71 \), (5) \( t^* = 7.16 \), (6) \( t^* = 7.60 \). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Fluid characteristics of simulated cases.

<table>
<thead>
<tr>
<th>Cases</th>
<th>( k )</th>
<th>( \nu )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.15</td>
<td>0.2333</td>
<td>0.0004</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>0.2333</td>
<td>0.0004</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.15</td>
<td>0.2333</td>
<td>0.0003</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.15</td>
<td>0.2333</td>
<td>0.0005</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.15</td>
<td>0.0667</td>
<td>0.0004</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.15</td>
<td>0.15</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

4.3. The effect of gravity

Another prominent parameter which is of fundamental importance in this research is gravity. To examine the effect of gravity, we consider evolution of droplet at different dimensionless times both in vertical and horizontal obstacles at constant \( Oh = 0.82 \) for \( Eo = 24 \) and \( Eo = 40 \). Results for a droplet impacting the obstacle are sketched in Figs. 12 and 13 for \( g = 0.0003 \) and \( g = 0.0005 \). In Fig. 12 by increasing \( Eo \) number, higher deformation of droplet is observed since the higher gravitational force enables the droplet to gain higher momentum. It can be found that for both two cases in this figure the longitudinal distance between filaments hanging from the obstacle is smaller than that of Figs. 8 and 10. As time progresses, in the case of lower gravity, filaments just approach each other and breakup does not occur until the described dimensionless times while in the case of higher gravity this phenomenon happens at the dimensionless time \( t^* = 7 \). Obviously, after disintegration at higher gravity acceleration the daughter droplets become closer to each other in comparison with previous case (Fig. 10). In the horizontal obstacle two filaments shown in Fig. 13 are also driven due to gravity and have less tendency to elongate towards the side walls of channel compared to two other cases (Figs. 9 and 11). Obviously, the maximum thickness of droplet in the lower gravity is higher in comparison with the case of higher gravity.

4.4. Wetting of droplet on a vertical obstacle

As droplet approaches the vertical obstacle, the shape of droplet is affected. Given droplet movement on a vertical obstacle, there is a vertical distance from upper tip of obstacle to the point where droplet detaches from each side of vertical obstacle, called wetting of droplet. Due to similarity of material used in both sides of the obstacle, the wetting on both sides is identical. In Fig. 14, we plotted the wettability of the droplet against the dimensionless time for the following cases presented in the Table 1. To better understand the comparison between these curves, each curve was shifted to the origin of coordinate with its own shifting value of \( T_0 \). Hence, onset of the curves is set in the dimensionless time of 0 and they are sketched until the time when the droplet is halved. As demonstrated in Fig. 14 it is of importance to mention that the highest and lowest grow rate of wetting has been clearly observed in the cases 5 and 2 respectively. Regarding the cases 1 and 4, we can notice the same grow rate of wetting for these two cases.
Fig. 13. Evolution of a falling droplet in channel with a horizontal obstacle, $k = 0.15$ and $\nu = 0.0667$, $(Oh = 0.24$, $Eo = 24$ and $Oh = 0.24$, $Eo = 40)$, $g = 0.0003$ (black color), $g = 0.0005$ (red color) at different dimensionless times: (1) $t^{*} = 5$, (2) $t^{*} = 6$, (3) $t^{*} = 7$, (4) $t^{*} = 8$, (5) $t^{*} = 8.5$, (6) $t^{*} = 9$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 14. Wetting versus dimensionless time for simulated cases.

4.5. Fluid thickness on a horizontal obstacle

Here, we analyze the fluid thickness of a falling droplet remaining on a horizontal obstacle, as shown in Fig. 15. The points on the curves that cross the vertical line of diagram correspond the fluid thickness when droplet impacts onto horizontal obstacle and the points at the bottom of curves illustrate the fluid thickness at the time that breakup happens. Hence, it is worthwhile to mention that as the droplet collapses on the obstacle, fluid thickness in all cases tends to reduce although there is a significant difference in the fluid thickness variations between some cases. A portion of fluid which remains on the top of the surface at the case 4 has the lowest variation of fluid thickness from the impact time to breakup time. Regarding the case 1, until almost $t^{*} < 7$ fluid thickness variations are the highest in comparison to the rest of the cases and after that case 3 holds the highest value.

5. Conclusion

In summary, LBM for two-phase flow was employed to study the deformation and breakup mechanism of a 2D falling droplet in a channel embedded with vertical or horizontal rectangular obstacle with two aspect ratios of 1:6 and 6:1. To model the droplet motion, He–Chen–Zhang model [23] was used. Evolution of the droplet within the channel was captured for some dimensionless time to understand its behavior before, during and after impacting the obstacle. The effect of variation of three parameters including surface tension, viscosity and gravity on droplet deformation and breakup time was totally investigated. It has been observed that the shape and position of generated daughter droplet basically depend on these mentioned parameters. It was found that the effect of two parameters, surface tension and viscosity on droplet motion is similar while gravity has opposite impact on droplet. Obviously, decreasing gravity and increasing viscosity as well as surface tension lead to an increase in breakup time. In addition, higher value of surface tension and viscosity, daughter droplets have a great tendency to turn into the circular shape while it is observed in low value of gravity. The results also show that increasing gravity and decreasing viscosity result in higher deformation rate of droplet. In other words, in both horizontal and vertical obstacles at relatively low $Eo$ numbers and high $Oh$ number, the droplet deforms slowly leading to an increase in the time of breakup. However, at higher $Eo$ number gravitational force prevail over the surface tension force and the droplet has more distortions. In this research, we also analyze the impact of wetting of droplet on a vertical obstacle and fluid thickness on horizontal obstacle. Future research can be conducted on the combined effect of horizontal and vertical
obstacle or effect of different geometrical configuration of obstacle on droplet deformation and breakup.

References


