Inverse optimization design of an impinging co-axial jet in order to achieve heat flux uniformity over the target object

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HIGHLIGHTS

• Inverse design of a co-axial impinging jet is performed to obtain heat flux uniformity over the target surface.
• Co-axial impinging jets have more potential to achieve uniform heat flux with respect to circular and slot impinging jets.
• A co-axial jet with a new geometry called GOF co-axial impinging jet is presented.
• GOF co-axial impinging jet improves the heat flux uniformity over the target surface.

ABSTRACT

In this study, co-axial impinging jet was optimized in order to obtain uniform heat flux over an isothermal heated surface by determining four design variables including geometrical and flow variables. The governing equations were solved using the finite volume method for a laminar, incompressible, and axisymmetric flow. The solution of inverse design problem was achieved by minimizing the root mean square of the difference between the local Nusselt number and the uniform design Nusselt number. A combination of pattern search and gradient-based methods was used for optimization. Also a co-axial jet with a new geometry was presented to improve the objective function and two design variables were added to the four previous variables. Optimization was performed for two jet configurations under 15 different conditions. Heat flux uniformity was obtained by these two jets with acceptable errors less than 2% for the outer jet to the target surface diameter ratios of higher than 0.6. The proposed co-axial jet showed its superiority at the small diameter ratios (0.4 and 0.3) and it reduced the error significantly (about 50%) for design Nusselt numbers of 7 and 10.

1. Introduction

During industrial heat transfer processes, achieving heat flux uniformity poses a very significant challenge. Heat flux uniformity reduces thermal stress and prevents undesirable deformations. It is an important necessity in many fields, such as glass production, food industries, paper dryer, and electronics cooling [1,2]. Heat flux uniformity can be reached by changing the geometrical parameters, fluid properties, and boundary conditions. Attaining this goal is possible by using inverse heat transfer techniques. Obtaining heat flux uniformity by inverse heat transfer method is accomplished in three modes of heat transfer. Most of investigations are performed in radiative heat transfer mode. Lemos et al. [3] applied inverse analysis to specify the geometrical configuration of filament heaters to attain uniform temperature and net radiative heat flux on the target surface. They determined the configuration which satisfied the two conditions on the design surface with an acceptable error. Kowsary et al. [4] compared the variable metric method and conjugate gradient method for optimization of a radiative design problem in order to obtain a specified temperature and heat flux distribution on the target surface. They showed that variable metric method is more efficient. Hosseini Sarvari et al. [5] computed the boundary condition in 3D radiative enclosures to obtain both a pre-scribed temperature distribution and radiative heat flux by Levenberg-Marquart optimization method. Some studies investigated both convective and radiative heat flux uniformity. For example, Birla et al. [6] used radio frequency for uniform heating of oranges and apples. They showed that, temperature uniformity in the fruits was significantly improved by rotation and movement of oranges and apples. Furthermore, Mossi et al. [7] used the inverse design method to obtain temperature and heat...
Impinging jets are widely used for cooling processes. Heat transfer characteristics of liquid and gas jets have been studied numerically and experimentally in recent years [8–11]. Impinging jets are studied mostly in turbulent regime because of their high heat transfer rates. However, laminar impinging jets are applicable for electronics cooling purposes [12–14]. Hosain et al. [15] numerically investigated the flow field and heat transfer of single and co-axial impinging jets, and (3) flexibility of a co-axial impinging jet, in this study. Celik [35] investigated the heat transfer of a co-axial impinging jet. Eren [34] investigated the local and average heat transfer coefficients of a co-axial impinging jet on a heated surface in turbulent regime experimentally. They showed that the heat transfer coefficient of a co-axial impinging jet is higher than a single circular jet with the same Reynolds number. Also they represented that the variation of local heat transfer coefficient of a co-axial impinging jet on the target surface becomes flatter with respect to a circular impinging jet. Celik and Bethenhausen [36] numerically studied the effect of diameter ratio, jet to impingement plate distance and the Reynolds number on the heat transfer coefficient of a co-axial impinging jet in turbulent regime. They showed that by increasing the inner to outer diameter of the co-axial jet from 0.105 to 0.55, local, stagnation and average heat transfer coefficients increase.

Impinging jets are used to obtain uniform heat flux. Local Nusselt number has a maximum in the stagnation point and decreases by getting away from this point. This gradient is very high for circular and slot impinging jets and it is the main challenge of these jets for achieving heat flux uniformity. Therefore, researchers have used moving nozzle to improve heat flux uniformity [37–40]. Also they have used moving nozzle to improve heat flux uniformity [41]. The gradient of local heat transfer coefficient of a co-axial impinging jet can be decreased compared to slot and circular impinging jets according to [34]. Co-axial impinging jets are more flexible and by changing the inner to outer diameter ratio and velocity, a single co-axial impinging jet can reach more uniform heat transfer compared to a circular or slot impinging jet.

Accordingly, due to (1) the importance of heat flux uniformity over the target surface, (2) limitation of a single circular or slot impinging jets, and (3) flexibility of a co-axial impinging jet, in this study, due to (1) the importance of heat flux uniformity over the target surface, (2) limitation of a single circular or slot impinging jets, and (3) flexibility of a co-axial impinging jet, in this study, it seems that co-axial impinging jets can be used as an alternative to circular impinging jets.
study, a co-axial impinging jet was used to obtain heat flux uniformity over the design surface. To the best of authors’ knowledge, co-axial impinging jets have not been used to achieve heat flux uniformity yet. The purpose of this research was to design the geometrical variables of a co-axial jet (inner to outer diameter ratio, jet to target surface distance) and flow variables (Reynolds number and inner to outer velocity ratio) to obtain uniform heat flux over a constant temperature surface. Also a co-axial jet with a new geometry was presented to improve the heat flux uniformity. The Inverse problem was converted into an optimization procedure and the optimum design was obtained.

2. Problem formulation

In order to obtain uniform heat flux, firstly co-axial impinging jet was used. A 3D view of this jet is illustrated in the upper left corner of Fig. 1(a). Axisymmetric boundary condition was utilized due to the geometrical configuration. A schematic diagram of the computational domain is shown in Fig. 1(a). The surface of the impingement wall was imposed by uniform temperature. The purpose was to make the heat flux, which was applied by the co-axial jet (circular jet with radius Ri and vi velocity and annular jet between radii Ri and Ro and with vo velocity) to the target surface (up to radius of Rt), uniform and with a specified quantity.

One of the advantages of co-axial impinging jets is that by changing the inner to outer diameter and velocity ratio, a broad range of the target surface can be cooled with uniform heat flux; whereas, slot and circular impinging jets have limitations. In other words, the designer has more variables at hand to obtain uniformity. However, if the outer jet diameter to target surface diameter ratio becomes less than a limit, co-axial impinging jets cannot obtain uniform heat flux either. That is to say the cooling flow of the co-axial jet does not access the endpoints (areas apart from the center) of the target surface. This region is not affected by the coolant and by changing geometrical and flow variables; the heat flux in these points does not change and remains a small amount. This causes a large error in heat flux uniformity.

To resolve the mentioned problem, a new geometry for the co-axial impinging jet was presented. To this end, unlike the conventional inner and outer co-axial impinging jets which impinge on the target surface vertically, the proposed outer co-axial impinging jet impinges on the surface with a specified angle. The outer jet flows between the surfaces of two cones, instead of two cylinders. The angle of these cones causes the outer jet to flow to the region where the heat flux is small in the conventional co-axial jets (areas apart from the center). The proposed jet is referred in this paper as guided outer flow (GOF) co-axial impinging jet. The 3D view of this jet is illustrated in the top left corner of Fig. 1(b). The computational domain of this jet is also shown in this figure.

2.1. Flow field calculation

Ambient air was used as the coolant fluid, similarly to usual jets. Air is assumed to be a Newtonian fluid and its properties were considered to be constant and have the values at the average temperature of inlet and target surface temperatures (25 °C and 70 °C, respectively). Flow is assumed to be axisymmetric and steady. The non-dimensional continuity, axial momentum, radial momentum, and energy equations are, respectively, given as:

\[ \frac{\partial v_r}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0, \]  

\[ \frac{\partial v_r}{\partial x} + \frac{v_r}{r} \frac{\partial v_r}{\partial r} = \frac{1}{Re} \left[ \frac{\partial^2 v_r}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) \right] - \frac{1}{Fr^2}, \]  

\[ \frac{\partial v_r}{\partial x} + \frac{v_r}{r} \frac{\partial v_r}{\partial r} = \frac{1}{Re} \left[ \frac{\partial^2 v_r}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) \right], \] and

Fig. 1. Schematic of computational domain and three dimensional; (a) co-axial jet (b) GOF co-axial jet.
\[ v_x \frac{\partial \theta}{\partial x} + v_r \frac{\partial \theta}{\partial r} = \frac{1}{Re Pr} \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right] \]  

Mean velocity of circular and annular jets was used to nondimensionalize velocity and pressure. The dimensionless variables are defined as:

\[ r^* = \frac{r}{D_0}, \]

\[ x^* = \frac{x}{D_0}, \]

\[ V_{\text{mean}} = \frac{V_0 A_i + V_o A_0}{A_i + A_0}, A_i = \frac{\pi D_i^2}{4}, A_0 = \frac{\pi (D_0^2 - D_i^2)}{4}, \]

\[ v_i^* = \frac{V_i}{V_{\text{mean}}}, \]

\[ v_o^* = \frac{V_o}{V_{\text{mean}}}, \]

\[ P^* = \frac{P - P_o}{\rho V_{\text{mean}} ^2}, \]

\[ \theta^* = \frac{T - T_w}{T_{\infty} - T_w}, \]

\[ Re = \frac{\rho V_{\text{mean}} D_0}{\mu}, \text{ and} \]

\[ Fr = \frac{V_{\text{mean}}}{\sqrt{g D_0}}. \]

Boundary conditions in dimensionless form for conventional co-axial impinging jet are illustrated in Fig. 2.

Boundary conditions for the GOF co-axial impinging jet are almost the same as conventional co-axial impinging jet with the difference being that for the outer jet. Therefore, the inlet velocity is applied as:

\[ v_i^* = \frac{V_o}{V_{\text{mean}}} \sin \alpha, \]

\[ v_o^* = \frac{V_o}{V_{\text{mean}}} \cos \alpha. \]  

Governing equations considering the boundary conditions were solved with the finite volume based software. SIMPLE algorithm was used for pressure-velocity coupling [42]. By solving the equations, velocity and temperature fields were obtained and the distribution of Nusselt number over the target surface was obtained. The Nusselt number is defined as:

\[ Nu = \frac{h D_o}{k_{\text{fluid}}} \]  

A structured grid was used. The grid near the target surface is much denser due to the high gradient of variables. The number of grid points was changed according to the changing of geometry during the optimization process. For the specified value of design variables \( (D_i/D_o = 0.4, Re = 400, D_i/D_o = 0.5, H/D_o = 2.5 \) and \( V_0/V_i = 2.5), \) four grids were considered. The grids’ specifications are reported in Table 1. Since the purpose of this study was the investigation of the Nusselt number distribution on the target surface, Nusselt distribution of four grids are plotted and compared in Fig. 3. The relative error with respect to the finest grid, which is defined as follows, was calculated and is represented in Table 1.

\[ \text{error}_{\text{grid}} = \left( \frac{\sum_{i=1}^{N} (Nu_{\text{grid}i} - Nu_{\text{finest grid}i})}{\text{Nu}_{\text{finest grid}}} \right)/N \]

Since the direct problem should be solved many times in the optimization process, short run time and acceptable error are the two parameters of choosing the appropriate grid. 105 \( \times 79 \) grid with 70 cells on the target surface was used for numerical solution due to its reasonable run time and very small error with respect to the finest grid (1.78%). Therefore grid independency was checked.

**2.2. The inverse problem**

Since the temperature of the target surface was set to be constant, considering the Newton’s law of cooling \( (q^* = h(T_w - T_{\infty})), \) for achieving uniform heat flux over the target surface, heat transfer coefficient on the surface should be uniform. Therefore uniform
Nusselt number distribution over the target surface (dimensionless form of heat transfer coefficient) yields heat flux uniformity. The inverse problem was converted to an optimization procedure [38,39], in which the objective function is formulated as a root mean square error. When the heat flux becomes completely uniform and equal to the desired magnitude all over the surface, the objective function becomes zero. In this situation, there are no constraints on the average values of the Nusselt number. However, when the goal is to reach a uniform distribution of Nusselt number on the surface, the average value of Nusselt number would get closer to the desired Nusselt number. Therefore, for a total of N grid points on the target surface, the objective function is defined as:

$$E_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{Nu_D \text{avg}_i - Nu_D}{Nu_D} \right) \times 100}^2.$$  \hspace{1cm} (18)

Outer co-axial jet diameter (Do) is considered as a fraction of the target surface. Therefore, Dp/Do and also the design Nusselt number were constant for each optimization procedure. The optimization problem was solved for various values of Do/Do and NuDo.

Four design variables for conventional co-axial impinging jets and their limits are as follows in dimensionless form:

1. Inner to outer diameter ratio (Di/Do), 0.1 < Di/Do < 0.9.
2. Jet to target surface distance to outer diameter ratio (H/Do), 0.5/(Di/Do) < H/Do < 1.1428/(Di/Do).
3. Outer (annular jet) velocity to inner (circular jet) ratio (vi/vi), 0.2 < vi/vi < 5.
4. Co-axial Reynolds number (Re), 50 < Re < 350.

Therefore, for each case of inverse problem, the above mentioned design variables were calculated to reduce the objective function as much as possible. For the GOF co-axial impinging jet, two design variables were added to the four variables presented above.

5. Inner to outer diameter of outer jet, (Dw/Do), 0.1 < Dw/Do < 0.9.
6. Outward angle of outer jet with axial direction (\(\alpha\)), 0 < \(\alpha\) < \(\arctan(1 - Dw/Do)\).

Limits of \(\alpha\) are defined in a way that with its minimum value (0), the jet is impinged on the surface perpendicularly, and at the maximum value, the angle is determined in a way that the outer jet is impinged on the end of the target surface. Also in the GOF co-axial jet, due to geometrical limitations, the constraint

$$D_l < D_w$$  \hspace{1cm} (19)

was considered.

Pattern search method [43] was used for the optimization problem. This method is an efficient approach to find the global minimum for problem types that have many local minimums. In the pattern search method, two types of movements are made to choose a new iterate. Exploring the local behavior of objective function by changing the parameters involved and, utilizing a larger search to improve the search direction. After each success in reducing the magnitude of objective function, the searching domain for each design variable would be doubled and, conversely, after each failure in reducing the magnitude of objective function, the searching domain would be reduced to half of its previous magnitude. For each optimization problem of the present investigation, the solution was assumed to be converged when the searching domain for each design variable reduced to \(10^{-6}\). The solution of the problem by using the Pattern search method quickly converges to values in the neighborhood of the minimum point, but it takes several iterations to find the minimum point exactly. Therefore, if the pattern search does not converge after a specified number of function evaluations (3 0 0), the process is continued by a gradient-based method, which works very efficient in the vicinity of the minimum point. In the gradient-based method, the central difference was used for computing the differentiation, and the solution was assumed to be converged when the searching domain for each design variable reduced to \(10^{-6}\) or when the number of function evaluation reached 1000.

Thus, the following procedure was implemented.

1. Initial guess is considered for four design variables. (Di/Do, H/Do, vi/vi and Re)
2. CFD solver computes the Nusselt number distribution on the target surface considering design variables of previous step.
3. The objective function is computed by Eq. (18).
4. Design variables are specified by means of the pattern search algorithm or gradient-based method (if iterations of pattern search method are terminated).
5. If the convergence criterion is satisfied, the process is stopped, otherwise it goes to step 2.

For the GOF co-axial jet, the same procedure was carried out, and two design variables (Dw/Do and \(\alpha\)) were added to the four previous variables.

3. Results and discussion

3.1. Verification

Since heat transfer of the co-axial impinging jet has not been studied in the laminar regime, results of a circular impinging jet in the laminar regime were compared with the present solution in this section. The circular impinging jet was selected due to its axisymmetric similarity to the co-axial impinging jet. The comparison was made with both experimental and numerical studies.

First, results were compared with experimental data of Scholtz and Trass [44]. The Sherwood number distribution in the mentioned paper was converted to the Nusselt number distribution by considering \(\sqrt{Re}\) coefficient for Re = 200. Furthermore, they defined the Sherwood number based on the radius of the target surface; whereas in this study, the diameter was used for the Nusselt number definition. Therefore, the experimental data values were doubled to address the use of diameter instead of radius. Results, plotted in Fig. 4(a), are compared at Re = 200 and the nozzle to surface distance to nozzle diameter ratio of 0.375. As can be seen, the results match with experimental data with acceptable error from the axis of symmetry to \(r/R_{nozzle} = 1\). The gap between results of this study and the experimental data for \(r/R_{nozzle} > 1\), is due to the fact that the jet in this study is confined; whereas, in the experimental setup of [44] it is not. Numerical results have a relative error of 7.72% with respect to experimental results.

In addition, the results of this study were compared with the numerical results of Chatterjee and Deviprasath [45] for a confined jet in Re = 100 and H/D = 0.25 in Fig. 4(b). The results have an acceptable relative error of 0.26%.

3.2. Convergence history

Heat flux uniformity was investigated for three design Nusselt numbers of 7, 10, and 13 and for each of them, five diameter ratios (Di/Do) of 1, 0.8, 0.6, 0.4, and 0.3 were studied. These 15 cases were examined in both the co-axial jet and the GOF co-axial jet and the optimization procedure was performed in each case.
The desired Nusselt numbers (7, 10, and 13) were chosen by considering the range of Nusselt numbers used in similar previous works in which laminar impinging jets were used for uniform cooling. The purpose of choosing these values (7, 10, and 13) is mainly based on the Forouzanmehr et al. work [38]. They used multiple impinging slot jets for reaching heat flux uniformity. Therefore, these values are chosen for comparing the results and, indeed, to show that uniformity in cooling of a hot surface could be achieved by using only one co-axial jet.

Due to the fact that there exist 15 convergence histories for design variables, objective functions, and local Nusselt distributions for co-axial and GOF co-axial jets, they are plotted only in a handful of cases.

- Local Nusselt number distribution

Convergence history of local Nusselt number distribution is plotted in one case for the two jet configurations as an example. These distributions for the design Nusselt number of 13 and diameter ratio of 0.8 for the co-axial jet and the design Nusselt number of 10 and diameter ratio of 0.6 for the GOF co-axial jet are illustrated in Fig. 5(a) and (b) for specified numbers of iterations, respectively.

- The objective function

Convergence histories of objective function for $N_{u0} = 10$ and various diameter ratios of the conventional co-axial jet are plotted in Fig. 6(a). They are also illustrated in Fig. 6(b) for $N_{u0} = 10$ and diameter ratios of 0.6 and 0.4 for the two jet configurations. Results indicate that satisfactory convergence is achieved after almost 300 iterations.

- Design variables

Convergence histories of four design variables ($D_i/D_o$, $H/D_o$, $V_o/V_i$, and $Re$) for the conventional co-axial jet are plotted in Fig. 7. In this figure, curves are depicted for different diameter
ratios for the case $N_u_D = 10$. Step variations in the convergence histories of design variables are a consequence of using pattern search algorithm. Also, drastic changes in the last steps of the condition in which $D_o/D_i = 1$, is a result of implementing the gradient-based method after the pattern search method in the vicinity of the optimum point. There is not this drastic change in the other cases, because the pattern search had reached to the optimum point and the gradient-based method converged quickly.

### Table 2
Dimensionless design variables and error in optimal state for co-axial jet.

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<th>Re</th>
<th>$E_{rms}$ (%)</th>
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### Table 3
Dimensionless design variables and error in optimal state for GOF co-axial jet.

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3.3. Results of the optimum state

Dimensionless design variables and objective functions (error with respect to full uniformity) in the optimum state for the co-axial jet and the GOF co-axial jet are given in the Tables 2 and 3, respectively. Inverse problem was solved for 15 cases in each table.

As can be seen in Table 2, the co-axial jet is suitable and has an acceptable error for $D_0/D_1 \geq 0.8$. The advantage of GOF co-axial jet
is obvious at lower diameter ratios (0.4 and 0.3) where the error is reduced significantly.

Local Nusselt number distributions in the optimum state for diameter ratios of 1, 0.6, and 0.4 for the co-axial jet are plotted in Fig. 8(a–c), respectively. This is also done for the GOF co-axial jet in Fig. 8(d–f). In each figure, Nusselt number distributions are illustrated for the design Nusselt numbers of 7, 10, and 13. Errors are shown in the right top corner of these figures. In Fig. 8, it is observed that the two jet configurations obtained uniform heat flux up to 0.6 diameter ratio. For \( \frac{D_o}{D_i} = 0.4 \), both of them are far from uniformity, but the non-uniformity error for the GOF co-axial jet is much less.

3.4. Discussion of optimum results

In this section, local Nusselt number distribution, objective function, and Reynolds number are compared in the optimum state at different diameter ratios and design Nusselt numbers for the two jet configurations. Nusselt number distributions for \( \text{Nu}_D = 10 \) and diameter ratios of 1, 0.8, 0.6, 0.4, and 0.3 are plotted for the co-axial jet in Fig. 9(a). In this figure, it is observed that the optimum local Nusselt number distribution gets far from uniformity by decreasing the diameter ratio. Variations of objective function versus decreasing diameter ratio for three design Nusselt numbers of 7, 10, and 13 are plotted in Fig. 9(b) and (c) for the co-axial jet and the GOF co-axial jet, respectively. It is observed that \( \text{Erms} \) generally increases as diameter ratio decreases. However, the objective function decreases when changing the diameter ratio from 1 to 0.8, as it is not designed for high diameter ratios. Y-axis of these figures is plotted in the logarithmic scale. It can be seen that the variation is almost linear in this diagram. Therefore \( \text{Erms} \) increases almost exponentially by decreasing the diameter ratio.

In addition, The amounts of objective functions for different desired Nusselt numbers in a specific situation \( \left( \frac{D_o}{D_i} \right) \) are in the same order of magnitude and there is not any significant difference between them (Fig. 9(b) and (c)). Therefore, the objective function does not have any meaningful trend with respect to the desired Nusselt number, and it varies arbitrarily. Besides, for the conditions \( \frac{D_o}{D_i} = 1, 0.8, \) and 0.6, the amounts of objective functions have relatively low order of magnitude. In other words, in these conditions, the intended uniformity had been almost achieved and therefore, the variation in the objective function with respect to the Nusselt number does not have any meaningful trend.

In Fig. 8(c), it is seen that due to a small value of diameter ratio (i.e. \( \frac{D_o}{D_i} = 0.4 \)), the Nusselt number in the optimum state significantly decreases at the points of target surface which are far from the jet nozzle. This is because the coolant cannot reach these points. This decrease can also be seen in Fig. 8(f) for the GOF co-axial jet, but the reduction is not as much and the Nusselt number comes closer to the design value.

For comparing the two configurations of jet, Nusselt distribution of the co-axial jet and the GOF co-axial jet are plotted simul-
Simultaneously for two cases of \( \text{Do}/D_t = 0.4, \text{Nu}_D = 7 \) and \( \text{Do}/D_t = 0.4, \text{Nu}_D = 10 \) in Fig. 10(a) and 10(b), respectively. It can be seen in Fig. 10(a) that the Nusselt number at \( r/R = 1 \) for the co-axial jet is 5.47; whereas it reaches 6.24 for the GOF co-axial jet. Therefore, the Nusselt number is 11% closer to the design Nusselt number of 7 at \( r/R = 1 \). Also in Fig. 10(b) for \( \text{Nu}_D = 10 \), the Nusselt number at \( r/R = 1 \) reaches 7.86 for the GOF co-axial jet, while it is 6.88 for the co-axial jet which is 9.8% closer to the design Nusselt number.

The variation of objective function of the two jet configurations by decreasing the diameter ratio for the design Nusselt numbers of 7, 10 and 13 are illustrated in Fig. 11(a–c), respectively. It is observed that at the diameter ratio of 1, the conventional co-axial jet works slightly better. By decreasing the diameter ratio, the objective function value of the GOF co-axial jet becomes much lower than that of the conventional co-axial jet. Therefore, the GOF co-axial jet shows its superiority at small diameter ratios.

The variation of Reynolds number in the optimum state for two jet configurations versus the design Nusselt number is illustrated in Fig. 13(a). As can be seen, by increasing the design Nusselt number, more cooling is needed, therefore, the value of Reynolds number increases. Also it is observed that although the GOF co-axial jet provides a better uniformity, it needs more coolant flow rate (higher Reynolds number).

The variation of Reynolds number in the optimum state by decreasing the diameter ratio is shown in Fig. 13(b) for both jets. It can be seen that the Reynolds number increases as diameter ratio decreases. Indeed, by decreasing the diameter ratio, Reynolds number increases due to better cooling of endpoints of the target surface. Also, similar to the previous figure, the Reynolds number of the GOF co-axial jet is higher than that of the conventional co-axial jet.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig11}
\caption{Comparison of the co-axial and GOF co-axial jets, objective function for (a) \( \text{Nu}_D = 7 \) (b) \( \text{Nu}_D = 10 \) (c) \( \text{Nu}_D = 13 \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig12}
\caption{Contours of flow field for \( \text{Nu}_D = 10 \) and (a) \( \text{Do}/D_t = 0.3 \) of co-axial jet, (b) \( \text{Do}/D_t = 0.4 \) of co-axial jet, (c) \( \text{Do}/D_t = 0.3 \) of GOF co-axial jet, (d) \( \text{Do}/D_t = 0.4 \) of GOF co-axial jet.}
\end{figure}
Advantages and disadvantages of the co-axial jet were compared with the GOF co-axial jet. The GOF co-axial jet showed its superiority at the small diameter ratios (0.4 and 0.3) that reduced the error significantly (about 50%) for Nu_D = 7 and 10. Since the co-axial jet could not cool the endpoints of the target surface efficiently, the GOF co-axial jet was used and the Nusselt number of endpoints got 10% closer to the design Nusselt number. Moreover, optimum local Nusselt number distribution stopped being uniform by decreasing the diameter ratio and Erms increased almost exponentially by decreasing diameter ratio. In addition, by increasing the design Nusselt number and decreasing the diameter ratio, more cooling was needed, therefore, the value of Reynolds number increased.

4. Conclusion

In this study, uniform heat flux over the target surface was achieved by a co-axial impinging jet. The inverse design problem was formulated as minimization of the root mean square difference between the local Nusselt distribution and the design Nusselt number as the objective function. The combination of pattern search and gradient based methods was used for optimization. A co-axial jet with a new geometry was presented to improve the heat flux uniformity. Optimization was performed for the design Nusselt numbers of 7, 10, and 13 and for each one, the outer jet to target surface diameter ratios of 1, 0.8, 0.6, 0.4 and 0.3 were considered. Heat flux uniformity was achieved by the two jet configurations for D_o/D_t ≥ 0.6 with acceptable errors less than 2%. The GOF co-axial jet showed its superiority at the small diameter ratios (0.4 and 0.3) that reduced the error significantly (about 50%) for Nu_D = 7 and 10. Since the co-axial jet could not cool the endpoints of the target surface efficiently, the GOF co-axial jet was used and the Nusselt number of endpoints got 10% closer to the design Nusselt number. Moreover, optimum local Nusselt number distribution stopped being uniform by decreasing the diameter ratio and Erms increased almost exponentially by decreasing diameter ratio. In addition, by increasing the design Nusselt number and decreasing the diameter ratio, more cooling was needed, therefore, the value of Reynolds number increased. Although GOF co-axial jet provided better uniformity, it needed a higher flow rate of coolant.

References