Methodology for Estimation of Local Convective Heat Transfer Coefficient for Vapor Condensation

MEHDI BAZARGANI and FARSHAD KOWSARY
Mechanical Engineering Department, University of Tehran, Tehran, Iran

This paper aims to present an effective two-dimensional inverse heat conduction technique and an experimental design for accurately estimating the local convective heat transfer coefficient of vapor condensation over a conical surface, given temperature measurements at some interior locations. The functional form for the heat transfer coefficient is not known a priori. The method uses a sequential procedure together with Beck’s function specification approach. Solution accuracy and the effects of experimental errors are examined using simulated temperature data. It is concluded that a good estimation of space-variable heat transfer coefficient can be made from the knowledge of transient temperature recordings using the proposed inverse heat conduction problem method. The method is also used in a series of numerical experiments to provide the optimum experimental design for condensation heat transfer investigation.

INTRODUCTION

There are several techniques used in evaluating the local convective heat transfer coefficient. Thermochromic liquid crystal methods utilize temperature maps obtained from liquid crystals applied to a constant heat flux surface and Newton’s law of cooling [1]. Optical measurements such as differential interferometry and infrared temperature mapping methods have also found widespread application [2]. They can be used to determine the local temperature gradients in the fluid and the heat transfer coefficient at the solid surface.

However, these methods either require expensive or delicate equipment or have limitations in terms of high temperatures and turbulent flow fields. An alternative method for estimating heat transfer coefficient is the inverse heat conduction technique. The inverse heat conduction problem solution is verified as a useful tool for obtaining the inner heat transfer coefficient [3]. The spatial distribution of internal heat transfer coefficients of turbine blade has been estimated using the inverse heat transfer technique [4].

The inverse heat conduction problem (IHCP) is a typical example of a mathematically ill-posed problem. There are several approaches used in solving IHCP, and researchers continue to develop more powerful methods. Least-squares techniques, regularization, filtering, and optimization methods have been used to treat the ill-posed nature of the problem. The finite-difference method in conjunction with the least-squares scheme and experimental temperature data has been used to predict the average heat transfer coefficient and fin efficiency on the fins of annular-finned tube heat exchangers and on the vertical square fin of the one-circular tube plate finned-tube heat exchangers [5, 6]. The most well-known methods are the sequential specification, Tikhonov regularization, combined function specification and Tikhonov regularization, and the conjugate-gradients methods. De Carvalho et al. [7] have used the conjugate-gradient method to estimate the unknown boundary surface heat flux in workpieces during grinding. The conjugate-gradient method has also been used to determine the overall heat transfer coefficient in a partially filled rotating cylinder [8]. A full description of fundamental concepts and an extensive bibliography and survey on IHCP methods can be found in Beck et al. [9], Alifanov [10], Özisik and Orlande [11], Tikhonov and Arsenin [12], Kuprisz and Nowak [13], Hensel [14], and Murio [15].

Little work is available in the area of inverse estimation of heat transfer coefficient in condensation. The inverse analysis of a heat transfer problem like that of condensation can be a powerful technique to estimate the unknown thermal boundary conditions. Hsu et al. [16, 17] successfully estimated the wall heat flux in film condensation on a vertical surface and a horizontal elliptical tube by an inverse model that is based on
film condensation thickness readings taken at several different points on the surface.

The purpose of the present work is to use the solution of a transient, sequential inverse heat conduction technique to estimate local convective heat transfer coefficient for vapor condensation on a conical surface and find an optimum experimental design. The input data (simulated temperatures) for IHCP are generated using an implicit finite difference (IFD) scheme. The temperatures at discrete regular times are polluted by adding random Gaussian errors, produced by a random generator.

ANALYSIS

The Direct Method

Due to the axisymmetry present in the geometry (see Figure 1), the governing differential equation of the transient heat conduction in cylindrical coordinates \((r, z)\), assuming constant thermal properties, is expressed as

\[
\frac{1}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{1}
\]

with the boundary conditions and initial condition

\[
\frac{\partial T}{\partial n} \bigg|_{\text{surf}-1} = \frac{\partial T}{\partial n} \bigg|_{\text{surf}-3} = 0 \tag{2}
\]

\[-k \frac{\partial T}{\partial n} \bigg|_{\text{surf}-4} = q(s, t) \tag{3}
\]

\[-k \frac{\partial T}{\partial n} \bigg|_{\text{surf}-2} = h_m (T - T_\infty) \tag{4}
\]

\[T|_{t_i} = 0 = T_0 \tag{5}\]

In the inverse problem the boundary condition on surface 4, \(q(s, t)\), is unknown, and to compensate for that, there is information available in form of a vector containing measured temperatures. This vector is denoted by \(Y\). As in real situations the measured temperatures \(Y_i\) are affected by errors, random errors are added to the produced temperatures from the solution of IFD. That is,

\[Y_i = T_i + \varepsilon_i \tag{6}\]

where \(T_i\) are calculated temperatures at sensor locations and \(\varepsilon_i\) is random error with Gaussian distribution, zero mean, and unit standard deviation \((\sigma)\). The generated errors are uncorrelated.

The convective heat transfer coefficient is calculated using the Newton’s law of cooling,

\[h(s, t) = \frac{q(s, t)}{T_{\text{sat}} - T(s, t)} \tag{7}\]

where the surface temperature \(T(s, t)\) is a by-product of the heat flux estimation; because of the diffusive nature of heat flow, the thermal response at an inner location of the heat conducting body is damped and lagged with respect to the active input at the boundary. This implies that in many cases the problem may present a low or insufficient sensitivity of measured temperatures to the unknown heat flux. The relationship between the thermal response and the unknown input can be expressed through a sensitivity matrix. The sensitivity matrix tends to be quasi-singular. This explains the principal difficulty of the IHCP.

The sensitivity coefficient is defined as the first derivative of the measured temperature with respect to the desired heat flux parameter. In a distinct time \(i\) at the sensor position \(j\) for the heat flux parameter of \(q_p\) at time \(m\), the sensitivity coefficient is given as

\[X_{jp}(m, i) = \frac{\partial T(s_j, t_i)}{\partial q_{pm}} \tag{8}\]

The governing differential equation for \(X_p\) with its boundary conditions and initial condition is readily derived from the heat equations, Eq. (1–5), by taking the derivative with respect to \(q_p\) as

\[
\frac{1}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial X_p}{\partial r} \right) \right) + \frac{\partial^2 X_p}{\partial z^2} = \frac{1}{\alpha} \frac{\partial X_p}{\partial t} \tag{9}
\]

\[-k \frac{\partial X_p}{\partial n} \bigg|_{\text{surf}-4} = \begin{cases} 1 & t_m - 1 \langle t_m \langle t_{m-1} \\ 0 & \text{otherwise on interval } p \tag{11a}\end{cases}
\]

\[-k \frac{\partial X_p}{\partial n} \bigg|_{\text{surf}-4} = \begin{cases} 0 & t_m - 1 \langle t_m \langle t_{m-1} \\ 0 & \text{otherwise on other intervals} \tag{11b}\end{cases}
\]
where surface 4 is divided to p subsurfaces (see Figure 2),

\[-k \frac{\partial T_p}{\partial n} \bigg|_{surf=2} = h_a X_p \]  \hspace{1cm} (12)

\[X_p = 0(t \leq t_{m-1}) \]  \hspace{1cm} (13)

**The Inverse Method**

The condensation surface is divided to several subsurfaces and the time-variable heat flux is estimated on each surface. Then using Newton’s law of cooling, the local convective heat transfer coefficient is calculated. The inverse method used in this paper is the sequential function specification method [9]. For stability of results, heat fluxes of r future time steps are temporarily assumed to be constant. It is assumed that heat fluxes from time 1, 2, \ldots, (m - 1) have been estimated and now the unknown heat flux at time m is to be evaluated.

The temperatures in any one-, two-, or three-dimensional body with temperature-independent thermal properties can be given in the standard form of

\[T = T_{\mid q=0} + Xq \]  \hspace{1cm} (14)

The symbol \(T_{\mid q=0}\) means the calculated temperature vector \(T\) at the sensor locations using estimated \(q\) vector of time 1, 2, \ldots, (m - 1) and setting \(q\) vector of r future times to zero.

For \(Np\) heat flux parameters, \(Ns\) temperature sensors, and \(r\) future times,

\[T = \begin{bmatrix} T(m) \\ T(m+1) \\ \vdots \\ T(m+r-1) \end{bmatrix} \]  \hspace{1cm} (15)

where \(T(i)\) is the vector of \(Ns\) measurements, which makes \(T\) an \(Ns \times r\) matrix. Also,

\[q = \begin{bmatrix} q(m) \\ q(m+1) \\ \vdots \\ q(m+r-1) \end{bmatrix} \]  \hspace{1cm} (16)

where \(q(i)\) is the vector of \(Np\) heat flux components, which makes \(q\) an \(Np \times r\) matrix. Further,

\[X = \begin{bmatrix} a(1) & a(1) & \cdots & a(1) \\ a(2) & a(2) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ a(r) & a(r-1) & \cdots & a(1) \end{bmatrix} \]  \hspace{1cm} (17)

**Figure 4** Sensitivity coefficient for \(q_6\) for two sensors located close to the active surface and far from the active surface.
where $X$ is an $N_s \times N_p$ matrix and

$$a(i) = \begin{bmatrix} a_{11}(i) & a_{12}(i) & \cdots & a_{1N_p(i)} \\ a_{21}(i) & a_{22}(i) & \cdots & a_{2N_p(i)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N_s1(i)} & a_{N_s2(i)} & \cdots & a_{N_sN_p(i)} \end{bmatrix},$$

$$a_{jp(i)} = \frac{\partial T(s_j, t_i)}{\partial q_p(1)}$$

(18)

where $a(i)$ is a matrix of $N_s \times N_p$.

For example, for the first two times (associated with $t_m$ and $t_{m+1}$), Eq. (15) becomes

$$T(m) = T(m)|_{q(m)=0} + a(1)q(m)$$

$$T(m + 1) = T(m + 1)|_{q(m+1)=0} + a(1)q(m + 1) + a(2)q(m)$$

(19)

where both these matrix equations represent $N_s$ scalar equations.

Note that there are $N_p$ unknown heat flux components at each time $t_m$. There are $N_s$ measurements at that time and $N_s$ must be no less than $N_p$. To produce stable results, we use $r$ matrix equations in a least-squares method. The sum of the square of the difference between the calculated and measured temperature is

$$S = (Y - T)\text{T}\, (Y - T)$$

(20)

with the temporary assumption of

$$q_1(m) = q_1(m + 1) = \cdots = q_1(m + r - 1)$$

$$q_{N_p}(m) = q_{N_p}(m + 1) = \cdots = q_{N_p}(m + r - 1)$$

(21)

The function to minimize is

$$S = (Y - T|_{q=0} - Zq)\text{T}\, (Y - T|_{q=0} - Zq)$$

(22)
where
\[
Z =XA
\]  
\[
A = \begin{bmatrix} A(1) \\ \vdots \\ A(r) \end{bmatrix}
\]  

For constant \( q \) assumption, \( A(i) = I_{Np \times Np} \), where \( I \) is the unit matrix. The matrix derivative of Eq. (22) with respect to \( q \) gives the estimator equation,
\[
\hat{q} = (Z^TZ)^{-1}Z^T (Y - T|_{q=0})
\]  

To calculate the vector of \( q \) in time, \( m \) is set to 1 and estimation begins with the calculation of \( T|_{q=0} \) using the direct problem and then Eq. (25) is applied. After that, \( m \) is increased by 1, \( T|_{q=0} \) is recalculated, and the estimator equation is used again.

**Experimental Setup Design**

The simulated experiment is conducted on a conical surface, over which the water vapor is condensed. The material of conical body could be from copper or aluminum. The default for material is copper. The base of the conical body is insulated using Teflon (as shown in Figure 3). In order to make a steady-state condition possible, the cone is internally cooled using circulated water. The distribution of the heat transfer coefficient of the cooling water on the inside surface of the cone is acquired using a finite-volume numerical algorithm. Physical dimensions of the cone are \( L = 15 \) cm, \( \gamma = 30^\circ \), \( b = 10 \) cm, and \( a = 4 \) cm. The volumetric flow rate of the required water is 100 l/h. Figure 3 shows the schematic diagram of the experimental setup. The experimental setup consists of a cone, a pump for circulating water, eight thermocouples that are all positioned at the same depth of 1 mm from the outer surface of the cone and 37.5 mm from each other, and a data acquisition system.

To design the best experimental setup, the sequential function specification method must be optimized. Therefore, parameters...
such as measurement time-step size ($\Delta t_{\text{meas}}$), the number of future time steps ($r$), and number of estimated unknowns must be chosen so to minimize the effects of noise. To estimate local heat transfer coefficient, the outer surface is divided into some subsurfaces. Increasing the number of subsurfaces gives a better resolution for the estimated local convective heat transfer coefficient. For this experimental setup, first, the number of subsurfaces was specified to be five. After that the results were analyzed. Then the number of subsurfaces was increased until it reached 10 and at every stage the results were analyzed. The optimum number of subsurfaces was specified to be 8 because increasing the number of subsurfaces creates instabilities in the inverse solution. After determining the number of subsurfaces, the number of thermocouples used to measure temperatures should be specified. If the number of thermocouples is chosen less than the number of subsurfaces (8), the inverse algorithm will be unstable. For the numbers 8 and 9, the results are close to each other; however, the numbers greater than 9 cause sensitivity coefficients to be correlated, and then the results will be worse. Thus, in this experimental design, the optimum number of thermocouples is determined to be 8. These thermocouples are located close to the external surface (active surface) in order to alleviate damage due to ill-conditioning of inverse estimation. If the sensors are located close to the active surface, the sensitivity coefficient is increased. Figures 4 and 5 show the sensitivity coefficient for $q_6$ and $q_8$ for two sensors located close to the active surface and far from the active surface. Distribution of calculated and simulated temperature at the situation of 8 sensors (sensors are positioned at the same depth of 1 mm from the outer surface of the cone and 37.5 mm from each other; see Figure 3) is shown in Figure 6.

### RESULTS AND DISCUSSION

There are two indices of error in the estimation in an inverse heat conduction problem. The first index of error is the unavoidable bias deviation (deterministic error). When the measurements are error free, the deterministic error measures how well the estimated heat transfer coefficient matches the exact heat transfer coefficient. The second index is the variance of estimations due to the amplification of measurement errors (stochastic error). The global effect of deterministic and stochastic errors is considered in the mean squared error or total error. The total error (RMS), the bias (D), and the variance (V) are defined by Eqs. (26)–(28), respectively:

$$\text{RMS} = \frac{1}{(\text{Nm} \cdot \text{Np})} \left( \sum_{j=1}^{N_p} \sum_{i=1}^{N_m} (h_j(i) - \hat{h}_j(i))^2 \right)^{1/2}$$  \hspace{1cm} (26)

$$D = \frac{1}{(\text{Nm} \cdot \text{Np})} \left( \sum_{j=1}^{N_p} \sum_{i=1}^{N_m} (h_j(i) - \hat{h}_j(i) |_{\sigma=0})^2 \right)^{1/2}$$  \hspace{1cm} (27)

$$V = \text{RMS}^2 - D^2$$  \hspace{1cm} (28)

![Figure 13](image.png)

**Figure 13** Comparison of the first and fifth estimated noisy and errorless local convective heat transfer coefficient with the exact data.
Figure 14  Comparison of the second and sixth estimated noisy and errorless local convective heat transfer coefficient with the exact data.

where $Nm$ is the number of measurements and $Np$ is the number of parameters being measured.

The best estimation is obtained from the minimization of total error (RMS), which gives the necessary balance between the two error sources. These errors measure the quality of the inverse solution and can be used to determine the critical factors in the experimental design; factors like the optimum number of future time steps, $r$, and the time step size, $\Delta t$.

The influences of time step size and $r$ can be seen in Tables 1 and 2, where variance and RMS errors show a clear dependence on the intensity of the noise ($\sigma$). Bias, on the other hand, is not very much affected by $\sigma$.

The main challenge of an inverse scheme is the treatment of measurement errors. If errors are amplified notably (i.e., a large increase in standard deviation $\sigma$), it would be more difficult for the inverse scheme to produce stable results.

The problem associated with the choice of $r$ is that while for $r = 1$ or $r = 2$ the estimation tends to be unstable; larger-than-optimum choices of $r$ lead to larger bias, which has an adverse effect on the quality of estimations. Increasing $r$ causes variance error to decrease while increasing the bias error. The time-step size has also a dramatic effect on the quality of final results. Very small time-step sizes result in undesirable fluctuations, whereas very large time-step size choices, while damping the fluctuations, will cause bias to grow significantly. This trade-off between bias and variance happens in inverse heat conduction problems. In this experimental setup a good choice of time step size is between 0.2 (s) and 0.5 (s) and the optimum $r$ is chosen to be 3 (Tables 1 and 2).

As mentioned previously, the condensation surface is subdivided into eight equally spaced surfaces: $h_i$, $i = 1, \ldots, 8$ denotes the heat transfer coefficient on each subsurface, starting with $h_1$, the heat transfer coefficient on the subsurface near the tip of the cone.

The method calculates the heat transfer coefficient using the transient temperature data. The results are illustrated in Figures 7–10, where a comparison of the estimated noisy and errorless heat transfer coefficient for vapor condensation over the copper conical surface and the exact data can be made (here, $\Delta t_{\text{meas}} = 0.2$ (s), $r = 3$, for noisy data: $\sigma = 0.01$). It can be seen that the time-dependent estimations have good quality and are in good agreement with the exact data (the exact data are obtained from the correlation provided by [18]). Maximum deviation of 7% occurs in the first component. The average deviation is approximately 5%.

In all cases, the steady-state $h$ value is attained for times beyond 150 seconds. The steady-state value of $h$ is that which is of more interest in practical applications. Figure 11 shows the estimated steady-state space-variable convective heat transfer coefficients for vapor condensation over the copper conical surface compared with the exact result. The total error (RMS) due to Eq. (26) in this estimation is 370.

In the present setup, the number of estimation parameters ($Np$) is 8. Although it is hoped that a higher spacewise resolution of $h$ can be achieved when using larger $Np$, there is the drawback space in which the estimation becomes unstable and

Figure 15  Comparison of the third and seventh estimated noisy and errorless local convective heat transfer coefficient with the exact data.

Figure 16  Comparison of the fourth and eighth estimated noisy and errorless local convective heat transfer coefficient with the exact data.

heat transfer engineering vol. 36 no. 9 2015
more sensitive to temperature noise. This is due to the effect that the sensitivity coefficients become more and more correlated yielding a more difficult inverse problem. The effect of \( Np \) on variance is demonstrated in Figure 12.

If the material of conical body is changed to aluminum, which has lower thermal diffusivity than copper, diffusion of heat in this material will be slower than for copper, and in the aluminum body, because of slower diffusion, the thermocouples have more time for recording temperature. Thus, in the same time of recording thermal data by thermocouples, less fluctuation in results will be noticed. The effects of material of conical body on bias, variance, and RMS error is shown in Table 3. As shown in Table 3, for equal time-step size, bias error in the estimation of local convective heat transfer coefficient on the surface, where the material has less thermal diffusivity, is increased, whereas variance error is decreased.

The estimation of noisy and errorless convective heat transfer coefficient for vapor condensation over aluminum conical surface in transient state (\( \Delta t_{meas} = 0.2 \) (s), \( r = 3 \), \( \sigma = 0.01 \)) is shown in Figures 13–16. The estimation of convective heat transfer coefficient for vapor condensation over aluminum conical surface in steady state (\( \Delta t_{meas} = 0.2 \) (s), \( r = 3 \), \( \sigma = 0.01 \)) is shown in Figure 17. In this case the total error (RMS) in the steady state due to Eq. (26) is 373.

**CONCLUSIONS**

The methodology that has been introduced for designing experimental setup for determining local convective heat transfer coefficient for vapor condensation in steady and transient state over a conical body gives results in close agreement with the exact local convective heat transfer coefficient. The major advantage of this method is that it can be carried out with simple and low-cost experimental equipment.

For achieving the optimum experimental design, each one of the different parameters like positions of sensors, number of sensors, number of estimated heat fluxes, number of future time steps, and the time-step size must be selected properly. Otherwise, the results tend to become unstable or include a large bias error.

**FUNDING**

The authors thank the Research Council of the University of Tehran (RCUT) for supporting this work.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>vector of unity matrix</td>
</tr>
<tr>
<td>a, b, L</td>
<td>shape geometric values</td>
</tr>
<tr>
<td>D</td>
<td>bias error</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>n</td>
<td>vector perpendicular to the surface</td>
</tr>
<tr>
<td>Nm</td>
<td>number of discrete measurements</td>
</tr>
<tr>
<td>Np</td>
<td>number of unknown parameters</td>
</tr>
<tr>
<td>Ns</td>
<td>number of sensors</td>
</tr>
<tr>
<td>q</td>
<td>heat flux</td>
</tr>
<tr>
<td>q</td>
<td>heat flux vector</td>
</tr>
<tr>
<td>r</td>
<td>number of future time steps</td>
</tr>
<tr>
<td>r, z</td>
<td>space coordinates</td>
</tr>
<tr>
<td>RMS</td>
<td>total error</td>
</tr>
<tr>
<td>S</td>
<td>sum of squares</td>
</tr>
<tr>
<td>s</td>
<td>coordinate of sensor placement</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>T</td>
<td>vector of calculated temperature</td>
</tr>
<tr>
<td>V</td>
<td>variance error</td>
</tr>
<tr>
<td>X</td>
<td>sensitivity coefficient</td>
</tr>
<tr>
<td>X</td>
<td>sensitivity coefficient matrix</td>
</tr>
<tr>
<td>Y</td>
<td>measured temperature</td>
</tr>
<tr>
<td>Y</td>
<td>vector of measured temperature</td>
</tr>
<tr>
<td>Z</td>
<td>vector of measured temperature</td>
</tr>
</tbody>
</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>thermal diffusivity</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>random error</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>cone angle</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>standard deviation</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>initial state</td>
</tr>
<tr>
<td>i</td>
<td>at time ( t_i )</td>
</tr>
<tr>
<td>j</td>
<td>location index</td>
</tr>
</tbody>
</table>
REFERENCES


Mehdi Bazargani obtained his M.S. in mechanical engineering from the University of Tehran in 2009. His research areas of interest include the use of numerical and analytical methods of solution of the heat transfer problems, simulation of HVAC system and energy of buildings, solution of inverse heat transfer problems, optimization of thermal systems, and numerical simulation of non-Newtonian fluid flow. Currently he is the head of the mechanical engineering department at Tarh Consulting Engineering Company.

Farshad Kowsary is a professor in the field of heat transfer at the University of Tehran, Iran. His research interests are in the area of inverse heat conduction. He has a sizable number of papers in this subject in reputable heat transfer journals. He is a major reviewer for the *Journal of Quantitative Spectroscopy and Radiative Transfer* as well as *Heat and Mass Transfer*. 