GPR random noise reduction using BPD and EMD

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Abstract

Ground-penetrating radar (GPR) exploration is a new high-frequency technology that explores near-surface objects and structures accurately. The high-frequency antenna of the GPR system makes it a high-resolution method compared to other geophysical methods. The frequency range of recorded GPR is so wide that random noise recording is inevitable due to acquisition. This kind of noise comes from unknown sources and its correlation to the adjacent traces is nearly zero. This characteristic of random noise along with the higher accuracy of GPR system makes denoising very important for interpretable results. The main objective of this paper is to reduce GPR random noise based on pursuing denoising using empirical mode decomposition. Our results showed that empirical mode decomposition in combination with basis pursuit denoising (BPD) provides satisfactory outputs due to the sifting process compared to the time-domain implementation of the BPD method on both synthetic and real examples. Our results demonstrate that because of the high computational costs, the BPD–empirical mode decomposition technique should only be used for heavily noisy signals.

Keywords: GPR, noise, filtering, inversion

(Some figures may appear in colour only in the online journal)

Introduction

Ground-penetrating radar (GPR) is a nondestructive geophysical technique to explore structures and subsurface features in shallow depth (up to few meters), which uses high-frequency radio waves in the 10 MHz to 2 GHz range.

Geophysical data processing is an inevitable step in accomplishing an applied study. Most of the signals are nonstationary; this nonstationary character is because of the signal change due to the signal propagation through the Earth; therefore, the analyses of time-variant processes are complex. As a consequence, GPR signal is nonstationary and time-variant, so it requires further consideration to provide reliable results, especially for denoising.

A GPR system consists of a transmitter and a receiver antenna. The apparatus moves along the terrain, and the transmitter antenna emits GPR pulses. When the incident pulse travels through the medium and the electrical/electromagnetic changes, some parts of these pulses will be reflected back to the Earth, and consequently, will be recorded by the receiver. The time delay and amplitude of the reflected pulse are used to address the heterogeneity.

Time-frequency decomposition such as wavelet transform could consider this behavior. Each time-frequency decomposition method has some disadvantages, and these disadvantages have encouraged researchers to find new methods for nonlinear and nonstationary analysis. One of the innovative methods to solve this problem is empirical mode decomposition. The significant advantage of empirical mode decomposition (EMD) is its ability to study the oscillations of the signal at a local level. This localization is very important for basis pursuit denoising (BPD) and its localization for each intrinsic mode function (IMF) provides reliable denoising results. EMD is widely used for signal processing. First, it was used to analyze ocean waves and was then immediately applied in biomedical engineering.

If we consider the Earth as a system the response of it consists of amplitude, phase and frequency alteration to address the change and discontinuity; to understand the system’s behavior we can consider it as a convolution of the
pulse and earth physical changes plus noise.

\[ G(t) = P(t) + E(t) + N(t). \]  

In equation (1) \( P(t) \) is a pulse, \( E(t) \) is earth reflection series and \( N(t) \) is additive white Gaussian random noise. Coherent noises have specific sources such as buried stream channels and multiples. Random noise does not have a specific source, and it may be due to boulders, animal burrows, and tree roots, near vehicles, buildings, fences, power lines, mobile antenna and other unknown sources. Random noise reduces data quality; this reduction causes difficulties in the further processing and interpretation. Therefore, noise attenuation is an important step in data processing. The attenuation of random noise is complicated and, because of its nature, numerous studies have been performed.


Chen and Ma (2014) combined f-x EMD and predictive filtering to attenuate random noise. Oskooi et al (2015) utilized various thresholding methods such as hard, soft, firm shrinkage and non-negative garrote thresholds for GPR noise reduction.

Huang et al (1998) proposed the EMD method and this approach provided a breakthrough in signal analysis. EMD empirically decomposes a complicated, nonstationary and nonlinear signal into a sum of finite intrinsic oscillatory components called IMFs. This composition is based on the nonstationary behavior of GPR signal and allows us to apply the basis pursuit denoising on each IMF. Chen and Donoho (1994) suggested the basis pursuit (BP) method for signal representation. The minimum L1 norm solution has been chosen among many possible solutions to \( y = Hx \). Thus, for GPR noise reduction, two types of real and synthetic data were used to test the proposed methods.

**Methodology**

It can represent a signal in IMFs. IMFs are considered as basic functions. Each IMF has distinctive frequency components, which potentially provides the geologic and stratigraphic information (Han and Baan 2013). Practically, the subsequent IMFs have lower frequency content than the first IMFs. IMFs must satisfy two conditions: (1) the number of extremums, and zero crossings must be equal or vary at most by 1. (2) In each point, the average value of the local maximum’s envelope and local minimum’s envelope is zero. These conditions must be established to ensure that each IMF has localized frequency content, which prevents frequency spreading because of the asymmetric waveform (Huang et al 1998). Figure 1 represents an IMF.

![Figure 1. One IMF of a signal.](image)

The EMD method is displayed as follows:

\[ s(t) = \sum_{n=1}^{N} c_n(t) + r_n(t). \]

Equation (2) shows that the decomposition divides the data into \( N \) components; each is defined by its time scale. The first part with the smallest time scale has the most rapid temporal change (higher frequency), as the decomposition procedure goes forward, the time scale increases and the average frequency of each mode decreases. The sifting process provides the IMFs. The application of the EMD algorithm for a signal \( x(t) \) is as follows:

1. Initialize: \( r_0 = x(t) \), and \( i = 1 \)
2. Extract the \( i \)th IMF \( c_i \)
   - (a) Initialize: \( h_{i(k-1)} = r_{i-1} \), \( k = 1 \)
   - (b) extract the local maxima and minima of \( h_{i(k-1)} \)
   - (c) interpolate the local maxima and the minima by cubic spline lines to form upper and lower envelopes of \( h_{i(k-1)} \)
   - (d) calculate the mean \( m_{i(k-1)} \) of the upper and lower envelopes of \( h_{i(k-1)} \)
   - (e) let \( h_k = h_{i(k-1)} - m_{i(k-1)} \)
   - (f) if \( h_k \) is an IMF and satisfies IMF’s conditions, then set \( c_i = h_k \), else go to step (b) with \( k = k + 1 \)
3. Define the remainder \( r_{i+1} = r_i - c_i \)
4. If \( r_{i+1} \) still has least 2 extrema then go to step (2) with \( i = i + 1 \) else the decomposition process is finished and \( r_{i+1} \) is the residue of the signal.

Figure 2 represents the flowchart of the EMD algorithm. Figures 3 and 4 provide the results of the application of empirical mode decomposition of the synthetic and real signals.

Note that EMD can be used as a temporal filter.

\[ x_{th}(t) = \sum_{j=t}^{N} C_j(t). \]

Many studies have been done to find new signal representations instead of signal decomposition as a superposition of some sine and cosine signals (Fourier transform). Representation should satisfy several conditions such as speed,
sparsity, perfect separation, stability and super-resolution (Chen et al 1998).

BP and BPD are two methods that satisfy the requirements of this research.

**BP**

BP can be expressed by the following equation:

\[
\text{argmin } \| x \| \text{ such that } y = Hx
\]

\[
\| x \| = \sum_{i=1}^{N} |x_i|.
\]

Despite the solution of other methods, the solution of this approach cannot be found explicitly and is achieved with an iterative algorithm. The efficiency of this technique depends on the sparsity of the original signal (Selesnick 2012).

**BP denoising**

Sometimes the signal is not pure, and is corrupted by additive noise:

\[
b' = b + z,
\]

where \(z\) is the noise and does not depend on signal \(b\). The recorded signal is \(b'\). So, the goal is to estimate the noise-free signal \(b\) with the information of \(b'\), in the form of \(y = Ax\).

An approximate solution is accomplished by:

\[
\text{Argmin } \gamma \| x \| + \frac{1}{2} \| Ax - y \|_2^2
\]

\[
\gamma = \sigma \sqrt{2 \log (\#D)},
\]

where \(\sigma\), is the noise level and \(\#D\) is the number of distinct vectors in the dictionary (Donoho 1995). BPD such as BP could be solved by an iterative algorithm. There are different algorithms to solve these kinds of problems. The algorithm used here is the split augmented Lagrangian shrinkage algorithm (SALSA).

The data are stored in a dictionary (a series of \(n\) atoms where each one has the length of \(m\)). The atoms in the dictionary are the elements of the signal that must be exhibited. Most dictionaries are over-complete, and their components are more than \(m\). The redundancy is generated by a combination of several dictionaries, but the goal of BP is to represent signal \(b\) with as few elements as possible; therefore, the best solution is the sparsest one.

The main problem for BPD is in minimizing both terms of equation (7). Minimizing the first term generates a sparse solution, and it reduces the information; so, the signal can be represented by less data, and the second term provides noise attenuation. BP and BPD are convex optimization problems. There are various algorithms to solve these kinds of problems. An appropriate algorithm must be converged fast enough to a solution and should have low computational cost (Selesnick 2012). The SALSA algorithm is used here.

**SALSA algorithm**

Some ill-posed imaging linear inverse problems such as denoising can be considered as a convex optimization problem, which includes minimizing a convex, but possibly non-smooth regularization function (Afonso et al 2011). SALSA, which was developed by Afonso et al (2011) is one of the efficient algorithms used to solve BP and BPD. It is a combination of an augmented Lagrangian and a variable splitting approach which solves inverse problems with sparse regularization. This method changes the constrained problem into an unconstrained one, and then the iterative algorithm is achieved by the application of the augmented Lagrangian method. This iterative method is flexible in solving different problems and is a fast convergence approach that makes it more reliable (Selesnick 2014).

First, the augmented Lagrangian is illustrated as:

\[
\text{argmin } E(z) \text{ such that } Cz - b = 0
\]

\[
L_\alpha(z, \alpha, \mu) = E(z) + \alpha^T (Cz - b) + \mu \| Cz - b \|^2_2,
\]

where \(\alpha\) is the Lagrangian multiplier.

This method searches a positive quantity for \(\mu\) that is similar to a step-like parameter. SALSA is used to solve the BPD. The problem is as follows:

\[
x_{\text{opt}} = \text{argmin } \frac{1}{2} \| y - Ax \|^2_2 + \| \lambda \odot x \|_1
\]

\[
\| \lambda \odot x \|_1 = \lambda_i x_i.
\]
Figure 3. Application of EMD on the synthetic signal.

Figure 4. Application of EMD on the real signal.
When all members of the vectors are \( \lambda \),
\[
\lambda_i = \lambda e^{R_+},
\]
the result is:
\[
\arg\min \frac{1}{2} \| y - Ax \|^2 + \lambda \| x \|_1.
\]
Variable splitting is applied to this formula
\[
\arg\min \frac{1}{2} \| y - Ax \|^2 + \lambda \| x \|_1
\]
such that \( u = x = 0 \).
Variable splitting introduces a new variable and separates the components of the objective function. In other words, it converts the coupling to a constraint.

The augmented Lagrangian is used to solve this formula:
\[
x, u \leftarrow \arg\min \frac{1}{2} \| y - Ax \|^2 + \lambda \| x \|_1 + \frac{\mu}{2} \| u - x - d \|^2.
\]
(11)
The vector \( d \) must be initialized former to the iteration. \( d \) usually is initialized with zero vector. \( \mu \geq 0 \) is the augmented Lagrangian penalty factor.

This minimization can be done for \( u \) and \( x \) separately:
\[
\begin{align*}
u & \leftarrow \arg\min \gamma \| \lambda \odot u \|_1 + \frac{\mu}{2} \| u - x - d \|^2 \\
x & \leftarrow \arg\min \frac{1}{2} \| y - Ax \|^2 + \frac{\mu}{2} \| u - x - d \|^2.
\end{align*}
\]

**Synthetic data**

Successful geophysical denoising depends on two important objectives, first noise attenuation, and second signal preservation; in this way to test the ability of the proposed method, synthetic examples could be useful. Due to kinematic similarities between the electromagnetic (radar) and seismic waves (Jeng et al 2009), synthetic GPR modeling is similar to synthetic seismic modeling; The Ricker wavelet for data modeling is used.

To generate synthetic data the Ricker wavelet with a dominant frequency of 50 MHz is convolved with a refraction coefficient series. The synthetic data are shown in figures 5(a) and 6(a).

After data generation, white Gaussian noise is added to them. The signal to noise ratio of data is 2 dB (figures 5(b) and 6(b)). To provide the noise influence, the amplitude spectrum of the clean and noisy sections are displayed (figures 5(f) and 6(f)).

Figures 5 and 6 show sections severely destroyed by noise, which created an event distortion that made it hard to follow the layer continuity. Then, the BPD was applied to the noisy sections in the time domain (figures 5(c) and 6(c)). The results showed that this approach cannot attenuate noise well, and created signal distortion. For a better comparison, the parameters were set constant for the BPD and BPD-EMD methods. Note that by changing the initial parameters BPD’s results could be more reliable.

In the next step, the BPD–EMD method is applied to the synthetic sections. First, EMD decomposes the signal, and then BPD is implemented on each IMF. As a consequence, for each IMF, the result is sparser. Obviously, this method attenuates noise well (figures 5(d) and 6(d)), and the details provided are better than those provided by the BPD method. Moreover, this approach increases coherency and continuity, and it is more successful in signal preservation.

For better comparison, a median filter as a regular filtering method for commercial purposes was applied to both data types. Figures 5(e) and 6(e) show that the the results are not satisfactory compared to figures 5(d) and 6(d). The power spectrum of all the results of figures 5(f) and 6(f) demonstrate the ability of the BPD–EMD method for denoising.

We can conclude that the median filter compared to the proposed methods does not have satisfactory results for denoising and signal preservation.

**Real Data**

After the synthetic test, the methods are applied to real data (figure 7(a)) to provide the advantages and disadvantages of the methods. The data are contaminated with unknown source noise.
BPD and BPD–EMD apply to this data. Figure 7(b) presents the BPD application to the real data. Comparing the results of figures 7(b)–(d) we can conclude that BPD–EMD attenuates noise well, and it was able to preserve the damaged parts, and although it decreased the distortion in the section, it was satisfactory. For better comparison some parts of the results of the BPD–EMD and BPD and the median filter were marked and the power spectrum supports our findings (figure 7(f)).

Based on the results of BPD–EMD method we can conclude that the random noise was attenuated significantly, and the continuity of events was improved. Furthermore, it is successful in preserving the signal.

**Conclusion**

This article introduced a GPR random noise reduction method using BPD in combination with empirical mode decomposition methods, and synthetic and real GPR data were used to test the ability of the proposed method for denoising. The
results of the synthetic and real data showed that BPD–EMD attenuated noise better than BPD; moreover, BPD produces signal distortions. For the real data example, the continuity of events was extended for both methods and as it showed, the events by BPD eliminated and disappeared but BPD–EMD retained those events. It is concluded that BPD–EMD is a powerful method for GPR random noise attenuation.

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