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Analysis of Raman scattering of self-focused Gaussian laser beam in plasma without WKB approximation

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The self-focusing and propagation of intense Gaussian laser beams in plasmas are investigated, and the explicit form of intensity of self-focused laser beams is derived without the use of WKB approximation. Propagation of self-focused laser beams in plasmas is strongly affected by Raman scattering and Brillouin scattering that are expected for hohlraum targets in inertial confinement fusion. The intensity of Raman and Brillouin scattered waves is derived in paraxial approximation where the effect of plasma temperature and Landau damping is considered through the kinetic theory of plasmas. The effect of plasma temperature and its density, as well as laser wavelength and its intensity, on self-focusing and spatial growth of scattered waves is considered. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4976850]

I. INTRODUCTION

Propagation of laser beams in plasmas is an important issue in many research areas such as X-ray lasers, inertial confinement fusion (ICF), plasma based accelerators, and harmonic generation.1–5 Recent development of high power lasers provides intense pulses with short duration time. Parametric instabilities, filamentation, self-focusing, self-modulation, and wake field generation are examples of physical processes, which are associated with laser propagation in plasmas. Investigation of these physical processes provides better understanding of and innovations in physics of plasmas.6–10 Raman scattering and Brillouin scattering, which can be interpreted as scattering of laser waves from plasmas and ion acoustic waves, are examples of parametric instabilities, and the knowledge of their growth rates and intensities is of importance.

Raman scattering and Brillouin scattering in magnetized plasma have been considered in various studies where the effect of constant magnetic fields on the growth of instability was investigated.11–13 The growth rate of Raman and Brillouin scattering in cold and homogenous plasmas has been evaluated in a number of plasma physics textbooks.14 The temperature of plasma electrons affects the growth rate and gain of Raman and Brillouin scattering due to its disturbance of ion-acoustic frequencies and corresponding Landau damping. Therefore, the kinetic theory of plasmas provides an appropriate model for the study of influence of temperature on the growth rate and gain. Raman scattering and Brillouin scattering were studied by Drake et al. using the kinetic theory.15 The derivation of explicit forms of the intensities of incident and scattered waves in Raman and Brillouin scattering and the effects of plasma and laser characteristics on these quantities are the scope of the present article. A high intensity laser beam propagating in a plasma can be self-focused due to exertion of ponderomotive forces on electrons. In this situation, the plasma acts as a concave lens due to the collection of electrons in high intensity areas. Self-focusing will modify the laser profiles and corresponding intensity. This provides a suitable way to guide lasers in long distances (much longer than the Rayleigh length), which has widespread applications in plasma experiments.16 The theory of self-focusing of a Gaussian laser beam in nonlinear media has been developed by Akhmanov et al.17 Their method is based on application of WKB and eikonal approximation along with eikonal expansion and derivation of a partial differential equation for the spot size of self-focused laser beams in nonlinear media. This approach, which is also followed by other studies (e.g., Refs. 18–20), does not provide any analytical expression for the spot size of self-focused laser beams in plasmas. Yariv and Yeh considered self-focusing and Kerr effects in nonlinear media and derived an analytic expression for the spot size of self-focused laser beams through the generalized ABCD law.21 By modifications of the dielectric and refraction constants, their method can be adopted to a plasma medium. Thus, for the first time, the intensity of propagating laser beams in a plasma medium can be computed analytically without any additional approximation such as WKB method. A high intensity laser beam will be scattered off of a plasma and ion-acoustic waves in Raman scattering and Brillouin scattering, respectively. The intensity of scattered waves is derived through the kinetic theory of plasmas.

The structure of the present article is as follows: In Section II, propagation, self-focusing, and Raman scattering of laser beams in plasmas are investigated without consideration of the WKB approximation method. The expressions for the intensity of laser and scattered waves are derived. A similar analysis for Brillouin scattering is presented in Section III. Section IV is devoted to the discussions of the last two sections along with the presentation of numerical analysis. Conclusions are drawn Section V.

II. RAMAN SCATTERING

Consider a linearly polarized laser beam propagating in plasma media. The normalized vector potential of the polarized laser beam is given by
\[ \tilde{a}_L = \frac{1}{2} a_L \frac{\partial}{\partial z} \exp \left\{ -i(k_0 z - \omega_0 t) \right\}, \]  

where \( a_L, \omega_0, \) and \( k_0 \) are the normalized vector potential, amplitude, frequency, and wave number of the laser beam, respectively. Propagation of the laser in the plasma will perturb its density and generates current density along the shifted frequency (Stokes) and corresponding scattered fields. Beating of laser and scattered fields feeds the perturbation and amplifies their amplitude. Thus, Raman scattering, as instability, will arise in the plasma. The total normalized vector potential will consist of laser and scattered waves and can be written as

\[ \tilde{a} = \tilde{a}_L + \tilde{a}_S, \]

where \( \tilde{a}_S \) is

\[ \tilde{a}_S = \frac{1}{2} a_{sc} \frac{\partial}{\partial z} \exp \left\{ -i(k_s z - \omega_s t) \right\}, \]

where \( a_{sc}, \omega_s, \) and \( k_s \) are the normalized vector potential, amplitude, frequency, and wave number of the backscattered wave, respectively. The total normalized vector potential satisfies the wave equation

\[ \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \tilde{a} = \frac{-4\pi q}{mc^2} J. \]  

Consider the laser intensity as \( a_L^2 = \frac{a_{sc}^2 dz}{W_{\infty}} e^{-\frac{x^2}{W_{\infty}^2}}, \) where \( W_{\infty} \) is the laser spot size and “d” is its value at the entrance plane. Insertion of normalized vector potential \( \tilde{a} \) and current density in Eq. (4), along with the use of paraxial approximation and Taylor expansion, leads to the following equations:

\[ \nabla^2 a_L - 2ik_0 \frac{\partial a_L}{\partial z} - \frac{3\omega_{pe}^2 a_L d^2}{4c^2 W_{\infty}^3} r^2 a_L = 0, \]  

\[ \nabla^2 a_{sc} - 2ik_s \frac{\partial a_{sc}}{\partial z} - \frac{\omega_{pe}^2 \delta n}{2c^2 n_0} a_L = 0, \]

where \( \delta n \) is the density perturbation due to Raman scattering. Dispersion relation of the laser is derived as

\[ k_0^2 = \frac{\omega_{pe}^2}{c^2}, \]

where

\[ \Pi = 1 - \frac{3 - \frac{3}{4} a_0^2}{\omega_0^2}, \]

and the dielectric function of the plasma is defined as

\[ \varepsilon = \Pi - r^2 \Delta, \]

where

\[ \Delta = \frac{3\omega_{pe}^2 a_0^2 d^2}{4c^2 W_{\infty}^3}. \]

Solutions of Equations (5) and (6) will return the intensity of incident and scattered waves, respectively.

### A. Intensity of the incident laser beam

Assume that \( a_L \) has the following form:

\[ a_L = a_0 e^{-ir^2 Q}, \]

where \( P \) and \( Q \) are two unknown functions of \( z \). Substituting for \( a_L \) in Eq. (5) results in the subsequent equations for \( P \) and \( Q \):

\[ 4Q^2 + 2k_0 \frac{dQ}{dz} + \frac{\omega_{pe}^2 \Delta}{c^2} = 0, \]

\[ \frac{dP}{dz} = -2i \frac{Q}{k_0}. \]

Eqs. (5), (12), and (13) will reduce to paraxial equations of the laser beam propagating in a linear medium if one sets \( \Delta = 0 \). Introducing a new parameter, “\( q \),” as

\[ q = \frac{k_0}{2Q}, \]

the following equation can be derived:

\[ \frac{d}{dz} \left( \frac{1}{q} \right) + \left( \frac{1}{q} \right)^2 + \alpha \left( \frac{1}{q} \right) \left( \frac{1}{q} \right)^2 = 0, \]

where

\[ \alpha = \frac{3\omega_{pe}^2 a_0^2 d^2}{16c^2}. \]

Eq. (15) is a nonlinear equation and can be solved analytically.\(^\text{21}\) The parameter \( q \) is a complex function with the following real and imaginary parts:

\[ \frac{1}{q} = K - iF. \]

Substituting for “\( q \)” in Eq. (15) and separating the real and imaginary parts lead to the following equations:

\[ \frac{d^2 K}{dz^2} + 6K \frac{dK}{dz} + 4K^3 = 0, \]

\[ \frac{dK}{dz} + (\alpha - 1)F^2 + K^2 = 0. \]

The analytic solution of Eq. (18) is given in Ref. 21

\[ K = \frac{1}{(z - B) + \frac{A^2}{B - z}}. \]

The complex part of \( \frac{1}{q} \) can then be easily derived as

\[ F = \frac{A}{\sqrt{\alpha - 1 \left( (z - B)^2 - A^2 \right)}}, \]

where
\[ A = \frac{k_0 d^2 \sqrt{x - 1}}{2[x - 1 - \theta^2]}, \]  
\[ B = \frac{-k_0 d^2 \theta}{2[x - 1 - \theta^2]}, \]  
\[ \theta = \frac{-k_0 d^2}{2R_0}, \]  
\[ d \text{ and } R_0 \text{ are the spot size and beam curvature at the entrance plane (} z = 0\text{), respectively. Therefore, parameters } Q \text{ and } P \text{ can be cast as} \]
\[ Q = \frac{k_0}{2} \left[ \frac{\sqrt{x - 1}}{B} - \frac{2i}{\sqrt[4]{x - 1}} \right], \]  
\[ P = -\frac{i}{2} \ln \left[ \frac{\sqrt{x - 1}}{B} - \frac{A^2}{2} \right] \]  
\[ + \frac{1}{2\sqrt{x - 1}} \ln \left[ \frac{z - B + A}{B - A}(A + B) \right]. \]  
The parameter \( Q \) can be re-arranged in the following conventional form:
\[ Q = \frac{k_0}{2} \left( \frac{1}{R} - \frac{2i}{k_0 W_{(z)}} \right), \]
where \( "W_{(z)}" \text{ and } "R_{(z)}" \text{ are the spot size and curvature of the laser beam. The explicit form of these quantities can be derived as}^{21} \]
\[ W_{(z)}^2 = \frac{2\sqrt{x - 1}}{k_0 A} [z - B] - A^2, \]  
\[ R_{(z)} = z - B + \frac{A^2}{B - z}. \]  
As \( x \to 0 \), the above equations will reduce to an electric field of the laser beam propagating in linear media
\[ P = -i \ln \left( 1 - \frac{2i}{k_0 d^2} \right), \]  
\[ Q = \frac{k_0}{2} \left[ \frac{2}{z + ik_0 d^2} \right]. \]  
When an intense laser beam propagates in a plasma, it will exclude electrons from high intensity areas and push them into low intensity regions. Therefore, the dielectric function of the plasma in the central section has a higher value and the plasma acts as a concave lens. This phenomenon is called self-focusing, which occurs in a nonlinear medium. The parameter \( "x" \) can also be defined as the ratio of laser power to critical power where the critical power is
\[ P_{\text{critical}} = 16.2 \frac{\omega_0^2}{\omega_0^2} (\text{GW}). \]  
This is consistent with what has been cited in Ref. 22. The self-focusing phenomena occurs in plasmas, if \( x > 1 \). Therefore, the laser beam intensity can be derived as
\[ I_{\text{laser}} = \frac{m v^2 e^2 a_0^2 a_0^2 d^2}{8\pi e^2 W_{(z)}^2 e^{-\frac{z^2}{W_{(z)}^2}}} \]  
\[ (33) \]

**B. Intensity of backscattered waves**

The solution of Eq. (6) requires the explicit form of \( \delta n \), which can be derived by integration of the perturbed distribution function. Consider the following perturbation for the distribution function:
\[ f = f_0 + \delta f e^{i\mathbf{k} \cdot \mathbf{r} - \omega t}, \]
where \( f \) is the distribution function of electrons and satisfies the Vlasov equation. The motion of ions is neglected since the pulse duration of the laser is much smaller than \( \omega_p^{-1} \). In this equation, \( f_0 \) and \( \delta f \) are unperturbed and perturbed distribution functions and \( \omega \) and \( k \) are the frequency and wave vector of plasma oscillations. By insertion of Eq. (34) in the Vlasov equation, the following equation is obtained:
\[ \frac{\partial f}{\partial t} + (\mathbf{v} \cdot \nabla) f + \left( \frac{e}{m} \mathbf{v} \right) \cdot \nabla f = 0. \]  
\[ (35) \]

The density perturbation of electrons is obtained by integration over velocity space
\[ \delta n = \int \left\{ \frac{-e \mathbf{E} - me^2 \nabla (\mathbf{a} \cdot \mathbf{a})}{i(\omega - k, \omega')} \right\} \frac{\partial f_0}{\partial \mathbf{v}} d\mathbf{v}. \]
\[ (36) \]

The explicit form of density perturbation can be derived by means of the Poisson equation and definition of electron susceptibility as
\[ \delta n = -\frac{c^2 m k^2 a_0 \lambda_e}{16\pi e^2 (1 + \lambda_e)}. \]
\[ (37) \]
By insertion of density perturbation in Eq. (6), the following equation is derived:
\[ \nabla_\perp^2 a_{sc} - 2i k_\parallel \frac{\partial a_{sc}}{\partial z} + \beta a_{sc} = 0, \]  
\[ (38) \]
where
\[ \beta = \frac{k^2 \lambda_e \omega_0^2}{8(\lambda_e + 1)}. \]  
\[ (39) \]
Let \( a_{sc} = a_s e^{-i\phi - i\varphi} \), where \( P \) and \( Q \) are two unknown functions of \( z \). Substitution in Equation (38) results in the following equations for \( P \) and \( Q \):
\[ 4Q^2 + 2k_\parallel \frac{dQ}{dz} = 0, \]  
\[ (40) \]
\[ \frac{dP}{dz} = -2i Q \frac{Q - \beta}{2k_\parallel}. \]  
\[ (41) \]
The intensity of Raman scattered waves is very small in comparison to the intensity of the laser beam, and the scattered waves are not self-focused. Therefore,

\[ P = -i \ln \left(1 - \frac{2i}{k_d z} \right) - \frac{\beta}{2k_0} z, \]  

\[ Q = \frac{k_s}{2 \left( z + ik_d z \right)^2}. \]

The spot size “\( W_z \)” and the curvature “\( R_z \)” of scattered waves are derived as

\[ W_z^2 = d^2 \left( 1 + \frac{4z^2}{k_d^2 a_0^2} \right), \]

\[ R_z = \frac{z}{1 + \frac{k_d^2 a_0^2}{4z^2}}. \]

Therefore, the intensity of scattered waves can be expressed as

\[ I_s = \frac{m_e^2 c^3 \omega_e^2 \alpha_0^2 d^2}{8\pi e^2 W_z^2} e^{-\frac{z^2}{4\pi \exp i \omega_0 t}}. \]

The exponent of the exponential term “\( \exp \left( \frac{Im(\beta)}{k_d z} \right) \)” is defined as the gain of Raman scattering, where \( G = \frac{\omega_e}{\omega_0} \omega_0^2 Im \left( \frac{1}{\omega_0^2} \right) \). Thus, the intensity varies exponentially with the factor of \( e^{-G z} \), as expected. In order to derive the explicit form of “\( Im(\beta) \),” through the use of the kinetic theory, the wave equation for scattered waves must be considered

\[ \left[ \frac{\omega_e^2}{\omega_0^2} - \frac{\omega_e^2 k_S^2 k_S}{c^2} \right] \delta S = -\frac{\omega_e^2}{\omega_0^2} \delta n \left( \frac{n_0}{\omega_0^2} \right) \delta L. \]

By insertion of density perturbation in Eq. (47), the following equation is obtained:

\[ \left( k_S^2 - \frac{\omega_e^2}{c^2} \right)  \delta S = \frac{k^2 a_0^2 \delta \varepsilon}{8(1 + \varepsilon)}. \]

The imaginary part of Eq. (48) results in the following equation:

\[ \text{Im} \left( \frac{\delta \varepsilon}{1 + \varepsilon} \right) = \frac{16\omega_0 \Gamma}{k_S^2 c^2 a_0^2}. \]

Consequently,

\[ \text{Im}(\beta) = \frac{2\omega_e \Gamma}{c^2}. \]

Here, \( \omega_e = \omega_0 - \omega \) and \( \omega \) is considered as a complex variable. The growth rate “\( \Gamma \)” can be derived by expansion of the plasma dispersion function for large arguments

\[ \left( \frac{\omega}{\sqrt{\omega^2/c^2}} \gg 1 \right) \]

\[ \left( k_S^2 - \frac{\omega_e^2}{c^2} \right) \left[ -\omega^2 + \frac{\omega_e^2}{c^2} \right] - \frac{\omega_e^2}{c^2} + \frac{i \sqrt{\pi} \omega^2 \omega_e^3}{2 \sqrt{2} k_S^2 (\varepsilon_0 T_e)^{3/2}} \]

\[ \times \exp \left( -\frac{\omega_e^2 a_0^2}{2 k_S^2 k_S} \right) \]  

\[ \frac{\omega_e^2 a_0^2}{4}. \]

which leads to the following equation:

\[ \Gamma(\Gamma + \Gamma_L) = \frac{\omega_0^2 \omega_e^2 a_0^2}{4 \left( \omega_0^2 + 3 k_S^2 k_S T_e \right)^{3/2}} \exp \left( -\frac{\omega_e^2 a_0^2}{2 k_S^2 k_S T_e} \right). \]

The solution of Equation (52) yields the growth rate

\[ \Gamma = -\frac{\Gamma_L}{2} + \frac{1}{2} \sqrt{\Gamma_L^2 + \frac{\omega_0^2 \omega_e^2 a_0^2}{\omega_0^2 + 3 k_S^2 k_S T_e}}. \]

III. BRILLOUIN SCATTERING

Brillouin scattering is a parametric instability initiated by scattering of electromagnetic waves from ion-acoustic waves. The intensity of the laser beam, which is derived explicitly in the previous section (Eq. 33), can be applied here, while scattered waves satisfy a modified equation. When a laser beam propagates in a plasma, it will perturb the plasma density. By insertion of perturbed density in the Vlasov equation (Eq. (35)), and taking advantage of ion and electron susceptibility, the following equation will be reached at for electron density perturbation:

\[ \delta n_e = -\frac{c^2 m_e^2 a_0^2 \delta \varepsilon}{16\pi e^2 (1 + \varepsilon + \varepsilon_h)}. \]

Therefore, backscattered waves satisfy the following equation:

\[ \nabla^2 a_{sc} - 2ik_s \frac{\partial a_{sc}}{\partial z} + \beta a_{sc} = 0, \]

where

\[ \beta = \frac{k_S^2 \varepsilon_h (1 + \varepsilon_h) a_0^2}{8(1 + \varepsilon + \varepsilon_h)}. \]

Assume that \( a_{sc} \) has the form of \( a_{sc} e^{-\rho - \beta z} \) and following the steps taken in Section II B, the intensity of scattered waves can be derived as follows:

\[ I_s = \frac{m_e^2 c^3 \omega_e^2 \alpha_0^2 d^2}{8\pi e^2 W_z^2} e^{-\frac{z^2}{4\pi \exp i \omega_0 t}}. \]

where
\( W^2(z) = d^2 \left( 1 + \frac{4z^2}{k_0^2d^2} \right), \) \hspace{1cm} (59)

\[ \text{Im} (\beta) = \frac{2\omega_p \Gamma}{c^2}. \] \hspace{1cm} (60)

Setting \( e^{\text{Im}(\beta)} = e^{G_z}, \) where \( G = \frac{k^2}{8\pi \omega_p^2} \text{Im} \left( \frac{z(1+z)}{1+z_0} \right) \) is defined as the gain of Raman scattering, the growth rate and gain of Brillouin scattering can be derived through the following dispersion equation of scattered waves:

\[ k^2 - \omega_p^2 + \frac{c^2}{c^2} = \frac{k^2 \gamma_e (1 + z_0) a_0^2}{4(1 + z_0 + z_0')}. \] \hspace{1cm} (61)

The expansion of electron susceptibility for small arguments \( \left( k \sqrt{\frac{a_0}{\omega_p}} \ll 1 \right) \) leads to the following equation for the growth rate

\[ \Gamma (\Gamma + \Gamma_L) = \frac{c^2 a_0^2 k_0 \omega_p^2}{2\omega_0} \left( \frac{k_B T_e}{m_e} \right). \] \hspace{1cm} (62)

Here, the magnitude of Landau damping of ion-acoustic waves is given by

\[ \Gamma_L = \frac{k \left( \frac{k_B T_e}{m_e} \right)}{2 \sqrt{1 + \frac{k^2 k_B T_e}{4 \pi n e^2}}} \sqrt{m_e}. \] \hspace{1cm} (63)

The solution of Equation (62) provides the Brillouin scattering growth rate

\[ \Gamma = -\Gamma_L \left( \frac{c^2 a_0^2 k_0 \omega_p^2}{2\omega_0} \right) \left( \frac{k_B T_e}{m_e} \right) \] \hspace{1cm} (64)

The gain of Raman and Brillouin scattering, as dominant factors in experimental studies, will be discussed numerically in Section IV, and the effect of plasma and laser characteristics on these parametric instabilities will be investigated.

**IV. DISCUSSION**

Self-focusing of laser pulses is a known phenomenon in nonlinear media where a plasma being such an example. Exertion of ponderomotive forces on electrons expels them from high intensity areas and pushes them outward from the central region of the beam. This leads to a change in the dielectric function of the plasma. In the present manuscript, the wave equation of laser pulses in a plasma medium is solved analytically without the use of WKB approximation. The modified spot size of self-focused laser beams given by Eq. (28) will be examined numerically. Figures 1–4 show the normalized spot size of the laser beam \( \left( \frac{W(z)}{d} \right) \) as a function of normalized distance of propagation \( \left( \frac{z}{z_R} \right) \) for different values of plasma frequency, laser intensity, beam waist, and laser wavelength, respectively. As was pointed out in Sec. II, an intense laser beam will be self-focused if the “\( a_0 \)” parameter is greater than 1. Self-focusing in a plasma medium occurs due to exertion of ponderomotive forces on electrons, which expels them from high intensity regions and pulls them into areas with less electrons. This process continues until a region with few electrons is formed. If plasma and
laser parameters are not properly matched, self-focusing will not occur and the plasma remains as a linear medium. Close inspection of Figures 1–4 indicates that the laser beam will be either self-focused or defocused depending upon appropriate selection of plasma and laser parameters. In the self-focusing regime, as the propagation distance increases, the spot size of the laser beam decreases and approaches a minimum. In this condition, an electron free region is formed and the laser beam will be defocused. This is due to the fact that the plasma acts as a linear medium. The location of laser beam minimum spot size also varies with variation in laser and plasma medium parameters. As Figures 1–4 indicate, the increase in plasma frequency, laser intensity, along with laser beam waist and wavelength, leads to shift in the location of minimum spot size, and it moves closer to the laser source. The findings of the present analysis on self-focusing of a laser beam in a plasma were compared with previous studies that were based on WKB and eikonal approximations. Fig. 5 displays the plot of normalized laser beam spot size \(\frac{W(z)}{d}\) versus normalized propagation distance \(\frac{z}{z_R}\) for different plasma temperatures.\(^{20}\) The close inspection of this figure reveals the reduction of laser spot size due to self-focusing as the laser beam propagates through the plasma. However, there are some regions where diffraction overcomes and the spot size increases. The comparison of the variation of laser beam spot size, shown in Figures 1–4, with that of Figure 5 indicates that there is a general agreement between the findings of the present article and those of Refs. 19, 20, 22 and 23.

Self-focused laser pulses in a plasma are subjected to different instabilities. Generally, a plasma medium is rich in wave phenomena and supports different types of waves. Raman scattering and Brillouin scattering are parametric instabilities, which lead to the growth of plasma and ion-acoustic waves. In the present article, the study is restricted to an unmagnetized plasma that supports electromagnetic, plasma, and ion-acoustic waves. These waves contribute to Raman and Brillouin scattering. Analytical studies of Raman and Brillouin scattering have shown that the growth rate of two types of parametric instabilities is more pronounced in the backward direction. Therefore, the study is focused on backward Raman and Brillouin scattering as a three-wave interaction. In previous sections (Secs. II and III), the expression for the growth rate, gain, and intensity of backward Raman and Brillouin scattering was derived through the kinetic theory of plasmas. The kinetic theory is the most appropriate theory for statistical analysis of plasmas where the effect of temperature on the gain and growth rate of Raman and Brillouin scattering can be investigated. The
analysis of the growth rate, intensity, and gain of both types of scattering is important due to their relevance in experimental studies of laser propagation in plasmas. When the intensity of the laser beam is sufficiently high, the growth rate of scattering will become more pronounced and a major portion of the laser beam will be scattered. Raman scattering and Brillouin scattering, as a nonlinear laser-plasma interaction, are important in ICF experiments, and understanding of the laser-plasma interaction in the coronal plasma of targets is crucial. The scattering processes preheat the target and generate super thermal electrons. It has been reported that Raman scattering and Brillouin scattering backscatter 35% and 25% of the incident laser energy in a NIF plasma, respectively. Therefore, a substantial portion of laser beams will not contribute in fusion due to backscattering. These instabilities also affect symmetric compression of the fusion target. The clear understanding of the physical mechanism of Brillouin and Raman scattering is essential for current ignition experiments. The optimal design of these experiments is based on reduction of the growth rate and gain of Raman and Brillouin scattering.

In Figs. 6–10, the effect of electron density and its temperature, as well as laser intensity and its wavelength, on Raman scattered waves is considered. According to the previous discussion in Sec. II, Raman scattered waves will be amplified due to occurrence of instability, and an exponential “Gain” factor appears in the explicit expression of intensity. The gain of Raman scattering increases with the increase in plasma density, laser intensity, and its frequency while decreases with increasing temperature. The increase in gain and intensity of scattered waves increases the reflectivity, which preheats the target. This generates super thermal electrons that are important challenges in ICF experiments. Figure 6 shows the variation of intensity of Raman scattered waves as a function of propagation distance “z” in a low density regime where five different plasma frequencies (100, 200, 400, 600, and 800 GHz) were considered. The increase in plasma frequency will increase the gain of Raman scattering. In Fig. 6(e), the intensity of Raman scattered wave reaches 6% of laser intensity for a plasma frequency of 800 GHz, while this ratio is below 0.01% for lower plasma frequencies (Figs. 6(a)–6(c)). The difference is due to exponential behavior of intensity, which amplifies this quantity in high gain conditions. The effect of plasma frequency in a

FIG. 6. Normalized intensity of Raman scattering $I_R$ as a function of normalized propagation distance $z/R$ for $a_0 = 0.1$, $\lambda_0 = 10.5 \mu m$, $K_\phi T_e = 1 \text{ keV}$, and five different values of $\omega_p$.

FIG. 7. Normalized intensity of Raman scattering $I_R$ as a function of normalized propagation distance $z/R$ for $a_0 = 0.1$, $\lambda_0 = 10.5 \mu m$, $K_\phi T_e = 10 \text{ keV}$, and three different values of $\omega_p$. 

high density region is considered in Fig. 7. The comparison between plots of Figs. 6 and 7 clearly indicates that the intensity of Raman scattered waves is increased in high gain regions where exponential behavior is dominant.

The effect of electron temperature on the intensity of Raman scattering is investigated in Fig. 8. The electron temperature affects Raman scattering in different ways. The frequency of plasma oscillation depends on thermal motion of electrons, and the temperature of the plasma will in turn modify the oscillation frequency. Furthermore, the Landau damping, which describes damping of plasma waves, is affected by electron temperature. Therefore, the dispersion relation of scattered waves, growth rate, and gain of Raman scattering depends on plasma temperature. The effect of plasma temperature on these quantities is discussed in Section II, and the corresponding relations were derived by means of the kinetic theory. Figures 8(a)–8(c) show the plots of intensity of Raman scattering versus electrons temperature. As the figures show, the increase in temperature reduces the gain of instability. Higher temperatures of electrons amplify the damping rate of Landau damping. This leads to reduction of the growth rate and gain of Raman scattering due to transfer of energy to particles rather than amplification of plasma waves. As the temperature of plasma electrons increases, the Landau damping rate accelerates and the amplification of scattered waves is diminished. Therefore, the gain of instability, which is the measure of the spatial amplification of the intensity of scattered waves, will decrease, and the intensity of Raman scattering is reduced by an increase in electron temperature.

The effect of laser wavelength and its power on the intensity of Raman scattering is investigated in Figs. 9 and 10. The frequency and beam power, as characteristics of the laser beam, modify the growth rate, gain, and intensity of Raman scattering. Higher laser intensity along with higher frequency will amplify the scattered waves, as shown in Figs. 9 and 10.

In Section III, the gain of Brillouin scattering was derived. The temperature of plasma electrons affects
dispersion relation of ion-acoustic and scattered waves, as well as the growth rate and Landau damping of ion-acoustic waves. Figure 11 shows the normalized intensity of Brillouin scattering versus normalized propagation distance. Three different temperatures of 1, 2, and 5 keV were considered. Figure 11 indicates that the gain and intensity of Brillouin scattering are reduced as electron temperature increases. This is due to the increase in the rate of Landau damping.

V. CONCLUSION

Propagation of a Gaussian laser beam in a plasma medium is investigated, and the explicit form of intensity of a self-focused laser beam is derived without additional approximation of WKB for the first time. Evolution of spot size and the effect of plasma and laser parameters on self-focusing of a laser beam are considered. The appropriate selection of laser and plasma parameters leads to either self-focusing or defocusing of the laser beam. A high intensity laser beam can be scattered in a plasma medium via two parametric instabilities known as Raman scattering and Brillouin scattering. The intensity of Raman and Brillouin scattered waves is derived analytically in paraxial approximation for the first time where the effect of plasma temperature and corresponding Landau damping was considered through the kinetic theory. The expression for gain, as an important parameter in experimental studies, appears in mathematical formalism of derivation of intensity of Raman and Brillouin scattering explicitly. An increase in plasma density, laser power, and its frequency leads to a rise in the intensity of scattered waves. However, higher electron temperature reduces the amount of scattered waves. Landau damping of plasma and ion- acoustic waves is the dominant dissipative process that affects instabilities in Raman and Brillouin scattering. The findings of the present study indicate that the increase in temperature amplifies Landau damping and decreases the gain and intensity of both Raman scattering and Brillouin scattering. These processes result in transfer of laser energy to particles and avoid temporal and spatial growth of scattered waves.