Fracture study of a ductile polymer-based nanocomposite weakened by blunt V-notches under mode I loading: Application of the Equivalent Material Concept

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**ARTICLE INFO**

Keywords:
Epoxy-based nanocomposite
Fracture
Equivalent Material Concept (EMC)
Mode I loading
Multi-walled carbon nanotube
Blunt V-notch

**ABSTRACT**

The onset of fracture is studied in a ductile epoxy-based nanocomposite containing round-tip V-notches both experimentally and theoretically under pure mode I loading conditions. Mechanical properties of the nanocomposite containing 0.1, 0.3, 0.5 and 1 wt% multi-walled carbon nanotubes (MWCNTs) are experimentally measured to find a desired amount of nanoparticles. Rectangular plates containing a central rhombic hole with four blunt V-shaped corners are utilized as the samples for fracture tests. Specimens with two different notch angles of 30° and 60° and various notch tip radii are tested under remote tension to obtain their load-carrying capacities (LCCs) experimentally. For theoretically predicting the experimental results, the Equivalent Material Concept (EMC) is reformulated and utilized. Two well-known brittle fracture criteria, namely the maximum tangential stress (MTS) and the mean stress (MS) criteria, are employed in conjunction with EMC for theoretical predictions. It is revealed that both the EMC-MTS and EMC-MS criteria could predict the experimental results well, without performing any elastic-plastic analyses of the nanocomposite specimens.

**1. Introduction**

Various types of notches, like V-, U-, and O-notches are extensively utilized in engineering components and structures with the aim to fasten two or more parts together, to transfer loads between components, etc. Depending upon the brittleness or ductility of material, the fracture phenomenon in notched members, normally consisting of the initiation and propagation of crack(s) emanating from the notch border, may take place suddenly or slowly [1]. Notches are widely observed also in polymeric components which are extensively utilized in different industries. Since polymeric components are usually subjected to direct or indirect mechanical loading with different magnitudes, their fracture investigation in the presence of notches is very important.

Polymer composites have attracted great interests in industrial and engineering applications such as automotive, construction materials, laminates, electronics and marine. The widest application of epoxy resins is perhaps utilizing them as a matrix for polymeric-based composite materials. Several studies have been performed by researchers to improve the mechanical properties of epoxy materials [2–6]. Various types of nanoparticles such as nano-diamond (ND) [7,8], carbon nanofibers (CNF) [9] and graphene (G) nano-platelets [10,11] have been proposed by researchers to add to the epoxy materials in order to improve their properties. Carbon nanotube (CNT), discovered in 1990s [12], is the most well-known nanoparticle which is extensively used as nano-reinforcement in polymeric materials. These wide applications are mainly due to very high aspect ratio, the unique atomic structure, and excellent mechanical, electrical and thermal properties of CNTs. Several researchers showed that CNTs can improve fracture toughness of polymer under different loading conditions [13–20]. However, reduction in the mechanical properties of polymer due to the addition of CNTs was also observed in some other cases [21]. Based on the brittleness or ductility of epoxy-based nanocomposites, they are prone to brittle or ductile fracture under different loading conditions. Almost all of the previous studies dealing with fracture in epoxy-based nanocomposites evaluated their brittle fracture behavior under different loading conditions, because most of epoxy materials exhibit brittle or quasi-brittle behaviors with linear elastic stress-strain relationships till final fracture, which usually takes place suddenly in a catastrophic manner. Hence, designers attempt to prevent sudden failure of nanocomposite components due to brittle fracture by utilizing appropriate brittle fracture criteria. Another practical way of preventing sudden fracture in epoxy-based nanocomposites is using ductile epoxy resins in...
their fabrication process. Since crack propagates in ductile materials slowly with considerable amount of plastic deformation around the crack tip, one has the chance to detect the crack before final rupture. To predict ductile fracture in epoxy-based nanocomposites weakened by notches, appropriate failure criteria must be developed and verified.

Elastic-plastic fracture mechanics (EPFM) is usually employed for studying the failure in cracked and notched engineering components exhibiting considerable ductile behavior. The main problem of EPFM analyses is that they are rather time-consuming and complicated. The Equivalent Material Concept (EMC) has been originally proposed by Torabi [22] in 2012, helps designers avoid EPFM analyses for investigating the fracture of ductile materials. In fact, in this new concept, a virtual brittle material is considered instead of the real ductile material and therefore, the linear elastic fracture mechanics (LEFM) criteria can be used to evaluate the fracture of the ductile material. For ductile failure prediction of notched specimens, EMC has been successfully used in conjunction with the stress-based brittle fracture criteria, such as the maximum tangential stress (MTS) and mean stress (MS) criteria [22,23] and also with the energy-based criteria, e.g. the Averaged Strain Energy Density (ASED) criterion [24,25]. The most recent work on the application of EMC in elastic-plastic failure estimation of notched members has been published by Cicero et al. [26]. It has been reported in [26] that a PMMA material with considerable amount of plastic deformation under tension could be equated with a virtual brittle material of perfectly linear elastic behavior thanks to EMC. They showed that the fracture predictions for U-notched PMMA specimens subjected to symmetric three-point bending by means of combining EMC with the theory of critical distances (TCD), i.e. EMC-TCD criterion, were successful.

In the present paper, first, the effects of MWCNTs on the mechanical properties of epoxy/MWCNTs nanocomposites are evaluated. Then, the fracture of nanocomposite specimens weakened by blunt V-notches and subjected to pure mode I loading conditions is studied. For this purpose, after conducting the fracture tests on the specimens with various V-notch geometries, the experimentally obtained load-carrying capacities (LCCs) are compared with the theoretically predicted ones. For theoretical predictions, EMC is reformulated and combined with MTS and MS criteria as the two well-known brittle fracture criteria. To have a better evaluation of the results, several finite element (FE) analyses are performed and some scanning electron microscopy (SEM) photographs are taken from the fracture surfaces and interpreted. It is shown that both the EMC-MTS and EMC-MS criteria could predict the experimental results well without any meaningful difference in the accuracy.

2. Experiments

2.1. Material characterization

An epoxy resin is selected as the matrix of the epoxy-based nanocomposite. It has been demonstrated that adding CNTs to the epoxy resins results in more brittle behavior of the obtained nanocomposite than the pure epoxy material [4,7,13]. Therefore, the pure epoxy material selected as the matrix of the nanocomposite should have considerable ductility in its tensile behavior. Consequently, the epoxy resin Araldite LY 5052 is selected as the matrix of the nanocomposite due to its wide applications in engineering. Moreover, the viscosity of this epoxy resin is low enough to allow appropriate dispersion of nanoparticles in the matrix. Aradur 5052 is also selected as a curing agent for the epoxy resin. As mentioned in [27], the effects of the two main parameters, namely the hardener concentration and the strain rate, on the stress-strain curve of the epoxy material are evaluated by performing a large number of standard tensile tests. These tests are designed to achieve a stress-strain curve for the epoxy material with considerable nonlinear portion. The stress-strain curves of the tested specimens with different hardener concentrations and strain rates are presented in Figs. 1 and 2, respectively. As shown in these two figures, by taking into account the maximum ductility as the constraint, the strain rate of 1 mm/min and the hardener concentration of 50 wt% are selected.

Tensile properties and fracture toughness of the pure epoxy resin obtained experimentally based on the standard codes ASTM-D638 [28] and ASTM-D5045 [29], respectively, are presented in the second column of Table 1.

To prepare the nanocomposite material, the functionalized multi-walled carbon nanotubes (MWCNTs) are added to the epoxy resin. According to the information provided by the supplier (Nanostructured and Amorphous Materials Inc. [30]), the lengths of the MWCNTs are between 10 and 30 µm, their diameters are between 10 and 20 nm, and the notch tip radius 2α, notch angle Kuters, plane-strain fracture toughness κ, ultimate tensile strength σ, mean value of tangential stress σ00, critical stress σc, maximum tangential stress MTS, mean stress MS, critical distance of the MS criterion d0, critical distance of the MTS criterion r.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Definition</th>
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<tbody>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>ρ</td>
<td>notch tip radius</td>
</tr>
<tr>
<td>2α</td>
<td>notch angle</td>
</tr>
<tr>
<td>Kcers</td>
<td>plane-strain fracture toughness</td>
</tr>
<tr>
<td>κ</td>
<td>ultimate tensile strength</td>
</tr>
</tbody>
</table>

![Fig. 1. Stress-strain curves of the epoxy resin for different hardener concentrations: (1) 65 wt% (2) 50 wt% (3) 38 wt% (4) 20 wt% [27].](image1)

![Fig. 2. The tensile stress-strain curves of the epoxy resin obtained at different strain rates: (1) 0.2, (2) 1, (3) 5, and (4) 10 mm/min [27].](image2)
the carbon purity of them is 95%.

Epoxide-based nanocomposites with four different percentages of nanoparticles as 0.1, 0.3, 0.5 and 1 wt% MWCNTs are prepared according to the following description. First, the specified amount of MWCNTs are dispersed in the epoxy resin by using the mechanical stirring. Then, for better dispersion of nanoparticles in the epoxy resin and to avoid the agglomeration of CNTs, the ultrasonic wave technique is used. Mixtures containing 0.1, 0.3, 0.5 and 1 wt% CNTs are sonicated around the notch border as a result of heating.

Moreover, to apply the sonication energy uniformly to the entire mixture, the mixture is stirred every 10 min using a small spoon during the sonication process. After well dispersion of CNTs in the epoxy resin, hardener is added to it gradually during the stirring of the mixture. To avoid the creation of bubbles, the mixture is degassed in vacuum and molded into the desired shape. Finally, according to the resin supplier recommendation [31], the molded components are cured in the ambient conditions for 24 h, followed by 4 h at 100 °C. Afterward, the desired specimens are fabricated by utilizing a high-precision 2D CNC water-jet cutting machine in order to avoid generating residual stresses around the notch border as a result of heating.

For determining the effects of CNTs contents on the mechanical properties of nanocomposite, dog-bone specimens of nanocomposite containing 0.1, 0.3, 0.5 and 1 wt% MWCNTs are tested under tensile loading according to ASTM-D638 [28]. The fracture toughness of specimens is also determined experimentally according to ASTM-D5045 [29]. According to the standard codes, each test is repeated at least three times. Therefore, 21 tests are conducted to obtain the tensile properties and fracture toughness of pure epoxy and nanocomposite materials. The properties of the tested pure epoxy and nanocomposite materials are presented in Table 1.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Pure epoxy</th>
<th>0.1% CNTs</th>
<th>0.3% CNTs</th>
<th>0.5% CNTs</th>
<th>1% CNTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa), E</td>
<td>2.41</td>
<td>2.74</td>
<td>2.88</td>
<td>3.01</td>
<td>3.2</td>
</tr>
<tr>
<td>Yield strength (MPa), σy</td>
<td>61.1</td>
<td>64.3</td>
<td>67.8</td>
<td>69</td>
<td>69.9</td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa), σu</td>
<td>71.2</td>
<td>76.3</td>
<td>82.1</td>
<td>83</td>
<td>86.8</td>
</tr>
<tr>
<td>Fracture toughness (MPa m^{1/2}), KIC</td>
<td>1.34</td>
<td>1.55</td>
<td>1.68</td>
<td>1.77</td>
<td>1.81</td>
</tr>
<tr>
<td>Elongation at the ultimate point (%)</td>
<td>12.2</td>
<td>11.5</td>
<td>11.2</td>
<td>9.8</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Tensile stress-strain curves obtained experimentally for the pure epoxy and nanocomposite materials with different MWCNTs weight percentages are also shown in Fig. 3. According to the data presented in Table 1, the mechanical properties of the tested epoxy resin are improved by adding the MWCNTs to it. It is obvious in Table 1 and Fig. 3 that by increasing the MWCNTs content in the nanocomposite material, the Young’s modulus, the tensile strength and the fracture toughness of the material increase. The same results have also been reported in some other research works [13,14,16]. However, the strain to failure for the nanocomposite decreases by increasing the nano-reinforcement content, indicating a ductile to brittle transition trend in the nanocomposite material. According to the stress-strain curves obtained for the nanocomposites (see Fig. 3), the best choice for the present work is the nanocomposite material with 0.3 wt% MWCNTs which has the tensile strength high enough and also considerable strain to failure. The main reason for this choice is that high strength and good ductility are excellent properties for appropriate structural materials.

2.2. Fracture testing of the nanocomposite material weakened by blunt V-notches

After determining the appropriate weight percentage of the MWCNTs in the nanocomposite material, the blunt V-notched specimens are fabricated from this nanocomposite. The test specimen is a rectangular plate of 4 mm thick containing a central rhombic hole with four blunt V-shaped corners. A schematic of the specimen including the dimensions and geometrical parameters is presented in Fig. 4. The parameter 2a in Fig. 4 is the notch angle which is considered to be equal to 30° and 60° in the experiments. Three different notch radii ρ of 1, 2, and 4 mm are also considered for fabrication of the notched specimens. These notch parameters (notch tip radii and notch angles) are selected according to the previous geometries reported in the literature (see for instance [23]).

The nanocomposite specimens weakened by V-notches are tested for fracture under uni-axial monotonic tension. The geometry of the specimens is quite symmetric and the direction of the applied tensile load P is perpendicular to the central slit bisector line. Therefore, the two blunt V-shaped corners of the rhombic hole experience pure mode I loading conditions. For each notch geometry, three samples are tested to evaluate the repeatability of the experimental results. Therefore, 18 fracture tests are totally conducted on the notched specimens. Some of the tested V-notched nanocomposite specimens before and after the fracture tests are shown in Fig. 5. It is clear in Fig. 5 that the fracture trajectory is along the notch bisector line, confirming the fracture of the V-notched specimens under mode I loading. The experimentally obtained load-carrying capacities (LCCs) of the V-notched nanocomposite specimens are summarized in Table 2 for different notch angles and notch tip radii. Note that in Table 2, the parameters P_i (i = 1, 2, 3) and P_{avg} denote the three LCCs in the repeated tests and the average of the three LCCs, respectively. It can be seen in Table 2 that for a constant notch angle, LCC increases as the notch tip radius increases. Moreover, seen in Table 2 is that for a constant notch tip radius, LCC also increases as the notch angle increases. Both the findings mentioned above are due to the decrease of the stress concentration around the notch tip.

A sample load–displacement curve obtained from the fracture tests on the nanocomposite specimens is presented in Fig. 6. The obvious considerable nonlinear portion before the peak point in this curve declares that at the onset of crack initiation from the notch tip, a significant plastic zone forms at the notch neighborhood, confirming well the ductile failure behavior of the specimens.

3. The Equivalent Material Concept for the tested nanocomposite material

The Equivalent Material Concept (EMC) was originally proposed by Torabi [22] in 2012 as a novel concept for ductile failure prediction. In
In EMC, fracture of a ductile material with elastic-plastic behavior is equated with that of a virtual brittle material with perfectly linear elastic behavior. Torabi and co-workers [22,23] could successfully predict the LCCs of notched ductile components under various in-plane loading conditions by utilizing EMC in conjunction with the stress-based brittle fracture criteria.

In EMC, the elastic modulus and the fracture toughness for the real ductile and virtual brittle materials are assumed to be equal. It is also assumed that both materials absorb the same amounts of strain energy density (SED) till the ultimate point of the tensile stress-strain curve. The power-law stress-strain relationship was used in the plastic zone of the ductile material and hence, a closed-form expression was obtained for the tensile strength of the equivalent material as follows [23]:

\[ \sigma_f^* = \sqrt{\sigma_Y^2 + \frac{2KE}{n+1}(\varepsilon_{u,\text{true}}^n - (0.002)^n)} \]  

(1)

The parameters \( \sigma_Y, \sigma_f^*, n, \varepsilon_{u,\text{true}}, E, \) and \( K \) in Eq. (1), are the yield strength, the tensile strength of the equivalent material, the strain-hardening exponent, the true plastic strain at the ultimate point, the elastic modulus, and the strain-hardening coefficient, respectively.

Eq. (1) has been applied to those ductile metallic materials exhibiting the power-law strain-hardening relationship. Therefore, the validity of this relationship should be verified for the tested nano-composite material. Hence, the true stress-true strain curve for the non-linear portion of the nanocomposite material is obtained in the logarithmic scale as shown in Fig. 7a. It is clear in Fig. 7a that the obtained curve does not follow a linear trend; therefore, the power-law equation is really inappropriate for the tested nanocomposite material.

An equation has been proposed by Bahadur [32] for expressing the stress (\( \sigma \))-strain (\( \varepsilon \)) relationship in the plastic region of polymeric materials as follows:

\[ \sigma = \frac{\sigma_0}{m} \varepsilon^{m} \]  

(2)

where the parameters \( \sigma_0 \) and \( m \) are the material constants. Based on the Bahadur’s expression, the true stress (in logarithmic scale) versus the true strain for the non-linear region of the tested nanocomposite material is plotted in Fig. 7b. It is obvious that this curve is almost linear, meaning that Bahadur’s expression can be well fitted to the non-linear portion of the tested nanocomposite. The values of \( \sigma_0 \) and \( m \) for the tested nanocomposite are obtained to be equal to 54.2 MPa and 1.31, respectively.

The total SED of the tested nanocomposite can be calculated by using Eq. (3) as follows:

\[ \text{SED}_{\text{tot}} = \frac{\sigma_f^2}{2E} + \frac{\sigma_0}{m} (e^{m\varepsilon_{u,\text{true}} - e^{0.004m}}) \]  

(3)

According to EMC, the SED values for both the virtual brittle material and the real ductile material should be equal. Hence

\[ \frac{\sigma_f^2}{2E} = \frac{\sigma_Y^2}{2E} + \frac{\sigma_0}{m} (e^{m\varepsilon_{u,\text{true}} - e^{0.004m}}) \]  

(4)

Consequently, the following expression can be used for calculating

Table 2
The experimentally obtained critical loads of the V-notched nanocomposite specimens.

<table>
<thead>
<tr>
<th>2( \alpha ) (deg.)</th>
<th>( \rho ) (mm)</th>
<th>( P_1 ) (N)</th>
<th>( P_2 ) (N)</th>
<th>( P_3 ) (N)</th>
<th>( P_{\text{avg}} ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1</td>
<td>3491</td>
<td>3672</td>
<td>3380</td>
<td>3514</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>3760</td>
<td>3663</td>
<td>3913</td>
<td>3779</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>3933</td>
<td>4081</td>
<td>4173</td>
<td>4062</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>3581</td>
<td>3737</td>
<td>3801</td>
<td>3707</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>3997</td>
<td>3807</td>
<td>3904</td>
<td>3904</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
<td>4280</td>
<td>4187</td>
<td>3998</td>
<td>4155</td>
</tr>
</tbody>
</table>

Fig. 4. Schematic of the blunt V-notched rectangular specimen.

Fig. 5. Some of the tested V-notched nanocomposite specimens before and after the fracture tests.

Fig. 6. Sample load-displacement curve for a blunt V-notched nanocomposite specimen.

Table 2
The experimentally obtained critical loads of the V-notched nanocomposite specimens.
the tensile strength of the equivalent brittle material $\sigma_f^*$:

$$\sigma_f^* = \sqrt{\frac{2 K_{IC}}{\pi}} \left( \varepsilon^{0.004} - \varepsilon_m \right)$$  \hspace{1cm} (5)

Substituting the values of the parameters into Eq. (5), the value of $\sigma_f^*$ for the tested nanocomposite is computed to be equal to 149.2 MPa. By using the obtained $\sigma_f^*$ and the fracture toughness of the tested nanocomposite $K_{IC}$ (or $K_C$) in the available brittle fracture criteria, the LCCs of the notched ductile nanocomposite specimens can be theoretically estimated.

4. Brittle fracture criteria

4.1. Maximum tangential stress (MTS) criterion

Erdogan and Sih [33] proposed the maximum tangential stress (MTS) criterion in 1963 to predict the brittle fracture of engineering structures in the presence of sharp cracks. The MTS criterion expresses that a cracked component fails by brittle fracture when the tangential stress at a specified critical distance $r_c$ ahead of the crack tip attains the material critical stress $\sigma_C$. The critical stress $\sigma_C$ is assumed to be a material property and normally equal to the tensile strength for brittle materials. According to MTS criterion, growth of the pre-existing crack happens radially, perpendicular to the maximum tangential stress.

In 2010, Ayatollahi and Torabi [34] have extended the original MTS criterion to V-notched components and developed a new criterion for predicting brittle fracture in rounded-tip V-notched specimens, called the RV-MTS criterion. According to RV-MTS criterion, brittle fracture in a blunt V-notched component happens when the tangential stress along the fracture initiation angle (zero for pure mode I loading) and at the critical distance $r_c$ from the notch tip reaches the critical stress $\sigma_C$. The critical distance in MTS criterion is assumed to be a material property and equal to $r_c = \frac{1}{2\pi} \left( \frac{K_{IC}}{\sigma_C} \right)^2$.

4.2. Mean stress (MS) criterion

Wieghardt [35] proposed the mean stress (MS) criterion in 1907. In MS criterion, the tensile stress over a specified critical distance $d_c$ ahead of the crack tip is first averaged. Then, if this average value attains the material critical stress $\sigma_C$, brittle fracture happens in the cracked component.

Ayatollahi and Torabi [36] proposed a novel failure criterion for brittle fracture of blunt V-shaped notches under pure mode I loading. This criterion was, in fact, an extended version of the original MS criterion for sharp cracks. According to MS criterion for blunt V-notches, brittle fracture takes place in a blunt V-notched component under mode I loading, when the average of the tensile tangential stress over the critical distance $d_c$ ahead of the notch tip reaches the critical stress $\sigma_C$. The critical distance in MS criterion is also assumed to be a material property and equal to $d_c = \frac{1}{2\pi} \left( \frac{K_{IC}}{\sigma_C} \right)^2$, which is apparently four times $r_c$.

The finite element (FE) analyses performed in this research are presented in the next section. The FE stress distributions around the blunt V-notches are obtained under pure mode I loading and MTS and MS failure concepts are directly applied to the FE models for predicting the experimentally obtained LCCs of the blunt V-notched nanocomposite specimens.

5. Finite element analysis

For simulating the nanocomposite specimens weakened by blunt V-notches, the corresponding two dimensional (2D) finite element (FE) models are created and analyzed under pure mode I loading. These simulations are performed in the ABAQUS software. Such FE analyses are conducted to predict LCCs of the notched nanocomposite specimens. As the thickness of the specimens is small compared to the length and width of them, FE models are analyzed under plane-stress conditions. The models are meshed with the eight-node plane-stress quadratic elements. In Fig. 8a, a sample mesh pattern for the whole specimen is shown. It is clear in Fig. 8b that refined meshes with the minimum size of about 0.01 mm are employed at the vicinity of blunt V-notches due to the existence of high stress gradient. As shown in Fig. 9, the typical distribution of the tensile tangential stress at the vicinity of a blunt V-notch is quite symmetric (see the butterfly wing shape), demonstrating that the V-notched nanocomposite specimen is under pure mode I loading conditions.

According to EMC, the critical stress $\sigma_f$ of the nanocomposite material should be replaced with the obtained $\sigma_f^*$ to theoretically predict

![Fig. 7. True stress-strain plots of the tested nanocomposite material for the non-linear region based on the (a) power-law and (b) Bahadur expressions.](image)

![Fig. 8. Mesh patterns for: (a) the whole specimen and (b) the vicinity of notch border.](image)
LCCs for blunt V-notched specimens by means of the EMC-MTS and EMC-MS combined criteria. To theoretically predict LCC for a notched nanocomposite specimen using EMC-MTS criterion, first, the specific point that locates on the notch bisector line at the critical distance $r_c = 0.02 \text{ mm}$ from the notch tip is determined. Then, for an arbitrary load (e.g. 100N), the tensile stress $\sigma_{\theta \theta}$ at this point is obtained. It was mentioned in Section 4 that brittle fracture of a blunt V-notched specimen occurs in accordance with MTS criterion when the tensile stress at the critical distance attains $\sigma_f^*$. Therefore, based on EMC-MTS criterion, LCC can simply be predicted as $P_{\text{EMC-MTS}} = \rho_{\text{EMC-MTS}} \times 100 \text{ N}$. The schematic of MTS failure concept for blunt V-notches under pure mode I loading is represented in Fig. 10.

For predicting LCC of a blunt V-notched nanocomposite specimen by means of EMC-MS criterion, unlike MTS criterion, the average of the tangential stresses over the critical distance $d_c$ ($\overline{\sigma_{\theta \theta}}$) should be calculated. The specimen fails by brittle fracture when the average stress $\overline{\sigma_{\theta \theta}}$ attains $\sigma_f^*$. Hence, according to EMC-MS criterion, LCC of any V-notched nanocomposite specimen tested can be obtained as $P_{\text{EMC-MS}} = \rho_{\text{EMC-MS}} \times 100 \text{ N}$. The schematic of MS fracture concept for blunt V-notches under pure mode I loading conditions is also depicted in Fig. 11.

### 6. Results and discussion

The experimental and theoretical LCCs of the V-notched nanocomposite specimens were obtained and their variations versus the notch tip radius are presented in Fig. 12. The discrepancies between the experimental and theoretical results are also presented in Table 3. It is clear in Fig. 12 and Table 3 that both EMC-MTS and EMC-MS criteria can successfully predict the experimentally obtained tensile LCCs for blunt V-notches.
the V-notched nanocomposite specimens. The average discrepancies of EMC-MTS and EMC-MS criteria are obtained to be 5.5% and 5.4%, respectively. Therefore, any of these two criteria can be employed for estimating the critical loads of ductile nanocomposite components weakened by blunt V-notches. Since EMC-MTS criterion is significantly easier than EMC-MS criterion to be applied to failure prediction, and also the accuracies of both criteria are almost the same, it is strongly recommended to be utilized in engineering design.

It is seen in the last column of Table 2 that the failure loads obtained experimentally for various notch geometries are not much different. It should be highlighted that the lower sensitivity of the fracture load of the specimens to the notch geometry is due to the specific properties of the tested nanocomposite for which the tensile stress-strain curve does not show considerable strain-hardening in the plastic region. Because all of the tested V-notched nanocomposite specimens failed following considerable amount of plastic deformation around the notch and considering that the level of strain-hardening is low for the material, the large size of plastic zone does not allow the fracture load to become considerably different for various notch geometries.

Since the notched nanocomposite specimens are simply tested under remote tension, a question may be raised in the mind that isn’t it possible to calculate the failure load easily by multiplying the tensile strength of the nanocomposite by the net area of the notched specimen, while the failure loads are not so different for various notch geometries? The answer to this important question is that this can be done for the present specimens; however, the aim of the present work is to propose some local failure criteria by which a wide range of notched ductile nanocomposite components and structures for which the net area may be generally meaningless can be analyzed. Such a poor sensitivity of the fracture load to the notch geometry has been recently reported in some papers regarding ductile failure of notched aluminum plates (see for example [23–25]). Moreover, it should be stressed that such insensitivity is not due to the global failure of the ligament by the plastic collapse, because not the entire ligament encounters plastic deformations at the onset of crack initiation from the notch tip.

To better recognize the failure of nanocomposite specimens in the presence of V-notches, the broken surfaces of the specimens are investigated by using the scanning electron microscopy (SEM) photographs. Some SEM photographs are depicted in Fig. 13 in different scales. These pictures correspond to the specimens broken under pure mode I loading. According to Fig. 13, the fracture surface of the nanocomposite is rough, confirming the existence of plastic deformations before fracture. Such a rough surface demonstrates that ductile fracture occurred in the tested specimen. For the specimens with the same notch angle, it is observed that by increasing the notch tip radius the fracture surface exhibits higher roughness. For the specimens with the same notch tip radius, the roughness of the fracture surface increases as the notch angle increases. By comparing the experimentally obtained LCCs of the tested specimens reported in Table 2, it is clear that LCC increases by increasing the notch angle and the notch tip radius, demonstrating that more energy is needed for fracturing the specimens. In fact, a fracture surface with higher roughness demonstrates more strain energy absorption during the fracture process.

In none of the SEM photographs taken from the broken nanocomposite specimens, smooth fracture surface is seen, meaning that there is not any brittle fracture in the tested nanocomposite specimens. The roughness of the fracture surfaces in the photographs taken is due to the considerable plastic deformations prior to the initiation of crack from the blunt V-notch tip. Moreover, the broken and pulled out nanofillers can be obviously seen on the fracture surface, meaning that the bridging mechanism happens in this region (see Fig. 13). The broken nanofillers appear with a very short length on the fracture surface near the notch tip, where the stress level is high.

![Fig. 13. SEM photographs of the rough fracture surface of blunt V-notched nanocomposite specimens.](image-url)
7. Conclusions

In this study, first, epoxy-based nanocomposites containing different weight percentages of MWCNTs were prepared and tested under tension to measure their mechanical properties. Then, the desired percentage of MWCNTs was determined to have a nanocomposite material with considerable ductility and high enough tensile strength. The blunt V-notched rectangular specimens with two different notch angles and various tip radii were fabricated from the finalized nanocomposite and tested under pure mode I loading conditions. After recording the LCCs of the specimens experimentally, for theoretically predicting the LCCs, first, the Equivalent Material Concept (EMC) was reformulated and a closed-form expression was developed for estimating the tensile strength of the equivalent material. Then, the reformulated EMC was utilized in conjunction with MTS and MS brittle fracture criteria. It was revealed that both EMC-MTS and EMC-MS criteria could predict the experimental results very well. By using the experimentally recorded load-displacement curves and the SEM photographs, it was demonstrated that the amount of plastic deformation around the notch blunt border at the crack initiation instance is considerable. Consequently, accurate prediction of LCCs for the blunt V-notched nanocomposite components subjected to pure mode I loading could be achieved by using EMC-MTS and EMC-MS criteria, without performing time-consuming and complex elastic-plastic analyses.

References