Property Modelling

Tensile fracture analysis of a ductile polymeric material weakened by U-notches


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ABSTRACT

Fracture of a ductile epoxy polymer containing U-notches was investigated both experimentally and theoretically under pure opening mode. Two various U-notched polymeric specimens with different notch tip radii were utilized in the experiments; one of them was loaded under remote tension and the other under three-point bending. The load-carrying capacities (LCCs) of the U-notched polymeric specimens were experimentally recorded. To predict the experimentally obtained LCCs, the Equivalent Material Concept (EMC), proposed originally by the first author, was reformulated for the ductile polymer tested. Then EMC was linked to two well-known brittle fracture criteria, namely the maximum tangential stress (MTS) and mean stress (MS) criteria. It is shown that both the EMC-MTS and EMC-MS criteria predict the experimental results well without needing elastic-plastic analyses of the polymer specimens.

1. Introduction

1.1. Epoxy resins

Epoxy resins are used intensively across a wide range of fields, such as electronics and automotive, marine and wind energy. They are also used as a matrix for polymeric composite materials, high-performance coatings, adhesives and paint. Such a wide range of engineering applications is due to their excellent mechanical properties, high adhesion to many substrates, low shrinkage and good heat and chemical resistance [1,2].

Considering the extensive applications of epoxy resins in engineering structures, determination of their mechanical properties, especially their fracture toughness, is very important. Epoxy resins can be reacted (cross-linked) with a wide range of hardeners or curatives, and the cross-linking reaction is commonly referred to as curing. Reaction of epoxy resins with suitable hardeners forms three-dimensional cross-linked thermoset structures with good properties. Their properties depend on the combination of the type of epoxy resins and the curing agents used [3,4]. As an important mechanical property, the fracture toughness of epoxy material is often affected by temperature, strain rate, additive concentration and other environmental variables [3–5]. In most cases, damage initiates from pre-existing defects or stress concentrators like voids, notches, holes, corners, free edges, etc. [6].

1.2. Fracture of polymeric materials containing cracks and notches

Under mechanical loading, brittle fracture behavior of epoxy resins weakened by cracks was studied in several researches [7–11]. Kinloch et al. [8] studied the microstructure and fracture behavior of a neat epoxy material and determined the onset of fracture, the type of crack growth and the detailed nature of the associated fracture surfaces. Fiedler et al. [9] studied the mechanical properties and the failure behavior of a plain epoxy resin by using specimens for tensile, torsion and compression tests fabricated from a plain resin slab.

While most of the researches on fracture behavior of epoxy resins deal with brittle fracture, there are a few papers in which the ductile fracture of epoxy materials are studied in the presence of cracks [12–15]. Fracture energy and yield strength have normally opposite trends with respect to each other, meaning that at high yield strengths, the fracture energies are relatively low and brittle fracture is the dominant mode of failure. At low yield strengths, the fracture energies are high, leading to a change in the mode of failure from brittle fracture to ductile fracture [15].

A large number of polymeric components contain notches of various geometries like V-, U-, and O-notches. Due to the stress concentration, crack(s) may initiate from the notch border and fracture may take place. In the presence of notches, brittle failure of polymethyl-methacrylate (PMMA) has been widely studied experimentally and
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>ρ</td>
<td>Notch radius</td>
</tr>
<tr>
<td>K_{SC}</td>
<td>Plane-strain fracture toughness</td>
</tr>
<tr>
<td>σ_{u}</td>
<td>Ultimate tensile strength</td>
</tr>
<tr>
<td>σ_{θθ}</td>
<td>Tangential stress</td>
</tr>
<tr>
<td>σ_{c}</td>
<td>Critical stress</td>
</tr>
<tr>
<td>σ_{θθ}^c</td>
<td>Mean value of tangential stress</td>
</tr>
<tr>
<td>MTS</td>
<td>Maximum tangential stress</td>
</tr>
<tr>
<td>MS</td>
<td>Mean stress</td>
</tr>
<tr>
<td>d_{c}</td>
<td>Critical distance of the MS criterion measured from the notch border</td>
</tr>
<tr>
<td>r_{c}</td>
<td>Critical distance of the MTS criterion measured from the notch border</td>
</tr>
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</table>

...theoretically under different loading conditions [16–29]. Dealing with epoxy resins and polystyrene, the notch resistance against brittle fracture has also been investigated by some researchers [3,4,12,30,31].

1.3. Ductile failure of notched components

The fracture of engineering materials with elastic-plastic behavior is usually analyzed by means of the failure criteria within the framework of elastic-plastic fracture mechanics (EPFM), which are rather time-consuming and complicated. Recently, Torabi and co-workers [32–39] proposed the EMC by which they could equate the real ductile material with a virtual brittle material and predict the load-carrying capacity (LCC) of the notched ductile specimens. Torabi and co-researchers [40–44] also demonstrated that EMC could be successfully linked to energy-based brittle fracture models, e.g. the Averaged Strain Energy Density (ASED) criterion. A large number of epoxy resins have elastic-plastic behavior and fail by significant strains. Therefore, the notch fracture resistance of ductile resins should also be investigated either theoretically or experimentally. To the best of authors’ knowledge, no paper or technical report is available in the literature dealing with ductile fracture of epoxy resins weakened by notches.

In the present work, it is attempted to study the fracture behavior of a ductile epoxy resin weakened by U-shaped notches. For this purpose, first, the effects of the hardener content and the strain rate on the tensile properties of a specific epoxy resin were experimentally evaluated and a mixture with significant ductility was obtained. Then, several test specimens containing U-shaped notches of various tip radii were fabricated from the obtained ductile resin and tested under tension and three point-bending. Finally, the well-known brittle fracture criteria of maximum tangential stress (MTS) and mean stress (MS) were linked to EMC to predict the experimentally obtained LCCs. It is shown that both EMC-MTS and EMC-MS criteria predict the experimental results well, without needing to perform elastic-plastic analyses.

2. Experiments

Two groups of fracture tests were performed. The first group was conducted for determining the effects of hardener content and strain-rate on the tensile stress-strain curve of the epoxy material and the second group was carried out on U-notched specimens. Details are presented below.

2.1. Characterizing the epoxy material

In this work, an epoxy resin with considerable nonlinear portion in the stress-strain curve is needed. Therefore, the epoxy resin Araldite LY 5052 which has wide engineering applications, was selected. The curing agent was Aradur 5052. Several tensile tests were conducted to experimentally evaluate the changes in the stress-strain curve of this epoxy material by changing the strain rate and hardener concentration.

For polymeric materials, the standard tensile test methods widely accepted and used to obtain the stress-strain behavior is ISO 527 [45].

There are two important parameters that affect the stress-strain curve of polymers; the hardener concentration and the strain rate. There are several researches in the literature reporting these effects [4,46,47]. Hence, different tests are designed and performed in this work to study the same effects on the selected epoxy material.

At first, for evaluating the effects of hardener concentration on the stress-strain curve of the epoxy material, the tensile specimens with four different hardener concentrations were fabricated, containing 20 wt%, 38 wt%, 50 wt% and 65 wt% hardener. To prepare the specimens, the hardener was added to the epoxy resin and the mixture degassed in a vacuum environment and molded. Eventually, on the basis of the resin supplier recommendations [48], the specimens were...
cured for 24 h at the ambient conditions followed by 4 h at 100 °C. As shown in Fig. 1, by increasing the hardener concentration from 20 wt% to 50 wt%, the ductility of the epoxy resin increases, but from 50 wt% to 65 wt% this ductility decreases. Thus, the epoxy resin with 50 wt% hardener concentration was selected in the present study.

After determining the appropriate hardener concentration, for evaluating the effects of strain rate on the stress-strain curve of the epoxy material, four different strain rates were considered in the tensile tests as 0.2, 1, 5 and 10 mm/min. As shown in Fig. 2, the maximum ductility is seen for the test with the strain rate of 1 mm/min. By increasing and decreasing this specific strain rate, the material ductility becomes smaller. Therefore, the strain rate of 1 mm/min was selected for the experiments.

The tensile properties and fracture toughness of the epoxy resin were experimentally determined according to ISO 527 [45] and ISO 13586 [49], respectively. To obtain the fracture toughness of the epoxy material experimentally, three compact-tension (CT) specimens were fabricated and tested according to ISO 13586 [49]. The properties of the tested epoxy material are presented in Table 1.

Table 2
The experimentally obtained fracture loads of the U-notched rectangular specimens.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>$P_1$ (N)</th>
<th>$P_2$ (N)</th>
<th>$P_3$ (N)</th>
<th>$P_{avg}$ (N)</th>
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<tr>
<td>1</td>
<td>2370</td>
<td>2415</td>
<td>2408</td>
<td>2398</td>
</tr>
<tr>
<td>2</td>
<td>2438</td>
<td>2441</td>
<td>2454</td>
<td>2444</td>
</tr>
<tr>
<td>4</td>
<td>2494</td>
<td>2489</td>
<td>2503</td>
<td>2495</td>
</tr>
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</table>

Fig. 3. Schematic of the rectangular specimen containing a central U-shaped notch.

Fig. 4. The U-notch neighborhood in a rectangular specimen before and after the fracture tests.

Fig. 5. Schematic of the U-SCB specimen loaded under pure mode I.

Fig. 6. The U-SCB specimen under pure mode I loading inside the test machine.
Table 3
The experimentally obtained fracture loads of the U-SCB epoxy specimens.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>P1 (N)</th>
<th>P2 (N)</th>
<th>P3 (N)</th>
<th>Pavg (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2894</td>
<td>2965</td>
<td>2848</td>
<td>2899</td>
</tr>
<tr>
<td>2</td>
<td>3686</td>
<td>3458</td>
<td>3612</td>
<td>3549</td>
</tr>
<tr>
<td>4</td>
<td>4708</td>
<td>4622</td>
<td>4660</td>
<td>4670</td>
</tr>
</tbody>
</table>

2.2. Fracture toughness testing of the epoxy material containing U-notches

After characterizing the epoxy material in the previous subsection, the second group of the fracture tests were performed on U-notched specimens. Two types of test specimen containing U-shaped notches were prepared to test under pure mode I loading conditions.

As shown in Fig. 3, the first test specimen was a rectangular plate of 4 mm thick containing a central bean-shaped slit with two U-shaped ends. Three notch radii 1, 2 and 4 mm were considered in the experiments. The specimens were tested under uni-axial monotonic tension. As the direction of the applied load P is perpendicular to the slit bisector line, two U-shaped ends experience pure mode I loading conditions. To evaluate the repeatability of the tests, for each notch radius, three samples were tested. It is clearly seen in Fig. 4b, the fracture trajectory is along the notch bisector line, demonstrating that fracture takes place under pure mode I loading. Table 2 presents the experimentally obtained fracture loads of the U-notched specimens. Note that in Table 2, the parameters P_i (i = 1, 2, 3) and P_avg denote the three fracture loads and the average of the three loads, respectively.

Fig. 5 shows the second test specimen, called the U-notched semi-circular bend (U-SCB) specimen, tested under symmetric three-point bending (TPB). For this specimen, the thickness was equal to 4 mm and three notch radii 1, 2 and 4 mm were considered in the experiments. Each test was repeated three times for the U-SCB specimen. The experimentally obtained fracture loads of the U-SCB epoxy specimens are summarized in Table 3. Fig. 7 also depicts the test machine. Each test was repeated three times for the U-SCB specimen containing U-shaped notches tested under symmetric three-point bending (TPB). For this specimen, the thickness was equal to 4 mm and the average of the three loads, respectively.

Dealing with the ductile epoxy material tested in the present study, the validity of the power-law strain-hardening relationship applied by Torabi and co-workers [32–39] should be verified. For this purpose, in Fig. 9a a linear trend, the power-law expression, which is well applicable to ductile metallic materials, seems to be inappropriate for the ductile epoxy polymer tested.

Bahadur [50] proposed an exponential expression for the strain-hardening behavior of polymers as follows:

$$\sigma = \sigma_0 e^{m}$$  \hfill (2)

where the parameters $\sigma_0$ and $m$ are the material constants. The plot of the true stress (in logarithmic scale) versus the true strain in the non-linear region for the epoxy polymer is depicted in Fig. 9b based on Bahadur’s expression. It can be seen that the plot is almost linear, meaning that Eq. (2) can identify well the non-linear portion of the standard tensile stress-strain curve of the epoxy polymer tested. Now, Eq. (2) is utilized to develop the EMC formulations for the epoxy polymer and to find the corresponding tensile strength of the equivalent material.

By applying Eq. (2), the expression of the total SED can be obtained as:

$$\sigma = \sqrt{\sigma_0^2 + \frac{2KE}{n+1} \left( e_\nu^{n+1} - (0.002)^{n+1} \right)}$$  \hfill (1)

where $\sigma_0$, $\sigma$, $e_\nu$, $E$, $K$ and n are the tensile strength of the equivalent material, the yield strength, the true plastic strain at the ultimate point, the elastic modulus, the strain-hardening coefficient and the strain-hardening exponent, respectively [39].

Dealing with the ductile epoxy material tested in the present study, the validity of the power-law strain-hardening relationship applied by Torabi and co-workers [32–39] should be verified. For this purpose, in Fig. 9a the true stress versus true strain for the non-linear region of the epoxy material is plotted on logarithmic coordinates. Since the plot does not follow a linear trend, the power-law expression, which is well applicable to ductile metallic materials, seems to be inappropriate for the ductile epoxy polymer tested.
By setting the SED values for both the real ductile and the virtual brittle materials to be equal, we finally have

\[
(SED)_{\text{tot}} = \frac{\sigma^2}{2E} + \frac{\sigma_u}{m} \left( e^{\text{max},\text{true}} - e^{0.003m} \right) 
\]

(3)

Such a \( \sigma^*_f \) can be utilized together with the material fracture toughness \( K_{IC} \) (or \( K_c \)) in various brittle fracture criteria for predicting the LCC of the U-notched polymeric specimens presented in section 2.

4. Brittle fracture criteria

4.1. Maximum tangential stress (MTS) criterion

The MTS criterion is a well-known failure criterion frequently used for brittle fracture assessment of engineering components. According to the MTS criterion, brittle fracture occurs when the tangential stress at a critical distance \( r_c \) ahead of the crack tip reaches the material critical stress \( \sigma_c \). Moreover, the pre-existing crack grows radially along a direction perpendicular to the maximum tangential stress.

The critical stress is a material property and usually assumed to be equal to the ultimate tensile strength for brittle and quasi-brittle materials. The critical distance \( r_c \) is considered to be also a material property and equal to \( 51 \):\n
\[
\sigma^*_f = \sqrt{\frac{\sigma^2}{2E} + \frac{2K_{IC}}{m} \left( e^{\text{max},\text{true}} - e^{0.003m} \right)} 
\]

(4)

Fig. 8. Sample load-displacement curves for (a) the rectangular and (b) U-SCB specimens.

\[
(SED)_{\text{tot}} = \frac{\sigma^2}{2E} + \frac{\sigma_u}{m} \left( e^{\text{max},\text{true}} - e^{0.003m} \right) 
\]

(3)

4.2. Mean stress (MS) criterion

According to the MS criterion, brittle fracture occurs when the mean value of the tensile stress over a specified critical distance \( d_c \) ahead of the crack tip attains the material critical stress \( \sigma_u \). The critical distance \( d_c \) is assumed to be the material property and it is obtained to be four times \( r_c \) as follows:

\[
d_c = 4r_c = 2 \left( \frac{K_{IC}}{\sigma_u} \right)^\frac{1}{2} 
\]

(6)

The U-notch mean stress (UMS) criterion uses the same mean stress (MS) failure concept for U-notches to generally predict brittle fracture in U-notched components regardless of the type of loading. In other words, brittle fracture takes place in a U-notch in accordance with the MTS criterion when the tangential stress at the critical distance \( r_c \) ahead of the notch border reaches the material critical stress \( \sigma_c \).

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\[\sigma^*_f = \sqrt{\frac{\sigma^2}{2E} + \frac{2K_{IC}}{m} \left( e^{\text{max},\text{true}} - e^{0.003m} \right)} \]

(4)

Fig. 9. True stress-strain of the tested polymeric material for the non-linear region based on the (a) power-law and (b) Bahadur expressions.

\[\sigma^*_f = \sqrt{\frac{\sigma^2}{2E} + \frac{2K_{IC}}{m} \left( e^{\text{max},\text{true}} - e^{0.003m} \right)} \]

(4)

Some other papers have also been published in which the MS criterion is utilized for predicting brittle fracture in U-shaped notches \([53-56]\). Eq. (6) has been frequently utilized in many researches for computing the critical distance of MS criterion for various notch geometries \([16,17,53,56,58]\).
In the next section, the finite element (FE) stress analyses of the tested U-notched polymeric specimens are presented. Since the polymeric material was equated in Section 3 with the virtual brittle material exhibiting perfectly linear elastic behavior, the FE analyses are performed under linear elastic conditions.

5. Finite element analysis

Several two dimensional (2D) FE models were created and analyzed to predict the LCC of the polymeric specimens. The specimens were simulated in the FE code ABAQUS. For both specimens, the analyses are performed under plane-stress conditions as a result of the slenderness of the specimens. Fig. 10 represents typical mesh patterns for both

Fig. 10. Mesh pattern for: a) the whole specimen and b) mesh pattern in the vicinity of notch tip.

Fig. 11. Contours of the tensile tangential stress at the vicinity of the U-notch border. The load is equal to 100 N.

Fig. 12. The MTS failure concept, schematically.
specimens. Eight-node plane-stress quadratic elements with reduced integration were used at the notch neighborhood. To achieve a reasonable convergence of the results, several analyses were repeated by different numbers of elements and the mesh-sensitivity was checked. In the vicinity of U-notches, high stress gradient exists, and hence, as shown in Fig. 10, refined meshes with the minimum size of about 0.01 mm were utilized in this region. For the U-notched rectangular model, the tensile load was applied to the nodes that lie on the upper

Table 4
The discrepancies between the theoretical and experimental results for the U-notched rectangular specimen.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>Discrepancy for EMC-MS (%)</th>
<th>Discrepancy for EMC-MTS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6.2</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Table 5
The discrepancies between the theoretical and experimental results for the U-SCB specimen.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>Discrepancy for EMC-MS (%)</th>
<th>Discrepancy for EMC-MTS (%)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2.9</td>
<td>3</td>
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<td>2</td>
<td>4.8</td>
<td>4.6</td>
</tr>
<tr>
<td>4</td>
<td>8.1</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 13. The concept of MS criterion, schematically.

Fig. 14. The variations of the fracture load versus the notch radius for the: a) U-notched rectangular specimen and b) U-SCB specimen.
is very similar to that from the MTS criterion, except that the average of the tensile stresses over the critical distance $d_c$ should reach $\sigma^*_f$. Therefore, the fracture load of the U-notched specimen dealing with MS criterion can easily be computed as $P_{MS} = \frac{\sigma^*_f}{\varepsilon_{ct}} \times 100 \text{ N}$. The concept of MS criterion is schematically represented in Fig. 13.

6. Results and discussion

The variations of the fracture load versus the notch tip radius for both specimens are depicted in Fig. 14. It is clear from Fig. 14 that both the EMC-MTS and EMC-MS criteria slightly underestimate the experimental results. Moreover, for both U-notched specimens, no meaningful difference can be realized between the accuracies of the two failure criteria, meaning that one can arbitrarily select any of these criteria for predicting the fracture loads. However, due to greater simplicity, the use of EMC-MS criterion is preferred in real engineering design.

Tables 4 and 5 present quantitatively the discrepancies between the theoretical and experimental results for the U-notched rectangular and U-SCB specimens, respectively. It is seen from the tables that the average discrepancies for the two specimens are 4.5% and 5.3%, respectively, meaning that both criteria are successful in predicting the LCCs of the ductile U-notched polymeric specimens.

A SEM (scanning electron microscopy) micrograph of fracture surface for a tested U-notched polymeric specimen is presented in Fig. 15. The fracture surface is rough, illustrating the formation of the plastic zone in this surface. Such a rough surface can be attributed to ductile, not brittle, fracture. The very smooth fracture surface associated with brittle fracture is seen in none of the polymeric specimens tested in this study. In fact, as the roughness of the fracture surface becomes denser and deeper, more strain energy is absorbed during the fracture process. The roughness of the fracture surface is formed due to the significant plastic deformations prior to crack initiation from the notch tip.

As mentioned in Section 2 (see Fig. 8), the load-displacement curves of both polymeric specimens obtained from the experiments reveal the plastic deformations in the specimens prior to crack initiation from the U-notch tip. The SEM micrograph in Fig. 15 demonstrates the same. To check to see if the two evidences above can also be confirmed by numerical simulations, a U-notched rectangular specimen with 1 mm radius and a U-SCB specimen with 2 mm radius were arbitrarily selected and analyzed by means of the FE method. Note that the FE models created are the same as those mentioned in section 5. However, the main difference is that the real material behavior (the elastic-plastic stress-strain curve of the epoxy material) is given to the FE code. As can be seen in Fig. 16, the plastic regions are distinguished by the butterfly wings. It is evident from both figures that the size of the plastic regions is significant with respect to the ligament size (particularly for the U-SCB specimen), demonstrating the failure of the polymeric rectangular
specimens within the moderate-scale yielding (MSY) and the U-SCB specimens by the large-scale yielding (LSY) regimes.

7. Conclusions
The effects of hardener concentration and strain rate on the stress-strain behavior of neat epoxy specimens were examined. The Equivalent Material Concept (EMC) was reformulated for the tested ductile polymeric material. Ductile failure of the U-notched specimens with two various geometries was studied experimentally under pure mode I loading conditions. Two types of tensile and three-point bending tests were conducted on the samples. The EMC was utilized in conjunction with the MTS and MS brittle fracture criteria to theoretically predict the experimentally obtained load carrying capacity of the specimens. It was revealed that both EMC-MTS and EMC-MS criteria could predict the LCCs of the ductile epoxy specimens very well. While the EMC-MS criterion provided a bit better predictions than the EMC-MTS criterion, no meaningful difference was found between the accuracies of the two criteria. By means of the experimental data, SEM micrograph and elastic-plastic finite element (FE) simulations, it was demonstrated that the tested specimens are broken with significant plastic deformation around the notch. By using the two combined criteria, it is possible to predict the tensile LCCs of ductile notched polymeric components without needing elastic-plastic analyses.

References

