Fair Allocation in Financial Disputes Between Public–Private Partnership Stakeholders Using Game Theory

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Abstract. Long-term contracts along with their various internal and external variables lead to inevitable changes in the financial estimation of public–private partnership (PPP) projects. In these cases, and during renegotiations, the excess of benefit/cost should be shared among the key stakeholders, including the private contractor, government, and end users, in terms of contract extensions, annual subsidies, and tariff adjustments, respectively. However, while the allocation of excess benefit/cost is an important factor in the successful execution of PPP projects, few methods have considered this issue. Moreover, these methods have rarely involved all three stakeholders and often have evaluated a limited number of possible solutions by qualitative techniques. To address the fair allocation of excess benefit/cost, this paper investigates some sharing mechanisms based on cooperative game concepts, including the core, the nucleolus, and the Shapley value. These mechanisms can improve the renegotiation regulations in PPP contracts and help decision makers manage renegotiations with better structure and supervision. The proposed allocation mechanisms are shown to be fair and practical approaches to managing the financial viability in PPP contracts.

Keywords: financial management • public–private partnership • fair allocation • game theory • shapley value

1. Introduction

To develop the country and create social welfare, infrastructure projects and public services are essential. Until recent decades, ownership and responsibility for infrastructure projects were carried out by the government. Recently, because of the increasing complexity of construction projects, the governments believe that private contractor innovation, investment, and skills could have a substantial impact in the implementation of public services and infrastructure (Button 2016). During the 1990s, significant growth in the volume of private investment was created, which was named public–private partnership (PPP). According to the British treasury (Arthur Andersen and Enterprise LSE 2000), a PPP contract is a long-term relationship between the government and a private contractor to encompass various modes of economic partnership such as concession, joint venture, contract management, franchise, lease, and outsourcing of public services.

There are several types of PPP contracts (Mouraviev and Kakabadse 2017). Some of the most important ones, like build–operate–transfer (BOT), build–rent–own–transfer, and build–own–operate–transfer are associated with the award of a concession period in which the private investor takes the right of utilization, exclusively. The four main phases in implementing these projects are determining the feasibility of the project, construction, operation, and transfer to the state (Shen et al. 2002). The second, third, and fourth phases of these projects can be found in the net present value (NPV) chart, which is shown in Figure 1.

In the second life cycle (operation phase), the private contractor utilizes the project, and anticipated incomes should be achieved. At this stage, private capital should come back and achieve expected profit. Nevertheless, because of the various factors in this long period, changes in contract conditions, project scope, and risk allocation are quite likely. This matter makes the successful implementation of the project difficult (Hart and Moore 1988, Dewatripont 1988). To lessen these concerns and to enhance the marketability of PPP projects for private contractors, governments guarantee a specified level of return on private investment. In this situation, different scenarios are adjusted for the serious risks, and with their occurrence, renegotiations are done (Shen et al. 2002, Cruz and Marques 2013, Domingues and Zlatkovic 2015).

The evidence from many research studies shows that renegotiation in PPP contracts is common and inevitable (Cruz and Marques 2013, Javed et al. 2014, Guasch et al. 2014, Domingues and Sarmento 2016). Because the concession period is long and the initial capital must be returned in this period, the possibility of renegotiation is very high. According to Guasch et al. (2014), renegotiations are frequently repeated in a project, and nearly 45% of them happen in the concession phase. Financial disputes are the main source of renegotiation and
Figure 1. Major Phases of PPP Projects with a Concession Period in Terms of Net Present Value Chart

usually will increase the cost of the project (Ho et al. 2015). Guasch et al. (2014, 2008) state that during 1990–2013, over 1,700 defined PPP projects in Latin American countries confronted financial disputes (55% and 75% in the transportation and water sectors, respectively). In the case of renegotiation during the operation, excess benefit/cost should be shared among the stakeholders of project. So, despite differing goals among stakeholders, they have to cooperate and take shared decisions to prevent the incompleteness of the contract.

According to the literature, project stakeholders can be divided into two major categories: inner and outer stakeholders. The first group contributes in project implementation, and the second exploit the project (Yuan et al. 2009, Newcombe 2003, Osei-Kyei and Chan 2017). In the literature of many researchers, the private contractor and government are classified as the main inner stakeholders, while the end users are the most significant outer stakeholders (Xie and Ng 2013, El-Gohary et al. 2006). The simplest solution for the sharing allocation problem would be to split the common cost equally or weighted with each stakeholder’s volume, which is not fair (Barron 2013, Frisk et al. 2010). Fair allocation of the excess benefit/cost among influential stakeholders in each period has an important role in the progress of the project and the quick resolution of disputes. A suboptimal allocation may create serious risks for the project structure and make it vulnerable.

While the allocation of excess benefit/cost is an important factor for the successful execution of PPP projects, few methods to date have considered this issue. Moreover, these methods have rarely involved all three stakeholders and have often evaluated a limited number of possible solutions by qualitative techniques. The main studies on dispute resolution and renegotiation are listed in Table 1. Most of these studies relate to the feasibility study and construction periods and model only the behaviour of the government and the private contractors. Xie and Ng (2013) and Xiong and Zhang (2014) considered the end users’ benefits in their models and introduced possible solutions to resolve disputes. Nikolaidis and Roumboutsos (2013) presented an expert-knowledge-based technique on stakeholder payoffs and power theory to identify potential renegotiation outcomes in PPP contracts. Sharafi et al. (2016) proposed a neural network method to estimate the cash flow (income and expenses) of projects during the operation period and calculated the difference between this estimation and initial predictions. Then, to resolve financial dispute and choose the best option among generated solutions, they used a min-max multiobjective approach.

Based on the review of the literature, there is no systematic method to fairly divide the excess benefit/cost among stakeholders, particularly during the operation period. The contribution of this paper is as follows. First, it provides a conceptual and computational framework for compensation of financial disputes and determining the role and capabilities of each stakeholder in the project. Second, based on these abilities and using the concepts of cooperative game theory, this paper presents some solution approaches to provide fair allocation of excess benefit/cost among stakeholders. The aim of this research is not to identify the best allocation paper, because choosing among different approaches is dependent on the features of the project and specific fairness criteria. Following, this financial dispute (excess benefit/cost) is examined in term of costs, and the procedure is similar for benefits.

The remaining sections are arranged as follows. Section 2 describes the renegotiation framework and characteristics related to stakeholders involved in the model. In Section 3, the proposed models are described for cost allocation between stakeholders. An illustrative example is presented in Section 4 along with numerical results. Finally, in the last section, some concluding remarks are presented.
Table 1. A Brief Review of the Main Studies on Renegotiation and Dispute Management in PPP Contracts

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Objectives</th>
<th>Project life period</th>
<th>The parties involved in renegotiations</th>
<th>Method(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xing and Wu (2001)</td>
<td>2001</td>
<td>Determine the type and capacity of power plant</td>
<td>Operation period</td>
<td>Private investor and the government</td>
<td>Stackelberg game theory</td>
</tr>
<tr>
<td>Ho (2006)</td>
<td>2006</td>
<td>Decision about bankruptcy or help contractor</td>
<td>Construction period</td>
<td>Government and private contractors</td>
<td>Dynamic game with complete information</td>
</tr>
<tr>
<td>Shen et al. (2007)</td>
<td>2007</td>
<td>Concession period determination</td>
<td>Feasibility study period</td>
<td>Public contractor and private investor</td>
<td>Game theory and bargaining</td>
</tr>
<tr>
<td>Cruz and Marques (2013)</td>
<td>2013</td>
<td>Identifying the exogenous factors for concession renegotiations</td>
<td>—</td>
<td>Government and concessionaire</td>
<td>Econometric analysis on a real database</td>
</tr>
<tr>
<td>Nikolaidis and Roumboutsos (2013)</td>
<td>2013</td>
<td>Identification the potential renegotiation outcomes</td>
<td>Operation period</td>
<td>Government and private contractors</td>
<td>Power theory and stakeholder payoffs</td>
</tr>
<tr>
<td>Xie and Ng (2013)</td>
<td>2013</td>
<td>Derive a set of noninferior solutions based on the expectations of stakeholders</td>
<td>Feasibility study period</td>
<td>Government, private contractor, and end users</td>
<td>Multiobjective Bayesian network models</td>
</tr>
<tr>
<td>Xiong and Zhang (2014)</td>
<td>2014</td>
<td>Analysis and prediction of future cash flow</td>
<td>Operation period</td>
<td>Government, private contractor, and end users</td>
<td>Time-series models</td>
</tr>
<tr>
<td>Domingues and Zlatkovic (2015)</td>
<td>2015</td>
<td>Determine factors for the renegotiation</td>
<td>—</td>
<td>Government and private contractors</td>
<td>Literature review and analyzing 9 European PPP projects</td>
</tr>
<tr>
<td>Song et al. (2015)</td>
<td>2015</td>
<td>Concession period determination</td>
<td>Feasibility period</td>
<td>Public contractor and private investor</td>
<td>Dynamic programming and fuzzy multiobjective</td>
</tr>
<tr>
<td>Xiong and Zhang (2016)</td>
<td>2016</td>
<td>Providing a flexible model for PPP contracts</td>
<td>Feasibility study period</td>
<td>Government and private contractors</td>
<td>Real option model using game theory</td>
</tr>
<tr>
<td>Sharafi et al. (2016)</td>
<td>2016</td>
<td>Cash flow forecasting and choosing the best solution among the available options</td>
<td>Operation period</td>
<td>Government, private contractor, and end users</td>
<td>Artificial intelligence and fuzzy multiobjective</td>
</tr>
</tbody>
</table>

2. Contract Modification During the Operation Period

To encourage private contractors to invest in a project, several financial guarantees are provided by governments. Least present value of revenue and minimum revenue are some of the certain indicators that measure these financial guarantees (Mouraviev and Kakabadse 2017, Xiong and Zhang 2014). With regard to the possibility of serious risks, if these financial indicators are driven under the corresponding level, renegotiation should be opened. In general terms, the dispute value is defined as the following equation:

\[
\Delta NPV = NPV^{(0)} - NPV^{(1)}. \tag{1}
\]

Here, \( NPV^{(0)} \) is the minimum level of NPV that is guaranteed, and \( NPV^{(1)} \) is the expected NPV, which is calculated on actual performance until the current time.

Renegotiation can lead to different results. Xie and Ng (2013) presented eight results including direct payment (lump sum or annual payment), tax benefits, changes in contract scope, change in rents for the operator, delay in investments, contract extension, contract reduction, and tariff adjustment. According to previous studies, common methods for compensation are contract extension, tariff adjustment, and annual subsidy (Xie and Ng 2013, Xiong and Zhang 2014, Domingues and Sarmento 2016). Furthermore, choosing any of the results influences the satisfaction of the main stakeholders, including the private contractor, government, and end users (Domingues and Sarmento 2016, Xie and Ng 2013). In these situations, decision makers must implement an order that covers \( \Delta NPV \) and concurrently provides social acceptance for the government, economic benefit for the private contractor, and public accountability for end users. It should be noted that in an actual situation, the final decision may be a combination of the results. Figure 2 illustrates a general renegotiation framework.

3. Measurement Allocation Problem Using Game Theory

Game theory interprets human behaviours as a set of strategies and intends to move individuals toward their goals in either a cooperative or competitive framework. Cooperative game theory has been widely applied in...
finding fair solutions such as common profit/cost allocation (Hennet and Arda 2008). Creating a coalition of players and allocating the excess value among its members are critical issues in applying cooperative games (Behzad and Jacobson 2016, Feng and Sivakumar 2016, Saberi et al. 2014). This section first provides some basic definitions of cooperative games and then focuses on solving the presented problem. Some sharing mechanisms, including the core, the nucleolus, and the Shapley value, are investigated to allocate the excess cost share to each stakeholder.

3.1. Preliminaries (Basic Definitions and Properties)

A cooperative game is shown with the pair $(N, V)$, where $N = \{1, \ldots, n\}$ is the set of players, and each nonempty subset of $N$ is named a coalition. For each coalition $W$, we shall specify the characteristic function $V(W) \subseteq \mathbb{R}$, which is the cost of coalition $W$. Every cost allocation is a vector of $x = (x_1, \ldots, x_n)$, denoting that cost $x_i$ allocated to player $i \in N$. Different criteria are defined for the fairness of the allocation; however, there is no scheme that satisfies all criteria in the literature. The most commonly used criteria are individual and group rationality (Saaty 2000). A cost allocation is said to be individual rational if no participant pays more than its own cost, when no coalitions are formed ($x_i \leq V(i)$).

A cost allocation should also have group rationality, which means that any decrease in the cost to a player must conform to an increase in the cost to one or more other players ($\sum_{i \in N} x_i = V(N)$).

The core is considered a set of payoff allocations that satisfies individual and group rationality. It makes a set of stable allocations such that no player likes to be out of the grand coalition, i.e., the set of all players. Suppose that the surplus function $e(W, x) = \sum_{i \in W} x_i - V(W)$ is a criterion of dissatisfaction of a certain coalition $W$, resulting from the allocation $x$. The core of each cooperative game, $C(N, V)$ or $C(0)$, is equal to Ho (2006):

$$C(N, V) = C(0) = \{x \in \mathbb{R}^n \mid e(W, x) \leq 0, \forall W \subseteq N\}. \tag{2}$$

There may be more than one allocation satisfying these properties and that might give a set of cost allocations. If the core contains only one member, which rarely occurs in most games, that is the solution. Otherwise, if the number of core members is high, the feasible set should be shrunk. Prominent solution concepts include the least core and the nucleolus (Barron 2013, Peleg and Sudhölter 2007). The $\varepsilon$-core, which is nonempty for the smallest possible $\varepsilon$, is named as the least core:

$$C(\varepsilon) = \{x \in \mathbb{R}^n \mid e(W, x) \leq \varepsilon, \forall W \subseteq N\}. \tag{3}$$

In the same way, if there is more than one member in the least core $C(\varepsilon)$, it should be cut down further. In other words, the maximum amount of surplus over whole allocations in $C(\varepsilon)$ should be minimized. This
solution is called the nucleolus. According to the characteristic function, this function finds the worst inequity in each stage and tries to identify an imputation $\bar{x} = (x_1, \ldots, x_n)$ that minimizes it. If the core is nonempty, the nucleolus is inside the core and unique (Schmeidler 1969).

The Shapley value is another popular scheme in the fair allocation of cooperative games that provides a unique allocation to a collection of players in coalitional games. Using the Shapely value, the cost allocation is done based on the importance of each player in the coalition. The Shapley value of each coalition is calculated using Equation (4), where $|W|$ is the number of members in the coalition $W$ (Shapley 1953):

$$SV_c(N, V) = \sum_{W : i \in W} \frac{(|W| - 1)!(n - |W|)!}{n!} [V(W) - V(W - i)].$$

### 3.2. The Application of Game Concepts in Dispute Management

According to the previous section, the main issue of this study is to determine the allocation of the excess cost ($\Delta NPV$) to each of three players. Table 2 presents the defined variables in the models. Obviously, the necessary condition for the existence of feasible solution is that the total sum of the maximum amount allocated to each player is more than the total dispute value, $\sum_{i=1}^3 U_i \geq \Delta NPV$. Also, in any solution, the total amount of allocated to players is exactly equal to dispute value, $\sum_{i=1}^n x_i = \Delta NPV$. It can be shown that in this case, the core is not empty, and the grand coalition is stable. So, the nucleolus always exists and is in the core. To calculate the core and the nucleolus, the characteristic function of each coalition should be formed in the game as defined in Equation (5). It means that if the coalition $W$ cannot tackle the excess cost, $\Delta NPV$, its characteristic function is equal to zero:

$$V(W) = \begin{cases} \sum_{i \in W} U_i - \Delta NPV & U_i \geq \Delta NPV \\ 0 & \text{otherwise.} \end{cases}$$

On the other hand, the Shapley value involves the expression $V(W) - V(W - i)$ in each term, and, according to Equation (4), only the terms with $V(W) - V(W - i) > 0$ need to be computed. There are three modes for each coalition that has been shown in Equation (6). First, the coalition $W$ with or without the player $i \in W$ can satisfy the excess cost, $\Delta NPV$ ($V(W) > 0$ and $V(W - i) > 0$). As a result we have

$$V(W) - V(W - i) = \sum_{j \in W} U_j - \Delta NPV - \left[ \sum_{j \in W, j \neq i} U_j - \Delta NPV \right] = U_i.$$

In the second case, the coalition $W$ only with the participation of $i$ can finance the excess cost, $V(W) > 0$ and $V(W - i) = 0$. So $V(W) - V(W - i) = \sum_{j \in W} U_j - \Delta NPV$. Finally, if coalition $W$ cannot finance the community, even with the presence of $i$, we have $V(W) - V(W - i) = 0$:

$$V(W) - V(W - i) = \begin{cases} U_i & V(W - i) > 0, V(W) > 0, \\ \sum_{j \in W} U_j - \Delta NPV & V(W - i) = 0, V(W) > 0, \\ 0 & V(W - i) = 0, V(W) = 0. \end{cases}$$

Note that in the first case of Equation (6), if $i \in W$ and $V(W - i) > 0$, automatically $V(W) > 0$. So, the Shapley value for each player is

$$x_j = \sum_{W : i \in W, V(W - i) > 0} U_i \frac{(|W| - 1)!(|N| - |W|)!}{|N|!} + \sum_{W : i \in W, V(W) > 0, V(W - i) = 0} \left[ \sum_{j \in W} U_j - \Delta NPV \right] \frac{(|W| - 1)!(|N| - |W|)!}{|N|!}.$$
Based on the proposed definition for the characteristic function of both the nucleolus and Shapley value, \( x_i \) refers to the portion of \( \sum_i U_i - \Delta NPV \) assigned to player \( i \). Consequently, the portion of \( \Delta NPV \) allocated to player \( i \) is \( y_i = U_i - x_i \).

The only remaining decision in using these mechanisms is determining the maximum share allocated to each of the players. Ho (2006) considered the political costs imposed on government due to financial subsidy. According to this research, the cost is mainly dependent on environmental conditions of project such as the importance of the project, the competition between contractors, causes of deviation from the schedule, etc. He concluded that the amount of annual subsidy cannot be more than a specified limit \( J \). So, if \( \Delta NPV_G \) denotes the government share of the total \( \Delta NPV \), it can be changed within interval \([0, U_G]\), where \( U_G \) is obtained from the Equation (8). In this equation, \( r \) is the discount rate, \( m \) is the current time, and \( T_C \) is the planned concession period:

\[
U_G = \sum_{i=0}^{T_C} \frac{J}{(1 + r)^t}.
\]

(8)

About the second stakeholder (private contractor), the share allocation is done via decreasing or increasing the period of concession. As mentioned in the introduction, the concession period is determined in such a way that the different costs during the second and the third phases of the project life (construction and O&M) are covered and achieve the expected profit. Figure 3 shows that during the operation phase, the NPV chart crosses the zero border (point A), and by reaching this chart to the guaranteed value \( (NPV(0)) \), the project is transferred to the government. Over time, the cost of O&M increases and the slope decreases. At point B, the NPV slope becomes zero and the economic life of the project ends. So, the concession period could be changed within interval \([A, B]\) and maximum share allocated to private contractors \( (U_C) \) is given as follows:

\[
U_C = \sum_{t=T_C+1}^{B} \frac{NPV_t}{(1 + r)^t}.
\]

(9)

Most of PPP contracts are infrastructure projects (highways, hospitals, power plants, etc.), and there are several alternatives for them. The demand for each mode is dependent on its utility function and different variables such as price, accessibility, and quality. There are several models to estimate the probability of each mode, and one of the most important one is multinomial logit model (Daly and Bierlaire 2003). If \( g_k \) denotes the utility function of mode \( k \), the probability of selecting this mode is calculated using the following equation:

\[
P(k) = \frac{e^{g_k}}{\sum_x e^{g_x}}.
\]

(10)

Therefore, by increasing tariffs for end users, the general tendency is reduced exponentially, and the overall income for the project in relation to tariffs will be parabolic, as in Figure 4. This means that the increase in tariffs leads to an increase in revenue to a certain level, and with further increase, total revenue decreases because of the reduced tendency of end users. So, the maximum share allocated to end users (third stakeholder) can be calculated using this multinomial logit function and procedures presented in Figure 4.
4. Computational and Practical Results

4.1. Profile Project and Solution Set

To examine the proposed model, this section uses data from a project presented in Xiong and Zhang (2014) with a little extra information. The presented project is set via a BOT contract. Its information is listed in Table 3. Eighteen years after operation of the project, the accumulated NPV is $4,573 million, and the estimated NPV for the end of concession period is $266 million, which is much less than the minimum guaranteed NPV. So the key stakeholders (private contractors, government, and end users) should undertake a part of the excess cost (ΔNPV = $3,302 million).

Suppose that the maximum amount of annual subsidy is $2,000 million, which the government can pay until 2033 (end of operation by the private contractor). It also assumes that by analyzing other alternatives for users, maximum annual earnings growth for the tariff change is $600 million. Furthermore, according to Table 3, economic useful life for this project is 38 years. With the extension of the concession up to 2033 (an increase of 10 years), $1,220 million will be added to annual cash flow, which will increase annually by 20%.

Figure 5 shows the border of the solution space for different levels of decision variables. Any point on the surface and the top of the shell can answer the question and make up ΔNPV = 3,302. As can be seen in Figure 5,

<table>
<thead>
<tr>
<th>Project indicator</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project construction time</td>
<td>5 years (from 1993 to 1998)</td>
</tr>
<tr>
<td>Cost of construction</td>
<td>$7,000 million</td>
</tr>
<tr>
<td>The period of concession</td>
<td>30 years (from 1993 to 2023)</td>
</tr>
<tr>
<td>Guaranteed minimum NPV</td>
<td>$3,320 million</td>
</tr>
<tr>
<td>Interest rate (r)</td>
<td>10% (annual)</td>
</tr>
<tr>
<td>Economic life</td>
<td>40 years (until 2033)</td>
</tr>
</tbody>
</table>

Source. Xiong and Zhang (2014).
the feasible space becomes more widespread by increasing the annual subsidy and the contract extension.

Moreover, with an increase of about 40% in tariff rates, need for funding the government subsidy and extending the contract is reduced.

4.2. Nucleolus and Shapley Value

To use the proposed mechanisms, the maximum share allocated to each of the stakeholders must be calculated. The values of $U_i$ can be written as follows:

\[
U_C = \sum_{t=19}^{30} \frac{2,000}{(1+r)^t} = 2,451 \text{ m}$, $U_C = \sum_{t=31}^{40} \frac{NPW_t}{(1+r)^t} = \sum_{t=31}^{40} \frac{1,220 \times (1.2)^{(t-31)}}{(1+r)^t} = 970 \text{ m}$, $U_u = \sum_{t=19}^{30} \frac{600}{(1+r)^t} = 947 \text{ m}$.

Since $\sum_i U_i = 4,368 \text{ m} > \Delta NPV = 3,302 \text{ m}$, there are feasible solutions for the allocation problem, and the core is not empty. According to Equation (2), the characteristic function of all possible coalitions are defined as follows:

\[
V(G) = V(C) = V(U) = 0, \\
V(GC) = 119, \quad V(GU) = 96, \quad V(CU) = 0, \\
V(GCU) = 1,066.
\]

So, the constraints associated with the core function for a given imputation $\bar{x} = (x_G, x_C, x_U)$ are $0 \leq x_C \leq 2,451$, $0 \leq x_U \leq 947$, $x_C + x_U \geq 96$, $x_C + x_U \geq 119$, and $x_C + x_C + x_U = 1,066$. Using some algebra and the substitution of $x_C = 1,066 - x_C - x_U$, these inequalities imply that

\[
C(0) = \{(1,066 - x_C - x_U, x_C, x_U) \mid 0 \leq x_C \leq 950, 0 \leq x_U \leq 945, 0 \leq x_C + x_U \leq 1,066\}.
\]

By plotting this region in the $(x_C, x_U)$ plane, Figure 6 is achieved. This means that this problem has many reasonable allocations. So, $C(\epsilon^1)$ should be used to shrink the core. By substituting $\epsilon^1 = -355.3$ instead of $\epsilon$, the least core is the set $C(\epsilon^1) = \{(1,066 - x_C - x_U, x_C, x_U) \mid x_C \leq 594.67, x_U \leq 589.67, x_C + x_U = 710.67\}$, which is shown in Figure 6 with a dark line.

Finding a single well-defined allocation requires calculating the nucleolus. A unique optimal solution for this model is $x_C = 355.3$, $x_C = 355.3$, and $x_U = 355.3$. Thus, based on the nucleolus mechanisms, the fair allocation of

**Figure 6.** The Core with Lots of Allocation and the Shrinking of the Core at $C(\epsilon^1)$

\[ \Delta NPV \] can be obtained as \( y_G = 2,095.7, \) \( y_C = 612.7, \) and \( y_U = 593.6 \) for government, private contractors, and end users, respectively.

The cost allocation based on Shapley values for this example, according to Equation (6), are

\[
\begin{align*}
V(G) - V(\Phi) &= 0, \quad V(C) - V(\Phi) = 0, \quad V(U) - V(\Phi) = 0, \\
V(GC) - V(C) &= 119, \quad V(GU) - V(U) = 96, \quad V(CU) - V(U) = 0, \\
V(GC) - V(G) &= 0, \quad V(GU) - V(G) = 0, \quad C(CU) - V(C) = 0, \\
V(GCU) - V(G) &= 0, \quad V(GCU) - V(C) = 970, \quad V(GCU) - V(U) = 947.
\end{align*}
\]

Using Equation (7), the Shapley values for stakeholders are \( x_G = 391.17, \) \( x_C = 343.17, \) and \( x_U = 331.66, \) and relative excess cost allocations are equal to \( y_G = 2,059.9, \) \( y_C = 627.8, \) and \( y_U = 615.3. \)

According to the results of each allocation mechanism, the behaviour of each stakeholder can be determined in the implementation of specified shares \( (y_i) \). For example, in the nucleolus, according to Equation (8), the government should pay an annual subsidy equal to 
$1,710 million for 12 years to supply its share \( (y_G = 2,095.7). \)

Using Equation (9), the share of the private contractor will be met with an increase in the concession period of 7.23 years (until early 2031). Finally, the tariffs should be increased by the amount that eventually adds 
$593.6 million to the project cash flow. Here, this amount will be achieved by an increase of 
$393 million in annual income from 2012 to the end of modified concession period, 2031.

4.3. Discussion and Managerial Results

Although the stakeholders involved in the PPP contracts have different financial objectives, their needs to complete and implement the project persuade them to cooperate. Figure 7 shows the power of two-member coalitions in funding excess cost, and the feasible solution set is marked in red. From Figure 7, (a) and (b), the region of possible solution is very restricted, in the absence of end users or private contractors, respectively.

**Figure 7.** Feasible Space of the Problem in the Absence of One of the Stakeholders Involved in the Contract
Furthermore, in the absence of government subsidy, other stakeholders do not have the ability to provide $\Delta NPV$ (Figure 7(c)).

A straightforward allocation approach is to distribute the total dispute value ($\Delta NPV$) according to a weighted measure based on the maximum share allocated to each of the stakeholders. This is expressed by the formula $y_i = w_i \times \Delta NPV$, where $w_i$ is equal to $U_i / \sum U_i$. Using this approach, the allocations can be obtained as $y_G = 1,852.8$, $y_C = 733.3$, and $y_U = 719.9$.

Table 4 shows the allocation results, according to the different described mechanisms. The results of the nucleolus and Shapley approaches are significantly different from those using the weighted method. Although the weighted method is easy to calculate and understand, it may provide allocations that are not fair (Barron 2013, Frisk et al. 2010). In fact, powerful players in the process of renegotiation are not the same, and the weighting method ignores this issue. Both the end users and the private contractor have about %22 of the weight ratio. However, since their presences in the coalition with the government are required to cover $\Delta NPV$, their powers increase and decrease their shares in the nucleolus and Shapley approaches, respectively. Similar to Littlechild and Thompson (1977), the nucleolus and the Shapley value propose cost allocations similar to those in the real data.

### Table 4. The Results of the Share Allocated ($Z_i$) and Proportion (in %) for Each Stakeholders in Different Mechanisms

<table>
<thead>
<tr>
<th></th>
<th>Weighed approach</th>
<th>Nucleolus approach</th>
<th>Shapley value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share</td>
<td>Proportion</td>
<td>Share</td>
</tr>
<tr>
<td>Government</td>
<td>1,852.8</td>
<td>56.1%</td>
<td>2,095.7</td>
</tr>
<tr>
<td>Private contractor</td>
<td>733.3</td>
<td>22.2%</td>
<td>614.7</td>
</tr>
<tr>
<td>End users</td>
<td>715.9</td>
<td>21.7%</td>
<td>591.7</td>
</tr>
<tr>
<td>Sum</td>
<td>3,302</td>
<td>1</td>
<td>3,302</td>
</tr>
</tbody>
</table>

5. Conclusions

Several objectives such as reducing cost, transferring risk, saving time, solving budgeting constraint problems, and providing higher quality make public–private partnerships popular. Public authorities may enhance the marketability of PPP projects by offering different financial guarantees. However, because of the long-term nature of these contracts, it is not possible to forecast the whole of future scenarios, and in most cases, preliminary estimates of financials change during the operation phase. To prevent the failure of PPP projects, renegotiations are frequently undertaken among the key involved stakeholders, including private contractors, government, and end users. In such situations, excess cost should be divided between them in the forms of contract extensions, annual subsidies, and tariff adjustments. Despite the prevalence of this condition in the execution environment, the literature still lacks a computational framework on allocation patterns. Existing methods rarely involve all three stakeholders and often evaluate a limited number of possible solutions by qualitative techniques.

The aim of this study is to address this research gap by investigating some sharing mechanisms based on cooperative game concepts including the core, the nucleolus, and the Shapley value. If the total sum of the maximum amount allocated to each of the players is more than the total dispute value, $\sum U_i \geq \Delta NPV$, the core is not empty and the nucleolus is introduced as a fair and stable solution. Following this, by defining a properly characteristic function, the Shapley value is introduced as the second fair solution. The proposed mechanisms were described in an example, and the share of each stakeholder was allocated. In addition, the results were compared with the simple weighting method, as a straightforward approach. The results of the nucleolus and Shapley approaches are similar. However, since the weighted approach does not consider the power of players in negotiation, its results are significantly different from those of our proposed mechanisms.

These mechanisms can improve the renegotiation regulations in PPP contracts and help decision makers to manage renegotiations with better structure and oversight. Developing a novel kind of bargaining game in which the players are allowed to bargain and decrease their shares can be introduced as a future study direction. Also, a model that could take into account the relative power of the situations of the stakeholders in the renegotiations is proposed as another future research topic. As some researchers noted, opportunism behaviours are one of the main reasons for renegotiation contracts (Ho et al. 2015). Providing a quantitative model to demonstrate these behaviours and their impacts on the earnings of stakeholders is another potential area for future studies. In recent years, many researchers have focused on incomplete contracts and partnership between stakeholders in
PPP projects (Domingues and Zlatkovic 2015). In this context, cooperation between stakeholders is important, and other cooperation models could be recognized as subjects for future work studies.

References


Author Queries

A1 Au: Please confirm/correct the title, and the the running head, which has been edited to 80 characters or less, including 3 author names, shortened title, and spaces.

A2 Au: Please confirm names, affiliations, and email addresses are okay as set. Please provide the postal code for University of Tehran.

A3 Au: Please confirm/correct edit to “To address the fair allocation of excess….”

A4 Au: Please provide any funding information, including grant numbers and granting institutions.

A5 Au: Please confirm keywords.

A6 Au: Please confirm heading levels are correct.

A7 Au: Please confirm/correct edit to “According to the British treasury….”

A8 Au: Please confirm edits to the sentence “Four main phases in implementing these projects are determining the feasibility of the project…..”

A9 Au: Please check that all figures and tables are set correctly.

A10 Au: Is “the end users the utilize the project…” meant?

A11 Au: Journal style is to change “due to” to “because of” where applicable. Please confirm.

A12 Au: Please confirm/correct edit to “over 1,700 defined PPP projects in Latin American countries confronted financial disputes.”

A13 Au: Please confirm/correct edits to the Table 1 caption. Can “Year” column be deleted since the years are mentioned in with the reference?

A14 Au: Please confirm/correct edit to “a neural network method.”

A15 Au: Please verify that all equations are set correctly. Note that journal style is to use italics for all variables. Please confirm throughout and mark where any changes are needed.

A16 Au: Please carefully review and confirm/correct edits to the sentence “Furthermore, choosing any of the results influences…..”

A17 Au: Please confirm/correct edits to the sentence “A cost allocation should also have group rationality…..”

A18 Au: Please confirm/correct edit to “It makes a set of stable allocations such that…..”

A19 Au: Please confirm/correct edit to “So, the nucleolus always exists.”

A20 Au: Please confirm/correct edits to “Ho (2006) considered…” and “He concluded that the amount…” below.

A21 Au: Define “O&M” at first use.

A22 Au: Is “the general tendency of end use” meant?

A23 Au: Is “the reduced tendency for end use” meant?

A24 Au: Please check that number appears correctly.

A25 Au: Please clarify the sentence beginning “Moreover, with an increase of about 40%…..” Is the following meant?: “However, with an increase of about…the need for government subsidy and to extend the contract is reduced.”

A26 Au: Is “the smallest core” meant?

A27 Au: Please confirm edit to “Thus, based on the nucleolus mechanisms….”

A28 Au: Please confirm/correct edit to “the share of the private contractor will be met with an increase in the concession period of 7.23 years.”

A29 Au: Confirm/correct edits to the two sentences beginning “The results of the nucleolus and Shapley approaches….”
Au: Please confirm/correct edit to “preliminary estimates of financials change during the operation phase.”

Au: Please confirm/reword edit to “renegotiations are frequently undertaken…. .”

Au: Please confirm edit to “address this research gap.”

Au: Please confirm edit to “The results of the nucleolus and Shapley approaches.”

Au: Please confirm edit to “and other cooperation models could be recognized as subjects for future work studies.”

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