Thermo-economic analysis of transcritical CO$_2$ cycles with bounded and unbounded reheats in low-temperature heat recovery applications

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**Abstract**

Performance of transcritical CO$_2$ Rankine cycles with and without reheat process is investigated thermo-economically in low-grade waste heat recovery applications. Two reheating scenarios are proposed to evaluate the effect of bounded and unbounded reheats on the cycle. In the first method, the constant available heat flow is distributed between the evaporator and the re heater via an optimized ratio, while in the second, the required energy for the reheat process is provided with optimized additional fuel consumption. The proposed cycles are modeled and optimized for source temperatures ranging from 150 to 300 $^\circ$C at fixed flow rate of 1000 kg/s. The results obtained from thermodynamic optimization indicate that reheat cycle with burning additional fuel leads to the largest power generations ranging from 14 to 57 MW depending on the source temperature, while the reheat cycle with heat stream division shows the weakest performance by producing 8–37 MW. In the thermo-economic optimization, the ratio of power output to the cycle total bare module cost has been maximized. Under these conditions, the reheat cycle with burners still shows the highest rate of power production, while economic indicators limit the power generation and introduce the simple Rankine cycle as the best option.

1. Introduction

The call for energy has been unremittingly increasing in the world for the past years, and this trend is believed to continue. Thermal power plants are one of the most important energy sources; however, most of them suffer from huge amounts of heat rejection to the environment. Thus, a promising approach to increase the efficiency of thermal power plants is adding a waste heat recovery process to them, so that more power is generated by capturing the previously-useless heat. Being free of fuel cost, such a solution gets even more attractive. Nevertheless, the biggest drawback associated with waste-heat-to-power processes is the low temperature of the medium, which is typically less than 350 $^\circ$C [1]. Liu et al. [2] concluded that the existence of hydrogen bond in molecules like water, which contributes to larger enthalpies of evaporation, is a disadvantage for cycles that work with low temperatures. With this regard, Dai et al. [3] found that the low-grade waste heat recovery cycles that run with organic working fluids are more efficient. It has been clearly demonstrated by Saleh et al. that fluids with low critical properties are the best choice for low temperature cycles [4]. This statement is already in accord with the findings of another work [5] which reveals that Organic Rankine Cycles (ORC’s) usually perform more efficiently when the working fluid works in the supercritical region. As it is also expressed by Lecompte et al. [1], there exists more than 1000 patents on supercritical cycles which reveal the importance of this issue.

CO$_2$ is a working fluid that benefits from most of the mentioned characteristics. Furthermore, it possesses a variety of favorable properties of a working fluid proposed by V. Maizza and A. Maizza [6], which include small specific volume, low viscosity, high thermal conductivity and stability, being non-combustible, non-toxic, non-corrosive, and compatible with engine material and lubricating oil. Being a natural substance, CO$_2$ is economically desirable, too. Yamaguchi et al. [7] demonstrated that although the first-law efficiency of transcritical CO$_2$ Rankine cycle is lower than that of a refrigerant-run cycle, it is more economical in terms of cost per net power output. This is in line with the results of a recent paper [8] which concludes that the exergetic efficiency of a transcritical
CO₂ Rankine cycle could be remarkably higher than other working fluids. In a research carried out by Shengjun et al. [9], the economic benefits of employing CO₂ as the working fluid have been well demonstrated. It was also shown that CO₂ transcritical cycle is more compact and environmentally friendly, and generates more power output than the R123 subcritical cycle [10]. Wu et al. [11] utilized genetic algorithm in order to optimize the influential parameters in a CO₂ transcritical power cycle with a pure thermodynamic approach. In this matter, they considered single-, double- and triple-stage cycles and took the maximum temperature and pressure of the cycle along with the exit temperature of the flue gases as the optimization parameters. Li et al. [12] performed an experimental study which revealed that CO₂ is effective in high-temperature supercritical states, and works stably in a transcritical cycle. In another experimental study on CO₂ transcritical power cycles [13], it is reported that with high pressure of 110 bar and low pressure of 46 bar, about 10 kJ/kg specific power output and thermal efficiency of 5% is achievable. Numerous efforts have been done to augment the efficiency of ORC’s; Lecompte et al. [1] mention that modified cycles with recuperators, regenerative turbine bleedings, reheaters, and vapor injectors are all attempts to maximize the temperature difference between heat addition and rejection processes, so that the cycle efficiency increases. Amongst these options, the recuperation technique has been studied widely in the literature for CO₂ transcritical cycles [10,14–17] and little attention has been paid to other options. For instance, in a recent research, Mondal and De focused on regenerative turbine bleeding with finite quantity of waste flue gas at 200 °C and 25 kg/s and found out that there exists an optimum value for bleed pressures with respect to maximizing the first law efficiency [18]. Also in 2002, Price and Hassani [19] studied three different configurations of ORCs in a solar application. Although they reported slightly higher efficiency for the reheater cycle, it is unbounded. Moreover, Baik et al. [20] considered the two limiting factors, but their study has only included one specific source temperature. In another relevant research [21], a two-stage reheater transcritical CO₂ Rankine cycle has been proposed for recovering heat from an internal combustion engine of a container ship. In this study, however, the heat sources used for the main heat absorption process and the reheater process are different and independent. The present work is aimed to investigate the potential benefits and/or drawbacks of adding reheater process to a transcritical CO₂ Rankine cycle that is used in waste heat recovery applications. It is theoretically easy to show that a Rankine power cycle with reheater process is more efficient than a simple Rankine cycle that works under the same conditions. However, this does not seem to be the

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**Nomenclature**

**Acronyms**

- ORC: Organic Rankine Cycle
- GA: Genetic Algorithm
- HEX: Heat Exchanger
- NTU: Number of Transfer Unit
- HP: High-Pressure
- LP: Low-Pressure
- FAR: Fuel-to-Air Ratio

**Symbols**

- T: temperature
- h: enthalpy
- m: CO₂ mass flow rate
- M: mass flow rate
- w: specific power
- W: power
- y: mass flow ratio
- C: heat capacity rate
- Cᵣ: heat capacity ratio
- D: shell diameter
- r: friction coefficient
- d: tube diameter
- Q: volume flow rate
- cᵢ: specific heat capacity at constant pressure
- fᵣ: temperature correction factor
- fᵢ: pressure correction factor
- fₘ: material correction factor
- F: fuel mass fraction
- A: surface area
- V: heat exchanger volume

**Greek letters**

- η: efficiency
- ε: effectiveness
- φ: equivalence ratio
- ζ: correction factor

**Subscripts**

- p: pump
- t: turbine
- is: isentropic
- i: inner
- o: outer
- max: maximum
- min: minimum
- source: hot source
- a: heat stream inlet
- b: main heat stream outlet
- b’: reheat heat stream outlet
- aux: auxiliary
- fuel: reacting fuel
- combustion: combustion reacting gases
- wᵢ: working fluid
- w: wall
- f: bulk flow
- s: shell
- TBM: total bare module cost
- DPI: direct permanent investment
- TPI: total permanent investment
- cont: contract fee
same in waste heat recovery applications where limited amount of heat is to be distributed between the main heat absorption process and the reheat process. In heat recovery processes, the source temperature and flow rate are both fixed, which result in more physical constraints and complexity. In such cases, the limitations in temperature drops and the coupling of the mass and energy equations may result in contrary solutions; that is less efficient cycles potentially capable of recovering more useful mechanical work from the heat source, or cycles with higher efficiencies but lower power outputs.

To address this controversial issue, we introduce and analyze three different cycle configurations at source temperatures ranging from 150 to 300 °C. The three configurations correspond to one simple transcritical CO₂ Rankine cycle, as the reference cycle for comparisons, and two distinct reheat cycles. In the first reheat arrangement, called plan A, the constant available heat is distributed between the main heat absorption process and the reheat process, and the distribution ratio is found via optimization. In the second approach, named plan B, the whole amount of available heat is transferred to the main heating process, and the required energy for the reheat process is provided by an external combustion source, which is assumed unbounded. Analyzing the two reheat methods enables us to clarify the effects of limited and unlimited available heat on the cycle performance. Once the governing equations of all the configurations are developed, optimizations are performed via genetic algorithm to find the optimum working conditions of each configuration at every source temperature. In the first step, the optimizations are conducted within a pure thermodynamic framework, where the net power output of the cycle is the maximized parameter. In the next section, the economic limits are taken into account, in order to make the results more realistic and practical. The optimized parameter in this approach is the invested money per kW of the output power.

2. Thermodynamic modeling and analysis

Due to their excellent thermodynamic performance, ORC’s and in particular transcritical CO₂ power cycles are known as promising choices for effectively recovering low-grade heat [12]. Low-grade heat refers to sources such as waste heat, geothermal, and low temperature concentrated solar collectors, which basically means low- and mid-temperature heat that has less exergy density and cannot be converted efficiently by conventional methods [25,26]. In this study, in order to conduct a comprehensive analysis for transcritical CO₂ power cycle, four heat source temperatures, i.e. 150 °C, 200 °C, 250 °C and 300 °C are selected to cover the whole range of low-grade heats. This wide temperature range is common among relevant efforts for optimization of ORC’s performance [27]. Also, the final exhaust temperature is governed by the dew point temperature of the combustion products at about 70 °C.

The reason why CO₂ has been chosen as the working fluid of this cycle is its favorable low critical temperature which allows it to be transferred to supercritical region easily. This feature contributes to a pinch-free gas heating process, and serves the cycle with a less irreversible heat transfer process. This not only improves the efficiency of the cycle, but augments its economic performance by means of cheaper heat exchangers required. The other advantage of CO₂ is its positive condenser pressure with a comparison to atmospheric pressure. No vacuum is induced in the condenser and thus, no de-aerator is needed for the cycle [28]; this poses a direct decrease in the equipment costs. This phenomenon is reflected especially in the heat exchanger area and its corresponding cost [9].

2.1. Transcritical CO₂ simple cycle

A conceptual configuration in the context of the T–s diagram of a transcritical Rankine cycle with its four basic processes is shown in Fig. 1. First, working fluid is pumped to high pressure (heat absorption pressure) from the saturated liquid at low pressure (condensing pressure). In the case of no reheat, the working fluid is then heated in one stage by absorbing the energy from the low-grade heat source in the gas heater (high-pressure heat exchanger). At this point, the expansion process occurs and the working fluid releases its energy to generate power in the turbine. Finally, the working fluid is cooled down and condensed by the ambient air in the condenser (low-pressure heat exchanger) to complete the cycle [29]. The mentioned cycle is referred to simple cycle in this study, since there is no additional component in the conventional Rankine cycle in order to improve the cycle performance.

Considering the basic thermodynamics, the following relation is written for the pumping process of the cycle:

\[ \eta_p = \frac{h_{26} - h_1}{h_2 - h_1} \]  

And the isentropic efficiency of the pump is defined as:

\[ \eta_p = \frac{h_{26s} - h_1}{h_{26} - h_1} \]  

Then,

\[ \eta_p = \frac{h_{26s} - h_1}{w_p} \]  

With Eq. (3), and the Engineering Equation Solver (EES) software, which serves as the property calculator, one can find the work input of the pump for the pressure increase required in the cycle. Similarly, by using the definition of the isentropic efficiency for the turbine:

\[ \eta_t = \frac{h_3 - h_{4t}}{h_3 - h_{4t}} \]  

Now, the net specific work could easily be calculated by:

\[ \eta_w = \frac{h_3 - h_{4t}}{w_t} \]  

Fig. 1. Schematic diagram of transcritical CO₂ simple cycle.
Applying the first law to the process ‘2–3’ of Fig. 1, i.e. the heat recovery occurring in the heat exchanger gives:

$$\dot{M}_{\text{source}}c_p(T_a - T_b) = \dot{m}(h_3 - h_2)$$

In which $T_a$ and $T_b$ are the inlet and outlet temperatures of the hot stream through the heat exchanger. Also, $\dot{M}$ represents the mass flow rate, and $c_p$ is the heat capacity of the hot stream which is assumed as that of air at the mean temperature. Similarly, $h_2$ and $h_3$ are the inlet and outlet enthalpies of CO2 in the heat recovery gas heater, and $\dot{m}$ is the mass flow rate of carbon dioxide in the cycle. The cooling process in the condenser occurs with the help of ambient air with a mean temperature of 15°C.

In order to analyze the effect of adding reheat process to transcritical CO2 Rankine cycles, two other conceptual cycles with two different approaches for the reheat application are defined and illustrated in the next section.

2.2. Transcritical reheat cycle configurations

It is known that the efficiency of Rankine cycle is increased by increasing the pressure during the heat absorption. In the case of constant maximum allowable temperature, the increase in pressure leads to increase in moisture content in the low-pressure stages of the turbine, while adding reheat process avoids this phenomenon [30]. However, moisture prevention is not an issue in transcritical CO2 cycles. In this study, the principal aim of utilizing reheat process is to augment the cycle efficiency in case of limited source temperature, as already shown in the literature [21]. To do so, an intermediate pressure is defined in which the working fluid is extracted from the high-pressure turbine and reheated in order to achieve acceptable enthalpy and generate power in the low-pressure turbine. As a result, the second turbine would receive the working fluid with a considerable value of enthalpy, despite the fact that it works under low pressures. This configuration of reheating with the same maximum temperature is presented in Fig. 2.

The primary concept of a reheat cycle seems not very complicated in the first view, however, when it comes to low-grade heat conversion cycles, it may encounter some difficulties. For instance, when waste exhaust gas, which is limited in terms of temperature and flow rate simultaneously, is the available heat source, one should distribute the available energy in such a way that both heat absorption processes, i.e. CO2 evaporation and reheating, receive sufficient amount of heat. This fact is the key factor in this study which leads to considering two designs for the reheat cycle in low-grade cases with limited flow rate, like the previous case study performed by the authors [31].

The first configuration, plan A, illustrated in Fig. 3, follows the idea of dividing the heat source stream into two streams by a defined ratio of "y". To explain more, $y$ represents the fraction of the main stream which is guided to the first vapor generator heat exchanger, while $(1-y)$ proportion of the stream is extracted for the reheat process in the second heat exchanger. This distribution is stated with the help of two governing equations:

**gas heater:**

$$y \times \dot{M}_{\text{source}} \times c_p \times (T_a - T_b) = \dot{m}(h_3 - h_2)$$

**reheater:**

$$(1-y) \times \dot{M}_{\text{source}} \times c_p \times (T_a - T_b) = \dot{m}(h_5 - h_4)$$

It should be noted that in Eqs. (8) and (9), the values of the heat capacities, $c_p$, are not the same, because each of them is calculated in its corresponding stream mean temperature.

The second cycle configuration, plan B depicted in Fig. 4, is practically different. In this plan, the primary heat source stream is
For the specific case of a stoichiometric methane-air reaction, \( F \) equals 0.055. In addition, the fuel-to-air ratio (FAR) and equivalence ratio are related as:

\[
\text{FAR} = \frac{\text{fuel mass}}{\text{air mass}} = \frac{F}{1 - F}
\]

(14)

\[
\varnothing = \frac{\text{FAR (actual mixture)}}{\text{FAR (stoichiometric mixture)}}
\]

(15)

Considering the adiabatic flame temperatures of the methane for different equivalence ratios along with the fact that very high temperature results in uncontrollable heat transfer, the equivalence ratio of 0.2 was selected for the burner reaction. This value will provide a lean combustion in order to limit the maximum temperature. With respect to the efficiency of the burner, flame temperature of 800 K would be appropriate and achievable [32]. Also, the fuel mass flow rate, which is a key parameter for the economic analysis of the reheat cycle, is calculated with regard to the mentioned reaction and equivalence ratio:

\[
\dot{M}_\text{fuel} = \frac{\dot{M}_\text{combustion}}{87.207}
\]

(16)

The advantage of studying the two mentioned ideas is that each case is representative of a concept for transcritical CO2 cycles. That is, when a limited source with constant exergy is available, two approaches for the reheat process are presented: dividing the constant exergy source or adding some additional source to the cycle.

Before any more discussion, it is necessary to state the main assumptions of the thermodynamic analysis for both simple Rankine and reheat cycles which are described below:

- The inlet temperature of the both low- and high-pressure turbines is equal. In other words, the maximum temperature of the cycle is limited to the source temperature and in the presence of reheat: \( T_5 = T_3 \).
- The pressure loss through each of the heat exchangers, i.e. main gas heater, reheater, and condenser, is assumed to be 5%.

In addition, some assumed data for the cycle and its components are presented in Table 1. The isentropic efficiencies and pressure losses are in accordance with the values in the literature [29,33]. It
has also been assumed that the heat loss in the subsystem is negligible [34].

3. Practical design modeling and thermo-economic analysis of the cycles

As it is obvious, there is always a broad distance between the ideal thermodynamic cycle and the practical one, and a lot of design constraints play important roles when it comes to construction. In other words, for an actual case, economic considerations and design constraints for construction take part in achieving an optimum performance. Sizing of the components and capability of the real heat exchangers may mostly affect the optimum power output of the cycle. Therefore, more detailed calculations get involved in this situation in which the heat exchanger performance would be the most influential.

3.1. Design of heat exchangers

Design of the heat exchangers depends on the approach which is chosen by the researcher or the engineer. In this study, effectiveness-NTU method is utilized in order to obtain the sizing of the heat exchangers. Selection of this method is due to its compatibility with popular types of the heat exchangers, such as cross flow or shell and tube heat exchangers. In the mentioned method, according to the dimensional analysis, three main non-dimensional parameters define the heat exchanger completely. In other words, it can be shown that for any heat exchanger [35]:

\[ \varepsilon = f(\text{NTU}, \frac{C_{\min}}{C_{\max}}) \]  

where C is the heat capacity rate of each of the streams. The parameter \( \varepsilon \) and the Number of Transfer Units (NTU) are also defined as follows:

\[ \varepsilon = \frac{q}{q_{\max}} \]  

\[ \text{NTU} = \frac{UA}{C_{\min}} \]  

where \( q_{\max} \) represents the maximum possible heat transfer rate and is equal to \( C_{\min}(T_h - T_c) \) and U is total heat transfer coefficient, and A is the heat transfer area. Neglecting the tube thickness, total heat transfer coefficient can be calculated as below:

\[ \frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} \]  

where \( h_i \) is the convection heat transfer coefficient inside the tube and \( h_o \) is the external convection heat transfer coefficient. Since the main focus of this study is the determination of the performance of the transcritical CO\(_2\) power cycle, and also the external conditions are controllable and assigned with respect to heat transfer methods regardless of the working fluid state, \( h_o \) is assumed to be constant and have a value of 100 \( \frac{\text{W}}{\text{m}^2\text{K}} \). Therefore, the convection heat transfer coefficient inside the tube is calculated for each of the heat transfer processes, i.e. vapor generation, reheat and condensation with respect to the state of CO\(_2\). Furthermore, it should be noted that the minimum pinch temperature difference is assumed 10° in all the heat exchangers.

3.1.1. Heat transfer calculation for single phase fluid

The studied transcritical cycle goes through single-phase heat transfer in the vapor generation stage and also in the reheat process, if utilized. In this case, the Dittus-Boelter correlation [36] is used to first measure the Nusselt number inside the tube, and consequently obtain the heat transfer coefficient.

\[ \frac{h_i}{k_f} = 0.023Re^{0.8}Pr^n \]  

where \( n = 0.4 \) for heating and 0.3 for cooling. Thus in this study, single-phase section of the condenser, \( n = 0.3 \) is the right choice, since CO\(_2\) flows inside the tube. This equation is reorganized with regard to Eq. (20), in order to calculate the heat transfer coefficient.

\[ h_i = 0.023 \frac{k_f}{D} \left( \frac{4(\frac{m}{\pi D})}{\ln \left( \frac{D}{D_0} \right)} \right)^{4/5} Pr^{0.3} \]  

in which \( m \) is the total mass flow rate of the CO\(_2\) and \( N_t \) is number of the tubes in each heat exchanger. It should be noted that when the fluid is in the supercritical region which is the case in the vapor generator and the reheater of this project, the Nusselt number is obtained by:

\[ \frac{h_i}{k_f} = 0.0183Re^{0.82}Pr^{0.3} \left( \frac{\rho_w}{\rho_f} \right)^{0.3} \]  

This correlation is suggested by Jackson and Fewster [37] in which “w” represents the properties at the tube wall temperature and “f” represents the properties at the bulk temperature of the fluid. On the other hand, NTU non-dimensional relation for n shell passes and 2n shell-and-tube heat exchanger is presented by Ref. [38]:

\[ \text{NTU} = n \times \text{NTU}_1 = n \left[ - \left( 1 + C_t^f \right)^{-0.5} \ln \left( \frac{E - 1}{E + 1} \right) \right] \]  

\[ E = \frac{2}{\ln \left( \frac{E - 1}{E + 1} \right)} - (1 + C_t) \]  

\[ F = \left( \frac{C_{\text{eff}} - 1}{C_{\text{eff}} + 1} \right)^{1/n} \]  

It should be noted that \( C_t \) represents the heat capacity ratio of the two fluid streams.

3.1.2. Heat transfer calculation for the two-phase fluid

In the present work, Shah [39] correlation is employed as a prediction for the heat transfer coefficient in the two-phase section of the condensation process. He defines \( h_{o1}, \) which stands for the heat transfer of the liquid phase flowing alone in the tube. This coefficient, with respect to cooling value for n, becomes in the form of Eq. (27), for this study.

<table>
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<th>Table 1</th>
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<tr>
<td>Parameter</td>
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<tr>
<td>Turbine Isentropic Efficiency</td>
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<td>Pump Isentropic Efficiency</td>
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<td>Condensing Pressure</td>
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<td>Condensing Temperature</td>
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<td>Source Stream Mass Flow Rate</td>
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\[ h_{LT} = 0.023 \frac{k_f}{D} \times \left[ \frac{4 \left( \frac{W}{A} \right)}{\pi D f} \right]^{4/5} P_{r0.3} \]  

And the heat transfer coefficient inside the tube for the turbulent regime is calculated by:

\[ h_i = h_{LT} \left( \frac{\mu_f}{T_{a_i} \rho_f} \right)^{0.0058 + 0.557 (\rho_i)} \left[ 1 - x_i^{0.8} + \frac{3.8 x_i^{0.76} (1 - x_i)^{0.04}}{p_i^{0.38}} \right] p_i^{0.4} \]

where \( p_i \) is the reduced pressure and \( x \) is vapor quality. On the other hand, in this case, the NTU relation for heat transfer is modeled such that \( C_t = 0 \) and is given by Ref. [38]:

\[ NTU = -\ln(1 - \epsilon) \]

It is worth noting that only a fraction of the condensing stage is governed by the mentioned equation. Therefore, the condenser surface area is the sum of two areas which are obtained by the single-phase and two-phase correlations separately.

In both cases, the heat transfer surface area, which plays an important role in the economic analysis, can be calculated at the end of this procedure as follow:

\[ A = \frac{NTU \times X_{min}}{U} \]

3.1.3. Electrical power consumption of the condenser fans

Another issue which should be addressed carefully in case of transcritical cycles is the electrical power consumption related to the condenser fans. This power is used by the fans which are responsible for air blowing inside the condenser and over the tubes. This power consumption imposes a strict limitation on the cycle thermo-economic performance and is one of the main agents of reduced power output in practical conditions. The utilized fans should overcome the pressure drop inside the heat exchanger which is calculated by Ref. [38]:

\[ \Delta p = N_t \chi \left( \frac{\rho V_{\max}^2}{2} \right) f \]

where \( f \) is the friction factor and \( \chi \) is correction factor. They are obtained with respect to maximum Reynolds number of the air flow inside between the tubes and geometrical configuration of the tubes [38]. Also, \( N_t \) specifies the number of rows of tubes. Therefore, the electrical power consumption of the fans will be computed by:

\[ W_{fan} = Q \cdot \Delta p \]

in which \( Q \) represents the volume flow rate of air crossing the tubes.

3.1.4. Calculations of thermodynamic properties for different conditions

In the case of thermo-economic analysis, evaluation of the working fluid thermodynamic properties, e.g., heat capacity, should be precisely taken into consideration. As it was expressed in the section of operating equations of the heat exchangers, these values directly affect the sizing and consequently the final cost. Therefore, in order to avoid any deviation from the practical design, the variation of thermodynamic properties in different processes should be analyzed, thoroughly. The most influential thermodynamic property that can affect the thermo-economic analysis is the heat capacity, and application of the \( e \)-NTU method for designing heat exchangers with variable heat capacity at constant pressure must be justified. Furthermore, it is interesting to note that in the supercritical region which occurs in the gas heating process, this variation goes far from the linear one. Thus, the integral method, i.e., discretization of the heat exchanger into the small subsections, in which the linear assumption is appropriate, could be utilized in this region. However here, to obtain a computationally efficient program, a sensitivity analysis is conducted in order to find out the necessity of the applying integral method. For better understanding, variation of heat capacity in constant pressure is given in Fig. 5. It should be noted that pressure and temperature ranges are assigned such that cover the practical operating conditions.

As it can be seen from Fig. 5, there is a peak in the specific heat capacity curve that occurs at lower temperatures, however, the assumption of linear variations is approximately valid for working temperatures above 130 °C. The results of the mentioned sensitivity analysis are presented in the first part of the thermo-economic optimization.

3.2. Cost estimation and economic consideration of different cycle configurations

The cost in the present study is defined as total bare module (TBM) cost or direct onsite costs of capital investment which is approximately equal to costs of working fluid, equipment and their installation. This cost is calculated by Eq. (33), as in Ref. [29]:

\[ C_{TBM} = \sum C_{BM} + \sum C_{spare} + C_{wf} \]  

In order to achieve this estimation, the magnitude of thermodynamic parameters, such as heat transfer and power, should be correlated with relevant component costs. In other words, price of a specific item, \( C_{BM} \), is stated as a function of its size, materials of construction, and design temperature and pressure [40,41], which is correlated by Eq. (34):

\[ C_{BM} = C_B \left( \frac{Q}{Q_B} \right)^M \frac{M}{f_p f_T} \]

where

\[ f_p = \begin{cases} 1 & \text{P=130 bar} \\ 1.5 & \text{P=150 bar} \\ 2 & \text{P=170 bar} \end{cases} \]

\[ f_T = \begin{cases} 1 & \text{P=130 bar} \\ 1.5 & \text{P=150 bar} \\ 2 & \text{P=170 bar} \end{cases} \]

Fig. 5. Variation of heat capacity with temperature in constant pressures.
As it is obvious, type of the equipment, its size, and range of operating temperature and pressure are the main influential factors in cost estimation. The cost of each component is calculated by Eq. (35) and Eq. (36).

\[ f_T = \frac{T}{400} + 0.85 \]  
\[ f_P = -10^{-5}P^2 + 0.0102P + 1.0278 \]  

In these relations, \( T \) and \( P \) show the maximum operating pressure and temperature of each of the equipment. Taking a look at the above data and considering the temperature and pressure range that our transcritical CO\(_2\) cycle is supposed to work at, it can be predicted that the maximum pressure will play a more important role in design and optimization, when economic factors are taken into account.

In addition, since the size of the pumps available in the correlated data are small in comparison to the average power required to pump CO\(_2\) in the designed power plant, some parallel 700-kW pumps are responsible for providing the desired pressure increase. Hence, the number of the pumps is also optimized in this study, which is calculated by Eq. (37):

\[ \text{Number of the pumps} = \frac{\text{Required pumping power}}{700} + 1 \]  

where bracket sign represents ceiling function of a non-integer number to ensure that the minimum power is provided. It should also be noted that one extra pump is considered. This pump mainly serves as a spare standby pump that is run in place of another pump which has failed. In this way, the mentioned spare cost is included in the bare module cost.

Spare components are taken into account for the case of any failure in the main process unit. In power plants, usually pumps are provided with spares, because they require frequent maintenance to prevent leaks [29]. In this study, the cost of spare components, i.e., \( C_{\text{spare}} \), is embedded in the bare module cost, as expressed before.

The mentioned data is sufficient to perform an acceptable economic analysis for the transcritical CO\(_2\) cycle; however, in the case of plan B, more considerations should be taken into account, due to its additional burning process. Although a shell-and-tube heat exchanger is used for the reheat process like in plan A, some burners have been added to the cycle and their corresponding size and cost should be calculated, as well. Authors employed the available constructed burners [42] and used their corresponding data which resulted in the selection of 5000-kW ones. Therefore, just the number of burners with Eq. (38) is obtained to estimate the associated additional cost.

\[ \text{Number of the burners} = \frac{\text{Required reheat heat transfer rate (kW)}}{5000} \]  

where again the bracket sign represents the ceiling function to ensure that the minimum required heating power is supplied to the reheat process.

In addition to the mentioned costs, it has also been shown that another important cost to deal with is the working fluid cost, \( C_{\text{wf}} \) [14]. The amount of working fluid is estimated as the liquid amount to fill the total process including equipment and piping [43]. Le et al. [29] have assumed the required working fluid amount to be the liquid amount to fill two times the volumes of heat exchangers of the ORC plant. Shell-and-tube heat exchangers are utilized in this work as the main heat absorber, reheater, and condenser.

Therefore, it is first necessary to design the shell-and-tube heat exchangers to find their dimensions, which are further utilized in calculating the working fluid cost. Equations for calculation of heat transfer surface area were presented in the previous section. On the other hand, shell diameter can be expressed by Eq. (39) in terms of required heat transfer area [44].
where PR is the tube pitch ratio, \( P_t/d_o \), which is usually chosen to be 1.25, 1.33, or 1.5. CTP is the tube count calculation constant that accounts for the difference between the shell section area and the section area filled by the tubes. And the values of tube layout constant, CL, depend on the angles that determine a specific layout for tubes.

To calculate the working fluid cost, the dimensions of the shell-and-tube heat exchangers are required to be determined. Following the approach introduced in the previous section, and assuming the shell length of the heat exchanger, \( L \), to be eight times its diameter, \( D_s \), as an engineering estimate, the final formula for calculating the volume of the each shell-and-tube heat exchanger is found to be:

\[
V = \frac{\pi D_s^2}{4} \times 8D_s = \frac{2\pi \times 0.4058 \times CL \times A_0/PR^2 d_o}{8 \times CTP}
\]  

Design constants and parameters are computed as described in Ref. [44]. By considering two pass shell, and angles of tube layout of 30 °, the heat exchanger parameters are given in Table 5.

With the help of this data along with the density of the liquid CO\(_2\) entering the pump, the required mass is obtained. Considering the price of CO\(_2\) to be 2.2 $/kg [45], the cost of working fluid, and consequently the total bare module cost are calculated.

With the above data, it is possible to calculate the total bare module cost \( C_{\text{CTBM}} \). Having this parameter along with the generated power of the cycle, the ratio of the cost over net power output is computed for each working condition and is further optimized to find the best cycle configuration.

For a more precise economic analysis, however, it is necessary to take more parameters into account, as in Ref. [29]. For this purpose, it is required to add the followings to the \( C_{\text{CTBM}} \).

\[
C_{\text{Site}} = 0.05 C_{\text{CTBM}}
\]

\[
C_{\text{Service}} = 0.05 C_{\text{CTBM}}
\]

Thus the direct permanent investment equals

\[
C_{\text{DPI}} = C_{\text{CTBM}} + C_{\text{Site}} + C_{\text{Service}}
\]

Considering 18\% of the direct permanent cost as the contract fee, the total depreciable cost will become:

\[
C_{\text{CTDC}} = C_{\text{DPI}} + C_{\text{Cont}}
\]

This is the primary cost that is assumed to be depreciated by a linear method within the 20 years of the plant application.

Besides, it is important to consider a start-up cost for the plant as follows

\[
C_{\text{Start-up}} = 0.1 C_{\text{CTDC}}
\]

All in all, the total permanent investment is calculated by

\[
C_{\text{TPI}} = C_{\text{CTDC}} + C_{\text{Start-up}}
\]

The plant according to its size is also associated with running costs that must be taken into account for a precise economic analysis. These costs may be summarized as follows

\[
C_{\text{Wage}} = 0.035 C_{\text{TDTC}}
\]

\[
C_{\text{Salary}} = 0.25 C_{\text{Wage}}
\]

\[
C_{\text{Material}} = C_{\text{Wage}}
\]

\[
C_{\text{Maintenance}} = 0.05 C_{\text{Wage}}
\]

\[
C_{\text{Insurance}} = 0.02 C_{\text{TDTC}}
\]

In case of plan B, where the reheat process is required with additional fuel cost, it is necessary to consider this cost, too. Knowing the fuel mass flow rate and assuming that the plant runs in 90\% of the year period, the annual fuel price is obtained. The price of natural gas is chosen as 0.128 $/kg [46].

Therefore, the total annual running cost of the plant is found as

\[
C_{\text{Running}} = C_{\text{Wage}} + C_{\text{Salary}} + C_{\text{Material}} + C_{\text{Maintenance}} + C_{\text{Insurance}} + C_{\text{Fuel}}
\]

4. Results and discussion

Having an increased number of design parameters in the studied project, choosing an appropriate algorithm in order to find the optimum operating conditions becomes crucial. In particular, a lot of thermo-economic parameters get involved when economic considerations are taken into account. As a result, an optimization algorithm is utilized to obtain the best working condition in both thermodynamic and thermo-economic approaches.

In order to perform the mentioned optimization, knowledge of governing equations and type of design variables is crucial. This knowledge leads to the selection of appropriate algorithms to solve the problem. Standard nonlinear programming techniques are computationally expensive; however, genetic algorithms (GA’s) are capable of finding the global optimum solution with a high probability and are well suited for solving such problems. Genetic algorithms are based on the principles of natural genetics and natural selection. The basic elements of natural genetics—reproduction, crossover, and mutation—are implemented in the genetic search procedure [48].

To perform the genetic algorithm in this study, each individual is characterized by three design variables values, i.e. maximum pressure, maximum temperature and the reheat pressure. According to the survival-of-the-fittest principle of nature, a function that is called fitness function is to be maximized, in this method of

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Shell and tube heat exchanger parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>CL</td>
<td>0.87</td>
</tr>
<tr>
<td>CTP</td>
<td>0.50</td>
</tr>
<tr>
<td>PR</td>
<td>1.5</td>
</tr>
<tr>
<td>( d_o ) (in)</td>
<td>1</td>
</tr>
</tbody>
</table>

N. Mirkhani et al. / Energy 133 (2017) 676–690
optimization. Therefore, this design problem is maximization, where in case of pure thermodynamic approach, the net power output, and in thermo-economic approach, the benefit-cost ratio equals to fitness function of each individual.

Reproduction is the first operation employed in order to select prosperous population to form a mating pool. In this study, a commonly used reproduction operator is utilized in which the probability of selecting the ith member for the mating pool, \( P_i \), is given by:

\[
P_i = \frac{F_i}{\sum_{j=1}^{n} F_j} \quad i = 1, 2, \ldots, n
\]  

(53)

where \( F_i \) denotes the fitness of the ith individual in the population of size n.

In this work, a crossover is carried out according to the position of the child with respect to the parents, and it has the capability of producing two children. So each parent (individuals sent to the mating pool in the previous step) could produce two children as follows:

\[
C = R_1 P_1 + (1 - R_1) P_2
\]  

(54)

\( R_1 \) is a random number between 0 and 1. Coding of the GA algorithm is performed in MatLab coupled with EES, in which the output of EES is imported as an input to MatLab, in order to modify the design parameters to achieve a better objective function in this complex design space. The results of applying this method to thermodynamic and thermo-economic analysis are presented, respectively, in the following sections. Finally, it should be noted that the implemented code is validated for both thermodynamic and thermo-economic approaches in the previous work of the authors [31].

### 4.1. Net power output optimization – thermodynamic approach

In this section, governing equations of the thermodynamic cycle are solved, and the three input parameters, i.e. maximum pressure, and maximum temperature of the cycle, and the reheat pressure are modified to achieve the highest power output. Table 6 represents the results for each source temperature and the three studied cycle configurations.

The data presented in Table 6 reveal that for all source temperatures the cycle efficiency of plan A is less than the other scenarios. This behavior is obvious due to the fact that in plan A, a considerable amount of the constant available waste heat is rejected to the atmosphere without being further utilized. Also, in this thermodynamic approach, plan B serves the best in case of net power output as expected.

To validate the obtained results, it is worth noting that the specific power output for the Rankine cycle is 25.17 kJ/kg for the lowest source temperature. On the other hand, Kim et al. [49] reported that the obtainable specific net power output for a low-temperature transcritical CO₂ cycle working between 57.7 and 200 bar is 21.4 kJ/kg, which is in accord with the calculated value in this work. In order to verify more accurately, published results for different heat source temperatures should be considered, too. For the case of 100 °C heat source, Balk et al. [23] have shown that a transcritical CO₂ cycle that works between 68 and 123 bars would have an efficiency of 5.6%, while in our case of 150 °C, the lowest studied heat source temperature, it is shown that a cycle with high pressure of 218 bars will have an efficiency of 11.6% under thermodynamic optimization. Variations are due to the higher heat source temperature in this study. As shown in Ref. [18], a transcritical CO₂ cycle with turbine bleeding is examined at a source temperature of 200 °C. Extrapolating their results for zero bleeding ratio, one finds an efficiency about 13.5%, which is exactly the same value obtained in our study for the case of 200 °C source temperature in our thermodynamic analysis. For the highest temperature ranges, we refer to Wu et al. [11] where the authors have used GA to optimize the performance of transcritical CO₂ power cycles that work with source temperatures between 250 and 500 °C. Taking benefit of a regeneration process between the turbine outlet and the pump outlet, they have depicted that for 300 °C, the cycle

Table 6
Optimum thermodynamic operating point of different cycles in the range of source temperature.

<table>
<thead>
<tr>
<th>Source Temperature (°C)</th>
<th>Cycle Configuration</th>
<th>Maximum Temperature (°C)</th>
<th>Maximum Pressure (bar)</th>
<th>Reheat Pressure (bar)</th>
<th>Power Output (kW)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150°C</td>
<td>Reheat Plan A</td>
<td>145</td>
<td>242</td>
<td>121</td>
<td>7882</td>
<td>10.52</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>148</td>
<td>312</td>
<td>141</td>
<td>14569</td>
<td>11.13</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>149</td>
<td>218</td>
<td>–</td>
<td>10897</td>
<td>11.59</td>
</tr>
<tr>
<td>200°C</td>
<td>Reheat Plan A</td>
<td>176</td>
<td>212</td>
<td>207</td>
<td>17731</td>
<td>12.12</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>197</td>
<td>457</td>
<td>185</td>
<td>27334</td>
<td>15.09</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>198</td>
<td>249</td>
<td>–</td>
<td>20270</td>
<td>13.53</td>
</tr>
<tr>
<td>250°C</td>
<td>Reheat Plan A</td>
<td>211</td>
<td>245</td>
<td>238</td>
<td>26912</td>
<td>14.02</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>239</td>
<td>473</td>
<td>166</td>
<td>41402</td>
<td>16.79</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>246</td>
<td>312</td>
<td>–</td>
<td>32259</td>
<td>17.47</td>
</tr>
<tr>
<td>300°C</td>
<td>Reheat Plan A</td>
<td>233</td>
<td>276</td>
<td>268</td>
<td>37121</td>
<td>15.42</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>281</td>
<td>503</td>
<td>188</td>
<td>56841</td>
<td>19.09</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>292</td>
<td>318</td>
<td>–</td>
<td>44204</td>
<td>18.75</td>
</tr>
</tbody>
</table>

Fig. 6. Variations of the optimum maximum temperature with source temperature for different cycles in thermodynamic approach.
produces power with an efficiency of 19.9%, which is higher than the value 18.75% found in our work. This little difference is supposed to be due to the existence of the regenerative process in their work. Also, comparison with a very recent work by Le et al. [50] shows that their proposed over-expansion transcritical carbon dioxide cycle can outperform simple and plan A cycles in thermodynamic point of view while plan B achieves higher specific power output between all of these configurations.

Some observations are interesting and may yield some valuable results about the reheat process in such transcritical cycles. First, the optimum maximum temperature shows a consistent increase and diverging trend for the three cycles as the source temperature increases. This phenomenon is illustrated in the Fig. 6. It is obvious that since plan A is not provided by any additional heat transfer and some of the available heat is wasted, the maximum temperature of this configuration cannot follow the temperature of the hot stream like plan B and simple Rankine configurations do.

Another interesting point is the behavior of the cycle efficiency and net power output in the vicinity of the optimum point for plan A. As it is mentioned, this configuration employs no extra heat and distributes the hot stream with the source temperature for the reheat process. Figs. 7 and 8 present the variation of the efficiency and power output, respectively, for this cycle, as the reheat pressure changes at the optimum maximum temperature and pressure.

The trend of the efficiency variation was predictable since the same concept had already been developed in Ref. [30] for water as the working fluid. However, the power variations show different trends at different source temperatures. As it can be seen, reheat pressure indicates an optimum value for the lowest source temperature i.e. 150 °C while the upper ones include a turning point in their curves.

This change in behavior has led to an interesting fact: in plan A, the reheat pressure increases until it reaches its maximum allowable value, which is set as 95% of the cycle maximum pressure in the numerical algorithm. Therefore, it can be found out that reheat process in upper source temperatures is totally useless, while it may present some advantages in the low temperatures. With taking a look at the optimum maximum temperature of the cycle, it can be concluded that a part of the hot stream was sufficient to make the CO₂ reach the source temperature, approximately. Thus, the other part of the stream will be guided to the reheat section to play an effective role. However, in higher temperatures, the hot stream has no more enough energy to heat up CO₂ to temperatures near the one of the stream.

In the conventional thermodynamic analysis, each cycle is first solved for unit mass flow rate, and then the mass flow rate is obtained for the arbitrary total power output or heat absorption. But, in our cycle, the constant temperature waste heat stream with fixed flow rate restricts the mass flow rate of the secondary cycle and couples the governing equations. This fact justifies the more net power output of the Rankine cycle than the plan A, although the reheat also yielded some advantages at this point. Table 7 summarizes the corresponding values of the distribution ratio “y” and the mass flow rate for the Rankine and plan A cycles. The difference in mass flow rates between two cycles is obvious.

Table 7 reveals that reheat process not only does not bring any advantages to the cycle when the heat source is limited, but also it leads to some drawbacks in temperatures higher than 200 °C.

On the other hand, the results of plan B contain some supplementary conclusions. Firstly, in this case, the excessive increase in the maximum allowable pressure is remarkable. This phenomenon represents that with respect to the unlimited heat source in the reheat process and the limited one in the vapor generator, the cycle is inclined to make the main heat absorption as small as possible and use the unlimited source in the reheat process. This is why the high-pressure turbine generates much less power than the low pressure one, in this case. Therefore, as it was predictable, additional arbitrary heat transfer in the reheat process raises the cycle power output in its thermodynamic analysis, since there is no constraint on parameters such as mass flow rate, maximum pressure, and fuel cost, all of which are of utmost importance in economic analysis and come into account in the next section.

4.2. Thermo-economic optimization

As it was stated earlier in section 3.1.4, a sensitivity analysis is carried out in order to investigate the effect of assuming linear variations of heat capacity in constant pressure. As a conservative
criterion, the ratio of the maximum possible integral average to the linear one for different source temperatures is presented in Table 8.

The results of thermo-economic optimization for the simple Rankine cycle and the change in the benefit-cost ratio are given in Table 9. As it is obvious, the effect of linear variations in heat exchanger calculations is negligible for source temperatures greater than 200 °C. Therefore, in order to implement a computationally effective program, for higher source temperatures, properties are calculated as the average of initial and final states of each of the heat transfer processes.

It is necessary to state that most ORC’s are employed as waste heat recovery cycles. In other words, they are usually an additional cycle which is coupled with the main plant. Therefore, although it may seem proper to optimize the rate of return, as in most industrial projects, the net power output to the bare module cost of the cycle is optimized, in this work. To explain more, investors are more inclined to know about the capital cost in these cases, and running cost may attract attention in the next place. In spite of this fact, since plan B would consist of tangible running costs due to fuel consumption, a comprehensive economic analysis is also carried out for each case at its optimum operating point. The corresponding results for the mentioned thermo-economic analysis are presented in Table 10.

Again, some attractive conclusions may be obtained with the help of these results. Firstly, it is clear that economic parameters mostly have affected operating conditions of plan B, and its maximum pressures have fallen remarkably. The limited amount of available heat transfer in the vapor generator has caused plan A and Rankine cycles to be restricted in the thermodynamic analysis, too. Therefore, the order of optimum parameters does not show an intensive variation. However, it is very interesting that both maximum temperature and maximum pressure have decreased in all of the cases. In other words, economic considerations show themselves in decreasing these maximum allowable parameters and explain the difference between the thermodynamic and the practical cycles. Fig. 9 depicts the maximum temperature of each cycle in different conditions which help the reader to comprehend both, the mentioned decrease in this parameter in comparison to Fig. 6 of thermodynamic approach and its relative magnitude in different cycle configurations.

In addition, trend of net power output is of interest in this approach since it can be a decision-making parameter for the cycle configuration. Fig. 10 indicates that plan B yields the highest power output in its optimum situation.

The objective function in this work is the ratio of the power output of the cycle to the total bare module cost, which is equivalent to the initial benefit-cost ratio for this project. The inverse of this maximized parameter, which represents the Dollars invested for each kW of obtained power and is more common in the literature, for each case is illustrated in Fig. 11.

Results shown in Fig. 11 indicate that for each source temperature, simple Rankine cycle has the least total bare module cost to power output ratio, and therefore, this configuration is more favorable in terms of economic analysis. However, it should not be neglected that plan B always produces more power in comparison to simple Rankine cycle. Also, Fig. 11 along with Table 10 reveals that under any circumstances, plan A does not represent any promising results. In fact, this cycle configuration has the lowest rate of power generation, and its initial investment is equal or larger than plan B with the highest power production. Therefore, as it was already shown in the pure thermodynamic analysis, adding a reheat process to a waste-heat-recovery cycle makes no benefits, but it also worsens the situation.

It could be concluded that the main competition is between simple Rankine configuration and plan B reheat cycle. The simple cycle configuration requires a way lower initial investment, while plan B produces higher power than the simple Rankine cycle. This higher power output of plan B is however associated with additional fuel consumption, and in fact, some of the costs of this configuration are also projected to the future. Therefore, as it was mentioned before, complete economic analysis for obtaining the rate of return and payback period is necessary in order to compare the cycles properly.

It is also necessary to compare our results with the ones in the literature to verify the acceptability of the results. Comparing our extrapolated values for ratio of total bare module cost to net power output with those in Ref. [12] for low source temperatures, it can be concluded that both works agree with the approximate value of 1700 $/kW for the heat source of 100 °C. Besides, Imran et al. [51] obtained specific investment costs of ORCs with different refrigerants in the order of 3000 $/kW. However, the corresponding values in this study for the CO2 transcritical cycle imply lower specific investments, which are in accord with the data reported by Ref. [9].

Table 9 summarizes the comprehensive economic analysis which is conducted for all cases in their optimum operating points. By taking a more precise look at the obtained values, it can be found...
out that in long term, Rankine cycle shows better overall economic performance and plan B has the worst situation, comparatively.

This weak performance of plan B under long-term analysis is due to the fact that this cycle configuration requires additional fuel consumption which increases the running costs. However, for source temperatures higher than 150 °C, plan B has also acceptable payback periods (less than 10 years), and this might persuade some of the investors to choose plan B instead of the simple Rankine cycle, especially in situations where higher power generation is more crucial than economic considerations.

Concerning the results of Table 11, one can compare the obtained data with the findings of Wang et al. [52] for the case of payback period optimization in waste heat recovery systems employing subcritical organic Rankine cycles with different working fluids. For the source temperature of 250 °C, they obtained an optimal 5.2-year payback period for R113, while in Table 11, it is shown that the optimal payback period for the mentioned temperature is 2.7 years. This improvement might be a contribution from the choice of CO2 as the working fluid, because it is a relatively inexpensive substance.

Finally, since the thermo-economic analysis tends to study the practical conditions, it is useful to present some achieved data for the optimum operating points of each cycle. For example, sizing of the components required pumping power, and separate power generations in high- and low-pressure turbines are the key parameters as the technical knowledge, which are crucial for each plant construction project. As a sample, Table 12 gives the mentioned technical data for two source temperatures in thermo-economic optimization.

According to Table 12, the highest mass flow rate is associated to plan B, which seems to be normal, because this configuration benefits from additional unlimited heat supply, and thus, it tends to have a higher mass flow rate. Moreover, it is observed that in both reheat configurations, i.e. plan A and plan B, the low-pressure turbine generates more power than the high-pressure turbine. Furthermore, regarding the heat exchanger areas, it can be seen that the areas of the main gas heater and the reheater decrease with the increase in the source temperature. This is due to the increased mean temperature difference in these heat exchangers, which results in less area required for the heat transfer. On the other hand, the condenser area is larger in 250 °C compared to 150 °C, simply because in this temperature, the condenser has to reject higher amounts of heat to the ambient. Also, at each source temperature, the condenser of plan B has the largest area, which is quite trivial due to the fact that this configuration receives more heat through the burners added to the cycle, and consequently more heat has to be rejected to the atmosphere.

<table>
<thead>
<tr>
<th>Source Temperature (°C)</th>
<th>Cycle Configuration</th>
<th>Maximum Temperature (°C)</th>
<th>Maximum Pressure (bar)</th>
<th>Reheat Pressure (bar)</th>
<th>Power Output (kW)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>Reheat Plan A</td>
<td>106</td>
<td>129</td>
<td>101</td>
<td>1815</td>
<td>5.96</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>106</td>
<td>179</td>
<td>113</td>
<td>3189</td>
<td>7.55</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>112</td>
<td>125</td>
<td>–</td>
<td>2036</td>
<td>6.71</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan A</td>
<td>146</td>
<td>143</td>
<td>118</td>
<td>3343</td>
<td>8.09</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>140</td>
<td>162</td>
<td>105</td>
<td>4270</td>
<td>8.66</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>157</td>
<td>145</td>
<td>–</td>
<td>3813</td>
<td>9.28</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan A</td>
<td>187</td>
<td>146</td>
<td>117</td>
<td>4578</td>
<td>8.95</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>185</td>
<td>164</td>
<td>122</td>
<td>5779</td>
<td>10.16</td>
</tr>
<tr>
<td>250</td>
<td>Rankine Cycle</td>
<td>199</td>
<td>143</td>
<td>–</td>
<td>5078</td>
<td>9.76</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan A</td>
<td>245</td>
<td>144</td>
<td>123</td>
<td>5813</td>
<td>9.31</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>239</td>
<td>164</td>
<td>93</td>
<td>7250</td>
<td>10.05</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>248</td>
<td>143</td>
<td>–</td>
<td>6314</td>
<td>10.15</td>
</tr>
</tbody>
</table>

Fig. 9. Variations of optimum maximum temperature with source temperature for different cycles in thermo-economic approach.

Fig. 10. Variations of cycle net power output with source temperature for different cycles.
5. Conclusion

In this work, the capability of adding reheat process in transcritical CO\textsubscript{2} Rankine cycles is investigated thoroughly by proposing two different reheat methods. The first method, plan A, utilizes the low-grade heat stream only and divides it into two fractions in order to be employed in the vapor generator and in the reheater. The second method, plan B, uses the entire low-grade heat stream in the main gas heater and adds additional fuel consumption for the reheater purpose. Optimization is carried out in two approaches of thermodynamic and thermo-economic methodologies. The simulations have been done in the whole range of the low-grade waste heat recovery applications, from 150 to 300 °C.

The optimized cycles with respect to net power output in thermodynamic viewpoint show some interesting results. Distribution of the constant and limited available heat in cases like waste heat recovery applications, yields no more tangible advantage, especially in relatively higher source temperatures. However, as it is predictable, employing some burners in the reheate process presents a remarkable increase in the net power output. Plan A generates 8 to 37 MW of power for the source temperatures from 150 to 300 °C, while plan B is capable of producing 14—57 MW of power, correspondingly. Due to the addition of excessive external heat in plan B, its efficiency is not necessarily higher than that of the Rankine cycle.

In the thermo-economic approach, the conditions and consequently the results are quite different. Thermo-economic optimization is conducted based on maximizing the ratio of the power output of the cycle to its total bare module cost. In this case, although plan B shows the highest power output in all the temperatures, simple Rankine cycle obtains the best benefit-to-cost ratio. Plan A does not represent any powerful performance and has the lowest power output and benefit-to-cost ratio. Therefore, simple Rankine cycle is the most economical cycle for applying to heat recovery processes, if capital cost is the only variable taken into account.

To elaborate a better comprehension, a more precise economic analysis over long term periods should be conducted, because the power generations and consequently the incomes are different from one cycle configuration to another, and also the investigated cycles, especially plan B, are involved with running costs. In this way, payback periods and rates of return are obtained for the optimum operating points of the cycles for all the source temperatures. Results indicate that in long term analysis, simple Rankine cycle still has the upper hand and represents shorter paybacks periods in comparison to plan B. However, it is worth noting that in higher source temperatures, plan B also benefits from acceptable payback periods. This means that one can still implement plan B and recover more power with a small reduction in the economic indicators of the plant.

To sum up, it can be concluded that bounded reheat process with no additional heat other than the fixed main heat source, as in plan A configuration, yields no effective improvement, either from thermodynamic point of view or in thermo-economic approach. In this case, it is better to launch a simple transcritical CO\textsubscript{2} Rankine cycle without any reheate. On the other hand, plan B, i.e. the cycle with unbounded reheat, is always the best cycle configuration in terms of power output generation. However, despite the higher power production, plan B represents a slightly weaker performance when economic conditions are applied, compared to the simple Rankine cycle. Nonetheless, plan B generates power outputs up to 50% higher than the simple cycle, and therefore, in cases where the additional capital and running costs associated with it are affordable for the project, plan B represents the best waste heat recovery option.

### Table 11

Economic parameters of the different cycle in the range of source temperature.

<table>
<thead>
<tr>
<th>Source Temperature</th>
<th>Cycle Configuration</th>
<th>Payback Period (year)</th>
<th>Rate of Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 °C</td>
<td>Reheat Plan A</td>
<td>10.7</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>6.5</td>
<td>19.2</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>5.2</td>
<td>23.9</td>
</tr>
<tr>
<td>200 °C</td>
<td>Reheat Plan A</td>
<td>8.2</td>
<td>15.6</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>3.4</td>
<td>33.7</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>4.1</td>
<td>26.9</td>
</tr>
<tr>
<td>250 °C</td>
<td>Reheat Plan B</td>
<td>6</td>
<td>20.2</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>2.7</td>
<td>39.1</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan A</td>
<td>4.3</td>
<td>26.4</td>
</tr>
<tr>
<td>300 °C</td>
<td>Reheat Plan B</td>
<td>8.6</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>3.1</td>
<td>38.5</td>
</tr>
</tbody>
</table>

### Table 12

Detail optimized parameters of the components for two source temperatures.

<table>
<thead>
<tr>
<th>Source Temperature</th>
<th>Cycle Configuration</th>
<th>Mass Flow Rate (kg/s)</th>
<th>HP Turbine Power (kW)</th>
<th>LP Turbine Power (kW)</th>
<th>Gas Heater Area (m\textsuperscript{2})</th>
<th>Reheat Area (m\textsuperscript{2})</th>
<th>Condenser Area (m\textsuperscript{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 °C</td>
<td>Reheat Plan A</td>
<td>128</td>
<td>1088</td>
<td>2109</td>
<td>29.3</td>
<td>0.6</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan B</td>
<td>185</td>
<td>2831</td>
<td>3857</td>
<td>35.4</td>
<td>3</td>
<td>38.9</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>137</td>
<td>3433</td>
<td>—</td>
<td>36</td>
<td>—</td>
<td>28.3</td>
</tr>
<tr>
<td></td>
<td>Reheat Plan A</td>
<td>154</td>
<td>1844</td>
<td>4815</td>
<td>13.8</td>
<td>0.4</td>
<td>45.3</td>
</tr>
<tr>
<td>250 °C</td>
<td>Reheat Plan B</td>
<td>170</td>
<td>2726</td>
<td>5852</td>
<td>24.4</td>
<td>0.7</td>
<td>46.7</td>
</tr>
<tr>
<td></td>
<td>Rankine Cycle</td>
<td>156</td>
<td>7170</td>
<td>—</td>
<td>17</td>
<td>—</td>
<td>42.9</td>
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