Multivariate simulation of block-support grades at Mehdiabad deposit, Iran

Seyed Ali Hosseini, Omid Asghari & Xavier Emery

To cite this article: Seyed Ali Hosseini, Omid Asghari & Xavier Emery (2017) Multivariate simulation of block-support grades at Mehdiabad deposit, Iran, Applied Earth Science, 126:3, 146-157, DOI: 10.1080/03717453.2017.1351652

To link to this article: http://dx.doi.org/10.1080/03717453.2017.1351652

Published online: 03 Aug 2017.

Submit your article to this journal

Article views: 41

View related articles

View Crossmark data
Multivariate simulation of block-support grades at Mehdiabad deposit, Iran

Seyed Ali Hosseini\(^a\), Omid Asghari\(^b\) and Xavier Emery\(^b,c\)

\(^a\)Simulation and Data Processing Laboratory, School of Mining Engineering, University College of Engineering, University of Tehran, Tehran, Iran; \(^b\)Department of Mining Engineering, University of Chile, Santiago, Chile; \(^c\)Advanced Mining Technology Center, University of Chile, Santiago, Chile

ABSTRACT

The evaluation of process performance within mining operations at Mehdiabad complex deposit, Central Iran, requires modelling the spatial variability of six cross-correlated grade variables: zinc (Zn) and lead (Pb) as the main products, silver (Ag) and copper (Cu) as byproducts, and iron (Fe) and manganese (Mn) as contaminants. To this end, the variables are first transformed into spatially uncorrelated factors, using the minimum/maximum autocorrelation factors approach, then the factors are simulated directly at the target block support. The result is a set of realisations (equiprobable scenarios) of the grade distribution within the deposit, which reproduce the joint dependence relationships and natural variability at all spatial scales. This model can decrease the level of uncertainty in Mehdiabad project development and consequently in production.

Introduction

Geostatistics is used for evaluating mineral resources and/or ore reserves and for further mine planning and beneficiation process schedule. In traditional applications, a metal grade is regarded as the central property of the study and the main objective is to distinguish between ore and waste (Tolosana-Delgado et al. 2015); in more recent researches, the objective of the modelling is to connect geometallurgical variables and spatially distributed geological information (Lishchuk 2016). In polymetallic deposits, the ore is no longer represented by a single-value element, but through a vector of more than one recoverable metal (main products and byproducts). Also, the contaminant elements exert a critical influence on process performance and smelting extraction. This perspective allows a quantitative insight based on their mineralogical composition and processing characteristics that modify and optimise the efficiency of downstream mining and processing steps or require additional treatment.

To assess the joint uncertainty of several co-regionallised variables in mineral deposits, such as the grades of main products, byproducts, and contaminants, the more promising approach is the construction of scenarios by multivariate conditional simulation, which requires modelling the spatial dependence relationships between the variables (Rondon 2012). To this end, one option is to fit a linear model of coregionalisation (LMC) (Goovaerts 1997; Chilès and Delfiner 2012). The major drawback of this approach is the difficulty in inferring and modelling cross variograms, together with the requirement of considerable computer processing capacity to construct large-scale block models (Boucher and Dimitrakopoulos 2009; Hosseini and Asghari 2016). Another option is to transform the original variables into independent ones; the dependence relationships are then restored when back-transforming the simulated independent variables. This alternative has been gaining popularity because of its better accuracy and computational efficiency when the number of variables to simulate increases (Rondon 2012). Several transformations allowing independent simulation have been proposed, e.g. principal component analysis (David 1988; Goovaerts 1993), stepwise conditional transformation (Rosenblatt 1956; Leuangthong and Deutsch 2003), projection pursuit multivariate transformation (Barnett et al. 2013), independent component analysis (Tercan and Sohrabian 2013), and minimum/maximum autocorrelation factors (MAF) (Switzer and Green 1984). Over the past few years, the latter has been increasingly used for simulating coregionalised variables (Lopes et al. 2011; Goodfellow et al. 2012; da Silva 2013; Da Silva and Costa 2014).

Another issue in the modelling of mineral deposits concerns the volumetric support of the target selective mining units (SMU) or blocks, which is usually much larger than the quasi-point support of the available drilling data. To address this issue, the usual approach consists in simulating the entire deposit at a point support on a fine grid, followed by a post-processing step to average the simulated values within the SMU to obtain a block-support simulation. In practice, this procedure can be cumbersome and requires a
significant amount of computer memory, especially when large deposits are simulated. An alternative is direct block-support simulation, which avoids simulating onto a fine grid that discretises the blocks. This idea was proposed by Journel and Huijbregts (1978) and was extended by Boucher and Dimitrakopoulos (2009, 2012) and Emery and Ortiz (2011) to the joint simulation of multiple elements.

This paper details the application of direct block simulation for the joint modelling of multiple geological variables at Mehdiabad, a world-class polymetallic deposit in Iran. The grades of six elements will be jointly simulated: zinc (Zn) and lead (Pb) as the main products, silver (Ag) and copper (Cu) as byproducts, and iron (Fe) and manganese (Mn) as contaminants. The result is a set of equiprobable representations of the deposit that allow transferring uncertainty of the resource prediction into risk in subsequent stages and can be used to assess the uncertainty about key aspects of the project. For example, the joint distribution of resource and contaminant variables helps to blend and design the plant and to control the ore quality to optimise the processing routes for ore blocks. At first, these variables will be transformed into normally distributed variables, then into MAF, which will be simulated directly onto a block support. Prior to the case study itself (Section “Application to a real case study: Mehdiabad Deposit”), we will detail the methodology of MAF transformation and direct block-support simulation in sections “Implementation of MAF” and “Block-support joint simulation with MAF”, respectively.

Implementation of MAF

Given a set of coregionalised variables, the MAF are defined as linear combinations of these variables, such that the first factor has the smallest spatial correlation and each additional factor is orthogonal (uncorrelated) to the previous one and has a higher level of spatial correlation. The final factor has the maximum spatial correlation of all the factors (Switzer and Green 1984; Desbarats and Dimitrakopoulos 2000).

The MAF can be constructed in two ways: (i) model-driven MAF that uses a two-structure LMC to derive a transformation matrix (Tran et al. 2006); (ii) data-driven MAF that performs two successive PCA decompositions without the need of fitting a LMC to the input data (Rondon 2012). Whichever approach is chosen, be it model-driven or data-driven, in general, the factors calculated are only approximately independent, as their cross variograms may not be identically zero (Mueller and Ferreira 2012), so it is advisable to check that cross-correlations are negligible. The algorithm for the implementation of the two MAF techniques is as follows.

Consider a second-order stationary vector Gaussian random field with \( K \) scalar components \( \mathbf{Y}(\mathbf{x}) = (Y_1(\mathbf{x}), \ldots, Y_K(\mathbf{x})) \), where \( \mathbf{x} \) stands for a generic point of the three-dimensional space, and assume that the direct and cross variograms of these components can be modelled by a two-structure LMC (Goovaerts 1997; Wackernagel 2003):

\[
\Gamma_Y(\mathbf{h}) = B_1 \Gamma_1(\mathbf{h}) + (B - B_1) \Gamma_2(\mathbf{h})
\]

where \( \mathbf{h} \) is a lag separation vector, \( \Gamma_Y(\mathbf{h}) \) is the \( K \times K \) matrix of direct and cross variograms at lag \( \mathbf{h} \), \( B \) is the correlation matrix and \( B_1 \) is the sill matrix associated with the first structure of the LMC (a symmetric positive semi-definite matrix).

Based on these assumptions, the MAF algorithm proceeds as follows:

1. Transform the original data into normal scores values, with a mean of 0 and variance of 1 by using a normal score transformation.
2. Compute the variance–covariance matrix \( \mathbf{B} \)
   \[
   \mathbf{B} = \text{cov} [\mathbf{Y}(\mathbf{x}), \mathbf{Y}(\mathbf{x})] = \mathbb{E} [\mathbf{Y}(\mathbf{x})^T \mathbf{Y}(\mathbf{x})]
   \]
3. Consider the spectral decomposition of the variance–covariance matrix \( \mathbf{B} \), that is
   \[
   \mathbf{B} = \mathbf{Q}^T \Lambda \mathbf{Q}
   \]
   where \( \Lambda \) is a diagonal matrix of eigenvalues and \( \mathbf{Q} \) a matrix of orthonormal eigenvectors. Make sure that the entries of \( \Lambda \) are sorted in a decreasing order.
4. Calculate a first rotation matrix \( \mathbf{A} \)
   \[
   \mathbf{A} = \Lambda^{1/2} \mathbf{Q}
   \]
   The PCA scores are obtained at each location \( \mathbf{x} \) as
   \[
   \text{PCA}(\mathbf{x}) = \mathbf{A} \mathbf{Y}(\mathbf{x})
   \]
   Owing to the multivariate Gaussian assumption, the PCA scores so obtained have a standard normal distribution.
5. From the fitted LMC (Equation (1)), calculate matrix \( \mathbf{V} \) as
   \[
   \mathbf{V} = \mathbf{A} \mathbf{B}_1 \mathbf{A}^T
   \]
   Alternatively, one can compute the experimental direct and cross variograms for the PCA scores at a suitable selected nonzero lag distance, as matrix \( \mathbf{V} \) (data-driven MAF).
6. Consider the spectral decomposition of \( \mathbf{V} \) as
   \[
   \mathbf{V} = \mathbf{Q}_1^T \Lambda_1 \mathbf{Q}_1
   \]
(7) Calculate the transformation matrix (factor coefficients) \( M \)
\[
M = Q_1 A
\] (8)
(8) Calculate the MAF factors as
\[
MAF(x) = MY(x)
\]
The forward transform is obtained through Equation (9). The back transform is computed through the inverse of matrix \( M \) as
\[
Y(x) = M^{-1} MAF(x)
\]
For detail of the MAF formalism, interested readers are referred to Rondon (2012) and Desbarats and Dimitrakopoulos (2000).

Block-support joint simulation with MAF

Geostatistical simulations can be upscaled to any support, but the size of the necessary fine scale model and the requirement to store and process the realisations can be a major drawback. On the other hand, one works with many variables that provide information about deposits. Joint block-support simulation with MAF allows for the fast generation of realisations and aims to alleviate the computational requirements during simulation (De Freitas Silva and Dimitrakopoulos 2015). The algorithm is

(1) For each of the \( K \) variables of interest, transform the original point-support data (random field \( Z_k(x) \)) into normal scores (Gaussian random field \( Y_k(x) \) with zero mean and unit variance):
\[
\forall k = 1 \ldots K, \quad Z_k(x) = \phi_k(Y_k(x)) \quad (11)
\]
(2) Transform \( Y(x) \) into orthogonal factors \( MAF(x) \) (Equation (9)).
(3) Fit the direct variograms \( \{ \gamma_1, \ldots, \gamma_K \} \) of the MAF to get a spatial structure model. Each MAF has a unit variance, hence its variogram should have a unit sill.
(4) Divide the simulation domain into non-overlapping blocks.
(5) Select a block \( v \) in the domain, randomly among the blocks that have not been previously selected:
   (a) At a set of points \( \{ x_1, \ldots, x_M \} \) discretising \( v \), simulate each MAF conditionally to the original point-support MAF data and to the previously simulated block-support data located in and around this block by the well-established sequential Gaussian simulation technique (Isaaks 1991). To this end, one needs to know the direct covariance functions of the point- and block-support MAF, which are as follows.

Point-to-point covariance:
\[
\text{cov}[MAF_k(x), MAF_k(x')] = 1 - \gamma_k(x - x') \quad (12)
\]
Point-to-block covariance:
\[
\text{cov}[MAF_k(x), MAF_k(v')] = \frac{1}{M} \sum_{i=1}^{M} (1 - \gamma_k(x - x'_i)) \quad (13)
\]
where \( \{ x'_1, \ldots, x'_M \} \) are a set of points discretising block \( v' \).
Block-to-block covariance:
\[
\text{cov}[MAF_k(v), MAF_k(v')] = \frac{1}{M^2} \sum_{i=1}^{M} \sum_{j=1}^{M} (1 - \gamma_k(x_i - x'_j)) \quad (14)
\]
(b) Average the simulated point-support MAF values over \( v \) to obtain block-support values that will be used for further conditioning along the simulation path, for each corresponding factor:
\[
MAF_k(v) = \frac{1}{M} \sum_{i=1}^{M} MAF_k(x_i) \quad (15)
\]
(c) Back-transform the simulated point-support MAF values within the block and average them to obtain simulated block-support
values for the original variables:

\[ Z_k(v) = \frac{1}{M} \sum_{i=1}^{M} Z_k(x_i) \]
\[ = \frac{1}{M} \sum_{i=1}^{M} \varphi_k(M^{-1}MAF(x_i)) \]  

(d) Repeat step (5) until all the blocks are simulated.

**Application to a real case study: Mehdiabad deposit**

**Deposit description**

The Mehdiabad deposit is a world-class Cretaceous deposit, located 116 km southeast of the city of Yazd in the Central Iranian tectonic block (Figure 1), one of the most important metallogenic provinces for zinc-lead mineralisation (Ghazanfari 1999; Ghorbani 2013). The deposit has been explored by various parties since 1960s. The geologic area of the Mehdiabad deposit is shown in Figure 2. Outcrops of different geological units have been distinguished in this area, mainly formed by the Sangestan, Taft, and Abkouh formations. The structural geology is characterised by faults that are one of the main controlling factors of mineralisation. The Mehdiabad deposit is divided into two parts: (1) the Mountain Ore Body (MOB) (also known as Calamine Mine) that represents the highest parts of the oxide ore mineralisation; and (2) the Valley Ore Body (VOB), the main part of the ore body located in a depression surrounded by hills and mountains (Reichert 2007).

**Mountain ore body (Calamine mine)**

The MOB wedged-in between the Black Hill Fault in the west and the Forouzandeh Fault in the east is located on a rough mountainside in the northwestern part of the deposit. It is completely oxidised. The...
The main sulphide minerals are galena, sphalerite, and pyrite. But no clear indications for the sulphide protor are present. The VOB consists of sulphides. The Taft formation hosts the main portion of the sulphide ore of the VOB, the supposed main sulphide mineral association of the MOB have been identified, which might be attributed to the thorough oxidation, folding, and faulting of the strata and solution collapse (Reichert 2007). The most important oxide ore minerals include hemimorphite, hydrozincite, smithsonite, goethite, in addition to small amounts of mimetite, hetaerolite, morphite, hydrozincite, smithsonite, goethite, and sauconite (Reichert et al. 2003). Three different stages of ore formation or alteration have been identified: (1) Precipitation of stage-1 hemimorphite (and possibly minor smithsonite, hydrozincite, goethite, and haematite) within the fault-zones and breccias; (2) Alteration of hemimorphite to hydrozincite, and precipitation of goethite/haematite; (3) Precipitation of type-2 hemimorphite as mineralisation within fractures and open spaces of the fault-breccia as well as the oxide ore without significant precipitation of goethite/haematite (Reichert et al. 2003).

**Valley ore body**

The VOB is located in a valley and is covered by an alluvial overburden of about 250 m. The Taft formation hosts the main portion of the sulphide ore of the VOB (Azari and Sethna 1994). The strata of the Taft formation are dolomitic and ankeritic limestone (Abkou formation) that are intensified by extensive brecciation (BRGM 1994) that is probably the result of emergence, paleo-karstification, and finally, the collapse of these strata (Reichert 2007). The main portion of the VOB consists of sulphides. The main sulphide minerals are galena, sphalerite, barite, pyrite, and traces of chalcopyrite (Azari and Sethna 1994) that occur as impregnation of the Taft formation of breccia and fill the interstitial space between the breccia fragments, and as matrix in a complex fracture and breccia system (Reichert 2007). Three different stages of (tectonic or collapse) displacement and mineralisation of the valley ore body can be interpreted: (1) paleo-karst and partly collapse of the limestone of the Taft formation; (2) the dolomitisation of the carbonate rock genetically linked with the emplacement of the sulphide ore and barite; (3) initiated with the oxidation of the sulphide ore that it is still alive (Reichert 2007).

**Presentation of the data set**

The Mehdiabad area is covered by 208 vertical and inclined exploration drill holes (Figure 2). The total length of diamond drilling in Mehdiabad is about 55,000 m and the drill hole spacing varies between 50 m and 100 m across the deposit. The data set used in this study comprises 3,706 core samples inside the main sulphide domain, regularised in interval lengths of 1 m, and assayed for Zn, Pb, Ag, Cu, Fe, and Mn grades. All the grades are known at all the samples, i.e. there is no missing information due to core recovery issues. The basic statistics for the available data in the main sulphide domain are shown in Table 1. To extract representative univariate statistics, cell declustering (Goovaerts 1997) is used. The Pearson and Spearman correlation coefficients between the elements are shown in Table 2.

**Normal scores and MAF transformations**

In the following, the joint simulation of Zn, Pb, Ag, Cu, Fe, and Mn will be limited to the main sulphide domain. A normal scores transformation is performed on each variable, and the normal scores data are then used to derive the MAF scores. Table 3 shows the

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>0.25Q</th>
<th>0.5Q</th>
<th>0.75Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn</td>
<td>%</td>
<td>3.643</td>
<td>4.055</td>
<td>0.000</td>
<td>32.500</td>
<td>1.033</td>
<td>2.438</td>
<td>4.841</td>
</tr>
<tr>
<td>Pb</td>
<td>%</td>
<td>1.453</td>
<td>1.625</td>
<td>0.001</td>
<td>23.600</td>
<td>0.440</td>
<td>0.983</td>
<td>1.938</td>
</tr>
<tr>
<td>Ag</td>
<td>g/t</td>
<td>34.990</td>
<td>43.351</td>
<td>0.000</td>
<td>561.229</td>
<td>11.560</td>
<td>22.850</td>
<td>42.225</td>
</tr>
<tr>
<td>Cu</td>
<td>%</td>
<td>0.147</td>
<td>0.496</td>
<td>0.000</td>
<td>19.959</td>
<td>0.005</td>
<td>0.050</td>
<td>0.194</td>
</tr>
<tr>
<td>Fe</td>
<td>%</td>
<td>20.156</td>
<td>9.413</td>
<td>0.538</td>
<td>44.800</td>
<td>14.690</td>
<td>22.711</td>
<td>26.980</td>
</tr>
<tr>
<td>Mn</td>
<td>%</td>
<td>3.971</td>
<td>2.164</td>
<td>0.000</td>
<td>14.150</td>
<td>2.460</td>
<td>3.878</td>
<td>5.427</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>0.25Q</th>
<th>0.5Q</th>
<th>0.75Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn</td>
<td>%</td>
<td>1</td>
<td>0.526</td>
<td>0.374</td>
<td>−0.090</td>
<td>−0.158</td>
<td>−0.005</td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>%</td>
<td>0.604</td>
<td>1</td>
<td>0.662</td>
<td>−0.066</td>
<td>−0.168</td>
<td>−0.138</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>%</td>
<td>0.519</td>
<td>0.763</td>
<td>1</td>
<td>0.066</td>
<td>−0.234</td>
<td>−0.242</td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>g/t</td>
<td>0.252</td>
<td>0.008</td>
<td>0.183</td>
<td>1</td>
<td>−0.023</td>
<td>−0.090</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>%</td>
<td>−0.036</td>
<td>−0.114</td>
<td>−0.159</td>
<td>0.055</td>
<td>1</td>
<td>0.833</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>%</td>
<td>0.185</td>
<td>−0.013</td>
<td>−0.126</td>
<td>−0.109</td>
<td>0.840</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>0.25Q</th>
<th>0.5Q</th>
<th>0.75Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn</td>
<td>%</td>
<td>1</td>
<td>0.622</td>
<td>0.529</td>
<td>−0.020</td>
<td>−0.081</td>
<td>0.135</td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>%</td>
<td>0.604</td>
<td>1</td>
<td>0.779</td>
<td>−0.001</td>
<td>−0.130</td>
<td>−0.026</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>%</td>
<td>0.519</td>
<td>0.763</td>
<td>1</td>
<td>0.206</td>
<td>−0.162</td>
<td>−0.130</td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>%</td>
<td>−0.252</td>
<td>−0.008</td>
<td>0.183</td>
<td>1</td>
<td>0.093</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>%</td>
<td>−0.036</td>
<td>−0.114</td>
<td>−0.159</td>
<td>0.055</td>
<td>1</td>
<td>0.821</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>%</td>
<td>0.185</td>
<td>−0.013</td>
<td>−0.126</td>
<td>−0.109</td>
<td>0.840</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Pearson correlation coefficients between the normal scores variables, which generally differ from that of the raw (untransformed) variables, whereas the Spearman correlation coefficients are not changed after the normal scores transformation (these coefficients measure the correlations of ranks and are invariant under a monotonic transformation of the data).

The matrix of direct and cross variograms is calculated for a lag separation distance of 30 m (omni-directional calculation) using the transformed data, which gives a unique solution for the derivation of the MAF, according to the data-driven procedure discussed in section “Implementation of MAF” (Vargas-Guzmán 2004; Da Silva and Costa 2014). The matrix $\mathbf{M}$ of factor coefficients is presented in Table 4. The MAF scores obtained using Equation (9) in Figure 3 are approximately normally distributed with mean 0 and variance 1. Figure 4 shows the experimental variograms of the factors, together with their respective fitted models, each with three basic nested structures.

### Table 4. Coefficients $\mathbf{M}$ of MAF for target domain.

<table>
<thead>
<tr>
<th></th>
<th>Zn</th>
<th>Pb</th>
<th>Ag</th>
<th>Cu</th>
<th>Fe</th>
<th>Mn</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF 1</td>
<td>0.462</td>
<td>0.074</td>
<td>-0.351</td>
<td>0.600</td>
<td>0.233</td>
<td>-0.494</td>
</tr>
<tr>
<td>MAF 2</td>
<td>-0.120</td>
<td>0.462</td>
<td>-0.640</td>
<td>0.309</td>
<td>0.446</td>
<td>-0.261</td>
</tr>
<tr>
<td>MAF 3</td>
<td>-0.210</td>
<td>0.552</td>
<td>-0.099</td>
<td>0.643</td>
<td>0.463</td>
<td>-0.116</td>
</tr>
<tr>
<td>MAF 4</td>
<td>-0.029</td>
<td>-0.034</td>
<td>-0.060</td>
<td>0.307</td>
<td>0.260</td>
<td>0.913</td>
</tr>
<tr>
<td>MAF 5</td>
<td>0.517</td>
<td>0.582</td>
<td>-0.040</td>
<td>-0.163</td>
<td>-0.509</td>
<td>0.326</td>
</tr>
<tr>
<td>MAF 6</td>
<td>0.783</td>
<td>0.518</td>
<td>0.030</td>
<td>-0.323</td>
<td>-0.064</td>
<td>0.093</td>
</tr>
</tbody>
</table>

Figure 3. Distribution (probability density) of the MAF factors.

Figure 4. Sample (dashed lines) and modelled (solid lines) variograms of MAF factors, along horizontal and vertical directions.
The fitted models are presented in Equations (17)–(22), where the horizontal and vertical ranges are indicated into brackets:

\[
\begin{align*}
\gamma_{MAF1} &= 0.202 \text{nugget} + 0.119 \exp(10m, 15m) \\
&\quad + 0.679 \text{sph}(120m, 180m) \\
\gamma_{MAF2} &= 0.188 \text{nugget} + 0.475 \exp(170m, 90m) \\
&\quad + 0.337 \text{sph}(250m, \infty) \\
\gamma_{MAF3} &= 0.243 \text{nugget} + 0.380 \exp(30m, 10m) \\
&\quad + 0.377 \text{sph}(350m, 160m) \\
\gamma_{MAF4} &= 0.364 \text{nugget} + 0.333 \exp(25m, 65m) \\
&\quad + 0.303 \text{sph}(350m, 90m) \\
\gamma_{MAF5} &= 0.332 \text{nugget} + 0.476 \exp(35m, 10m) \\
&\quad + 0.192 \text{sph}(350m, 60m) \\
\gamma_{MAF6} &= 0.052 \text{nugget} + 0.244 \exp(35m, 60m) \\
&\quad + 0.704 \text{sph}(370m, \infty)
\end{align*}
\]

The models were checked using leave-one-out cross-validation, which consists in predicting (by simple kriging) each factor at each data location using the data in a neighbourhood centred at the target data location. In this case, the neighbourhood is an ellipsoid with major axis of 100 m along the direction of main continuity (horizontal plane or vertical direction, depending on the factor under consideration) and minor axis of 50 m along the orthogonal direction. The results (Table 5) demonstrate a low variance of prediction errors, a high correlation between true and predicted values, a lack of bias of the predicted values (mean error close to zero), and a lack of correlation between predicted values and errors (conditional unbiasedness), which validates the fitted variogram models.

As for the experimental cross variograms between the factors (Figure 5), they show that these factors can be considered as spatially uncorrelated and therefore simulated independently.

**Block-support simulation**

The target domain is simulated 100 times using the proposed joint block-support simulation algorithm. Each realisation contains 19,350 blocks, each of 25 m × 25 m × 10 m, with simulated values of the Zn, Pb, Ag, Cu, Fe, and Mn grades. The block discretisation is set to 5 × 5 × 2; this discretisation level is deemed sufficient, as a finer discretisation does not bring much difference in the calculation of point-to-block and block-to-block covariances. The maps of the first realisation for a selected section at northing 3,484,300 m are displayed in Figure 6 for the zinc, lead, silver, copper, iron, and manganese grades. Also the expected

<table>
<thead>
<tr>
<th>Table 5. Cross-validation statistics.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>MAF 1</td>
</tr>
<tr>
<td>MAF 2</td>
</tr>
<tr>
<td>MAF 3</td>
</tr>
<tr>
<td>MAF 4</td>
</tr>
<tr>
<td>MAF 5</td>
</tr>
<tr>
<td>MAF 6</td>
</tr>
</tbody>
</table>

**Figure 5.** Experimental cross variograms between MAF factors.
grade models can be constructed by averaging the 100 realisations on a block-by-block basis (Figure 7).

Table 6 shows statistics on the simulated grades for one realisation and for the average of 100 realisations. Owing to the support effect (Chilès and Delfiner 2012), the simulated grades should have the same averages but a smaller variability than the original data at quasi-point support, which can be corroborated by comparing the mean values, standard deviations, minimum, and maximum values in Tables 1 and 6. This smoothing effect is more pronounced when considering the expected grades, insofar as the average of 100 realisations is less dispersed than each individual realisation. Also, because it smoothes out the small-scale variability, the change of support from the drill cores to the blocks tends to enhance the Pearson correlation.

Figure 6. Maps of simulated block-support grades for realisation #1.
coefficients between the grades, in comparison with the original data, whereas the Spearman correlation coefficients remain quite similar (Figure 8). Figure 9 displays the tonnages and mean grades above different cut-offs applied to the zinc grade for all the realisations, together with the curves associated with the original data at the sample support. One observes that the tonnage curve of the data decreases faster at low cut-offs and slower at high cut-offs than that of the realisations, which reflects the greater dispersion of the data grade in comparison with the block-support grade (support effect), although, at a zero cut-off, the mean grades for both the point-support data and the block-support realisations are comparable. The spread of the tonnage and mean grade curves associated with the realisations is an indicator of our uncertainty in the recoverable resources within the simulated region of the deposit. The expected resources are indicated with the dashed

Figure 7. Maps of expected block-support grades (average of 100 realisations).
Curves, corresponding to the average of the 100 realisation curves.

**Conclusions**

This paper presented theoretical and practical aspects of an efficient framework for the joint simulation of multiple correlated variables at a block support, based on a transformation into MAF. The methodology has been applied to model the spatial variability at Mehdia-bad deposit, where the grades of zinc (Zn) and lead (Pb) as the main products, silver (Ag) and copper (Cu) as byproducts, and iron (Fe) and manganese (Mn) as contaminants have been simulated within the main sulphide domain.

The MAF approach simplifies multivariate block-support simulations in mineral deposits by transforming cross-correlated variables into independent factors that can be simulated separately. Such an approach required fitting only six variograms for the factors instead of 21 direct and cross variograms that would be needed for a full model of coregionalisation. Also, since the simulated point-support values are not stored and only block-support information is retained, this approach reduces CPU time and memory storage requirements with respect to traditional approaches.
The output of this study is a set of multiple equally probable scenarios of the Mehdiabad deposit that model the spatial variability of the grades of main products, byproducts, and contaminants. The scenarios can be further used for assessing the uncertainty of in situ resources, evaluating process performance within mining operations and decreasing the level of uncertainty in the project development and consequently in production (Baumgartner et al. 2011).

### Acknowledgements

Constructive comments from two anonymous reviewers helped improving the manuscript. The last author acknowledges the funding from the Chilean Commission for Scientific and Technological Research, through Project CONICYT PIA Anillo ACT1407.

### Disclosure statement

No potential conflict of interest was reported by the authors.
References


