Fuzzy control of asymmetric plan buildings with active tuned mass damper considering soil-structure interaction

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A B S T R A C T

In recent decades, many researchers have conducted studies on structural control to improve the safety and serviceability of high-rise buildings or towers against earthquake and strong wind. Since most buildings have a sort of asymmetric plan they experience torsional effects when subjected to earthquake and subsequently the torsion would increase the structural response. Also, such buildings might be constructed on soft soil where the Soil-Structure Interaction (SSI) influence would be important.

Thus in this paper, considering the SSI effect a mathematical model is presented to obtain the seismic performance of an irregular multi-story building having two Active Tuned Mass Dampers (ATMD) at center of mass on the top floor. The model is employed to investigate the seismic response of 10 and 15-story asymmetric plan buildings in different cases using fuzzy logic and LQR forces for those two ATMD. Moreover, two TMDs are used at the same location for result comparison. Obtained results demonstrated that using two ATMDs on the top floor would reduce induced structural response in the 10 and 15-story buildings even in those on soft soils.

1. Introduction

In recent century, several strong earthquakes have reportedly caused loss of life and severe damage to the buildings located in near-fault zones. Most buildings in such regions are not robust enough against the strong earthquakes. Hence, research on near-fault seismic ground motions influencing on the performance of engineering structures, has become an important and attractive area in the earthquake engineering field. Control of structural response due to seismic loads in various structures has been the focus of many research studies in the recent decades. Various seismic control devices presented to help structures under seismic loads can be classified as active, passive, hybrid and semi-active control. The active control methods are efficient for a wide excitation frequency range and transient vibrations. An active control system is a system which typically consists of the sensors for measuring structural responses and requires a large power source for operation of electrohydraulic or electromechanical actuators which supply control forces applied to the structure. The location of the sensors might be remote from the active control system [1]. Since the active control can be adjusted to work with a number of vibration modes, it is an ideal choice for the multi-story buildings [2].

One of the most efficient active control devices particularly for towers and high-rise structures is Active Tuned Mass Damper (ATMD) system. One of the major advantages of ATMD is that a relatively small mass can be used to reduce structural response and induced to have high efficiency [3]. In addition, active control forces are applied to move this small mass and significantly induce secondary inertia force against structural vibrations [4]. Several studies on using ATMD can be found in structural engineering literature [4–7].

There are various classical and robust control algorithms to reduce high-rise building responses. Control theories such as LQR, LQG, H\textsubscript{2} and H\textsubscript{\infty} optimal control methods are currently used for active control of earthquake response of structures [8]. However, all of the conventional control theory must first obtain acceptable mathematical model for the real structure and then design the control law. Since civil engineering structures with multi-degree of freedom systems are complex, finding an exact mathematical model to describe the behavior of the structure would be very difficult. This is why intelligent algorithms such as fuzzy logic are typically used to determine control forces.

Fuzzy logic theory was first proposed by Zadeh [9] that has been under research for many years in various fields of civil engineering. In structural engineering, the fuzzy theories have been applied by several researchers [9–11]. The most powerful advantage of fuzzy control is that the control force is not obtained by mathematical model and also, it can tolerate the uncertainties of earthquake excitations and structural response sensors, thus, the result would be a controller system with an adequate inherent robustness. The processing stage is based on a collection of logic rules in the form of IF-THEN statements by incorporating human experience and
expertise into the fuzzy logic control (FLC) can be constructed for the complex structural control systems. The FLC algorithms have been used for the active control of civil engineering structures by several researchers [12–16], Pourzeynali et al. [4] used ATMD control systems for multi-story building response reduction. They combined genetic algorithms and fuzzy logic to design and optimize different parameters of the ATMD control system to obtain the best results in reducing the building response under earthquake excitations. The results acquired from their proposed control algorithm showed that integration of the genetic algorithms and FLC has highly effective impact on reduction of structural response. Guclu and Yazici [17] designed fuzzy logic and PD controllers for a multi-story structure with ATMD to reduce earthquake-induced response of structure. Their results showed that the fuzzy logic controllers perform well under different earthquake records. To reduce a multi-story building structural response, Kaveh et al. [18] adopted a mass damper and a semi-active Magneto-Rheological (MR) damper working together in parallel. For reducing structural responses, semi-active TMDs were used such that a proper input voltage of the MR damper was calculated by an optimized fuzzy controller to increase its efficiency.

In reality, most buildings have irregular plans inducing rotation around their vertical axis when subjected to earthquakes that would further increase the structural response. The torsional effect cannot be considered in two-dimensional frame, thus three-dimensional (3D) structure model must be used. In such a model, in addition to the lateral displacement in both x and y directions, there is a large rotation around z direction [19–22]. Yanik et al. [23] presented a new performance index for the 3D structures with a fully active tendon controller implemented in one direction of the building. The performance of the proposed control algorithm was compared to a classical linear optimal control algorithm under several far and near-fault earthquakes. Jiang and Adeli [24] presented an innovative nonlinear control model for active control of 3D building structures considering both geometrical and material nonlinearities. To predict the structural response, they presented a dynamic fuzzy wavelet neuroemulator. Numerical validations in both time and frequency domains showed that the new neuroemulator provides accurate prediction of structural displacement responses, required in neural network models for active control of structures. Lin et al. [25] illustrated the vibration control effectiveness of TMDs for multi-story irregular buildings under bi-directional horizontal earthquake excitations. Daniel and Lavan [26] presented a formal optimization methodology for the performance based design of multiple TMDs for three-dimensional irregular buildings. By constraining both inter-story relative displacements and accelerations in the allowable values, the total weight of all TMDs could be minimized. Yanik et al. [27] proposed a new simple approach for controlling 3D structural systems against seismic vibration. The approach was arranged by modifying the stiffness and damping parameters of the structure. To obtain the additional stiffness and damping parameters, they solved the Riccati equation using the optimal control gain matrix.

On the other hand, some buildings are resting on soft soil and thus considering the soil-structure interaction (SSI) effect can be important. The dynamic characteristics of structures like natural frequencies, damping ratios and mode shapes would be changed by the SSI effects [28]. The complexity of soil behavior has induced the development of many idealized models of soil behavior based on the classical theories of elasticity and plasticity for considering the SSI effects. Assuming the supporting soil behavior as a linear elastic continuum is the simplest type of idealized soil response, so that, it can be assumed that the deformations are as linear and reversible. Various types of soil models can be expressed as, Elastic Half-Space Models, Winkler’s Model [28], two-parameter elastic models like Pasternak [29,30] and so on. Lin et al. [31] and Nazarimofrad and Zahrai [2] investigated the SSI effect on the vibration control effectiveness of active tendon systems of an irregular building subjected to earthquake excitations. In their model, in addition to the lateral displacement in both x and y directions, there is a large rotation around z direction. Nazarimofrad and Zahrai [2] also developed a mathematical model to obtain the seismic response control of an irregular multi-story building using active tendons by LQR algorithm. In addition, the SSI effect was considered by changing structure mass, stiffness and damping matrices. Results showed that active tendons on the building resting on soft soils have low effect. Farshidianfar and Soheili [32] investigated the optimized parameters for TMDs by ant colony optimization method to reduce both the maximum displacement and acceleration of the floors in a multi-story building under earthquake excitation considering SSI effects. The design variables in the optimization were the TMD mass, damping coefficient and spring stiffness.

In previous work by Nazarimofrad and Zahrai [2], a mathematical model was developed to investigate the seismic performance of an irregular multi-story building resting on soft soil using active tendon system and LQR algorithm. However, few studies might be found on the ATMD in the 3D buildings with asymmetric plan and there has been no research on fuzzy logic control of ATMD in torsional response reduction of a 3D building considering SSI effects. As previous researchers have shown that fuzzy logic control demonstrates high performance [4, 7 and 18], this paper proposes a fuzzy logic controller (FLC) to evaluate the active control force in two-ATMD controller for a 3D building with asymmetric plan considering SSI effect. The fuzzy logic control used in this study works fine separately in x and y directions, nonetheless the algorithm is successful in creating proper active forces to reduce the 3D building model responses.

2. Mathematical model of the building

Equation of motion of a building under bi-directional horizontal earthquake excitations with n-story as three-dimensional model with SSI effect and existence of two ATMD at center of mass on the top floor (Figs. 1 and 2) can be written as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{f}(t) + \mathbf{G}\mathbf{U}(t)$$

(1)

where M, C, and K are (3n + 7)×(3n + 7)-dimensional matrix of mass, damping and stiffness defined later as Eqs. (4,13,15). The terms of “3n” shows that each story has three degrees of freedom. The terms of “7” shows two degrees of freedom for TMD and five degrees of freedom for stiffness and dashpot of soil are considered. All matrices are derived using combination of some references [2,31]. In such references, the modeling approach of three-dimensional structure with TMD and SSI effects has been presented.

According to Fig. 3, \(\mathbf{u}(t)=[x_1 y_1 \dot{\theta}_1, ... , x_n y_n \dot{\theta}_n, x_{n+1} y_{n+1}, \dot{\phi}_n, \dot{\varphi}_n, x_{3n} y_{3n}]^T\) is the 3n-dimensional response vector with respect to the ground except

![Fig. 1. Multi-story building under torsional effect with two ATMD systems on the top floor.](image-url)
that is with respect to the top floor. Vector $\{x_i, y_i, \theta_i, x_{ik}, y_{ik}, \theta_{ik}\}$ is the response vector and $\{x, y, \theta, x_k, y_k, \theta_k\}$ is response of the degree of freedom of soil and vector $\{x_i, y_i, \theta_i\}$ is the displacement of two ATMDs. $u(t)$ and $u(t)$ are the response vector velocity and acceleration of each story respectively. $\Gamma$ is the $(3n+7)*2$-dimensional location matrix of controllers defined by Eq. (2). Two ATMDs assumed at center of mass on the top floor (Figs. 1 and 2) work independently; one moves only in $x$ direction and the other moves just in $y$ direction. $U(t)$ is the $(2*1)$-dimensional active control force vector applied by each ATMD, separately, and is described as Eq. (3).

$$\mathbf{\Gamma} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \end{bmatrix}^T$$ (2)

where $\mathbf{0}$ is the zero matrix with $(3n)*1$ dimension.

$$U(t) = [u_{dx}, u_{dy}]^T$$ (3)

In this study, control forces are applied to the ATMD in two $x$ and $y$ directions using fuzzy logic and LQR method for comparison purposes. $f(t)$ is the vector of the ground acceleration in terms of the time. $\eta$ is $(in + 2^*1)$-dimensional matrix that is the influence vector described by Eq. (15). The earthquake can induce vibration of the structure only in one direct direction or in two directions. The building structure is usually modeled with rigid floor diaphragms having three degrees of freedom at each floor, i.e., lateral displacements in two perpendicular directions and also a rotation around a vertical axis for the third dimension. Fig. 3 shows the first story of a multi-story building model with SSI effect.

According to this model, the mass matrix of an $n$-story three-dimensional building with ATMD considering SSI effect can be expressed as

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{x1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_{x1} & 0 & 0 & 0 & 0 & 0 & h_{x1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{ax} & 0 & 0 & 0 & I_{ax} & 0 & 0 \\ m_1 & 0 & 0 & m_2 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{x1} & 0 & 0 & l_{x2} & 0 & 0 & 0 & 0 & I_{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{x2} & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ m_1 & 0 & 0 & m_2 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{x1} & 0 & 0 & l_{x2} & 0 & 0 & 0 & 0 & I_{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{x2} & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ m_1 & 0 & 0 & m_2 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{x1} & 0 & 0 & l_{x2} & 0 & 0 & 0 & 0 & I_{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{x2} & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ m_1 & 0 & 0 & m_2 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 & m_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{x1} & 0 & 0 & l_{x2} & 0 & 0 & 0 & 0 & I_{y1} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{x2} & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_a & 0 & 0 & 0 & 0 & m_a & 0 & 0 \end{bmatrix}$$

$$ \sum_{j=1}^{n} (k_{ij}\Gamma - \gamma_i)^2 + k_{ij}\Theta_i - \theta_i)^2$$ (8)

$$k_{ij}$$ and $k_{ij}$ are lateral stiffness of each resistant elements (i.e., columns) in $x$ and $y$ directions, respectively, and, $x_i$ and $y_i$ are coordinates of resistant elements in $x$ and $y$ directions, respectively. Index $j$ is the label of resistant elements in each story and size$k$ is the number of the whole resistant elements in each story. The coordinates for center of stiffness of each story in $x$ and $y$ directions can be determined as

$$X_k = \frac{\sum_{j=1}^{n} x_i x_k}{\sum_{j=1}^{n} x_i}, \quad Y_k = \frac{\sum_{j=1}^{n} y_i x_k}{\sum_{j=1}^{n} x_i}$$ (9)

After determining stiffness matrix for each story, the total stiffness matrix for the whole structure with ATMD can be assembled considering SSI effect as

Where,

- $m_i$ = mass of story $i$, $h_i$ = height of story $i$,
- $I_{xi}$ = mass moment of inertia around $x$ direction,
- $I_{yi}$ = mass moment of inertia around $y$ direction,
- $I_{x}$ = mass moment of inertia around $x$ direction,
- $I_{y}$ = mass moment of inertia around $y$ direction,

The mass moment of inertia can be obtained as follows where "i" indicates the desired story.

$$I_{xi} = \sum_{j=1}^{n} \left( \frac{m_j}{12} (a^2 + b^2) + m_j (\bar{x} - x_i)^2 + (\bar{y} - y_i)^2 \right)$$

$$I_{yi} = \sum_{j=1}^{n} \left( \frac{m_j}{12} (b^2) + m_j (\bar{x} - x_i)^2 + (\bar{y} - y_i)^2 \right)$$

$$I_{y} = \sum_{j=1}^{n} \left( \frac{m_j}{12} (a^2) + m_j (\bar{x} - x_i)^2 + (\bar{y} - y_i)^2 \right)$$ (5)

Where $a$ and $b$ are the length and width of each slab panel in the floor diaphragm. Also $\bar{x}$ and $\bar{y}$ are center of mass and $j$ is the label of each panel slab in diaphragm. So $X$ and $Y$ that indicate center of mass of the whole diaphragm, can be expressed as

$$X = \frac{\sum_{m_j} x_j}{\sum_{m_j}}, \quad Y = \frac{\sum_{m_j} y_j}{\sum_{m_j}}$$ (6)

In Eq. (5), $m_j$ and $m_o$ are the mass of ATMD in $x$ and $y$ directions, respectively. Also, the stiffness matrix of each story of three-dimensional building can be expressed as a $3^*3$ dimensional matrix according to Eq. (8).
Building with ATMD and considering SSI effect can be expressed as

\[
C = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(13)

where 0 is the (1*3n+5)-dimensional matrix of zero and \(0^T\) is its transpose and matrix of \(C_{rx}, C_{ry}\) and \(C_{θr}\) can be expressed as

\[
C_{rx} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(14)

Where 0 is the (3n-3*1)-dimensional matrix of zero. The influence vector of earthquake \(η\) can be expressed as

\[
V_x = \sum_{i=1}^{n} k_{xi} \cdot Dx_i + k_{adj.} \cdot Δθ_i
\]

(16)

Where \(Dx\), \(Dy\) and \(Δθ\) are the lateral relative displacements in x and y directions and rotation around z direction, respectively. Base shear is the sum of all story shear forces.

### 3. Fuzzy logic and LQR controller

In control theory [2], Eq. (1) can be conveniently rewritten in state-space form as

\[
\dot{Z}(t) = AZ(t) + Bu(t) + B_f(t)
\]

(17)

Where \(A\) is ((6n+14)*(6n+14))-dimensional presented by:

\[
A = \begin{bmatrix}
0 & 1 \\
-1M^+K^* & -1M^+C^*
\end{bmatrix}
\]

(18)

where 0 and 1 are the zero and identity matrix with ((3n+7)*(3n+7)) dimension, respectively. \(B_u\) is ((6n+14)^2)-dimensional matrix given by

\[
B_u = \begin{bmatrix}
0 \\
M^+1\Gamma
\end{bmatrix}
\]

(19)

where 0 is the zeros matrix with ((3n+7)*2) dimension. \(B_f\) is ((6n+14)*1)-dimensional matrix presented by

<table>
<thead>
<tr>
<th>Direction</th>
<th>Swaying</th>
<th>rocking</th>
<th>twisting</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>(\frac{4π}{T})</td>
<td>(\frac{4π}{T})</td>
<td>(\frac{4π}{T})</td>
</tr>
<tr>
<td>stiffness</td>
<td>(k_s = \frac{4π^2}{T^2})</td>
<td>(k_r = \frac{4π^2}{T^2})</td>
<td>(k_t = \frac{4π^2}{T^2})</td>
</tr>
<tr>
<td>damping</td>
<td>(c_s = \frac{4π^2}{T}\rho V_s)</td>
<td>(c_r = \frac{4π^2}{T}\rho V_r)</td>
<td>(c_t = \frac{4π^2}{T}\rho V_t)</td>
</tr>
</tbody>
</table>

Table 1: Stiffness of springs and values of dashpot [28,34].
\[
\begin{align*}
B_r &= \begin{bmatrix}
0 \\
-M^{-1}\eta
\end{bmatrix} \\
\text{where } \mathbf{0} \text{ is the zeros matrix with } (3n+7)\times1 \text{ dimension. }
\end{align*}
\] (20)

\[
\begin{align*}
\mathbf{Z}(t) &= \begin{bmatrix}
\mathbf{u}(t) \\
\dot{\mathbf{u}}(t)
\end{bmatrix} \\
\mathbf{\dot{Z}}(t) &= \begin{bmatrix}
\dot{\mathbf{u}}(t) \\
\ddot{\mathbf{u}}(t)
\end{bmatrix}
\end{align*}
\] (21)

For active control (Fuzzy logic and LQR controller), Eq. (17) can be rewritten as

\[
\mathbf{\dot{Z}}(t) = A\mathbf{Z}(t) + B_r(\mathbf{B}_u\mathbf{U}(t) + \mathbf{B}_s\mathbf{F}(t))
\]

\[
\mathbf{F}_P(t) = \mathbf{B}_u\mathbf{U}(t) + \mathbf{B}_s\mathbf{F}(t)
\] (22)

Where \(\mathbf{U}(t)\) is active control force (Fuzzy logic or LQR) that will explain later and \(\mathbf{B}_s\) is identity matrix with \((6n+14)\times6n+14\) dimension. After simplification, Eq. (22) can be expressed as

\[
\mathbf{\dot{Z}}(t) = A\mathbf{Z}(t) + B_r\mathbf{F}_P(t)
\] (23)

For the analysis of structural response in the state space, in addition to the Eq. (23), also the Eq. (24) should be defined.

\[
y = E\mathbf{Z}(t) + L^*\mathbf{F}_P(t)
\] (24)
where $E$ and $L$ matrices can be expressed as

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -M^T K & -M^T C \end{bmatrix}$$

(25)

where 0 and I are the zero and identity matrices with $(3n+7)\times(3n+7)$ dimension, respectively.

$$L^* = \begin{bmatrix} 0 \\ B_a \end{bmatrix}$$

(26)

where 0 is the zero matrix with $(3n+7)\times(6n+14)$ dimension.

Equations of motion of structures are modeled by Simulink of MATLAB for time history analyses [33]. The Dormand–Prince method that is more powerful than Runge–Kutta, with variable-step size and 1e-3 relative tolerance was used to conduct numerical simulations and the time delay factor is assumed as 20 ms in this study according to [22]. In the process of actual control force production and applying the force to the structures, some delay might occur called the time delay. A transport delay block was located after the production of control force in Simulink. In Fig. 4, the block diagram of Simulink can be observed. In the figure, the input earthquake records are located in the left-hand multiplied by $B_a$ matrix, and considered as the input of state-space block. Then, the output of state-space block is multiplied by the gain matrix of controller. The result would be the active control force that is multiplied by $B_a$ matrix. Finally, that would be as the extra input of state-space block. This loop is implemented up to 20 s.

In addition to time-domain analysis, transfer functions in the frequency domain are analyzed. In this study, transfer functions are the Fourier transforms of structure responses in x, y and z directions.

4. Designing the fuzzy logic controller (FLC)

A FLC can be incorporated into a closed-loop control system similar to conventional controllers. The fuzzy rule defines the relationship between input and output fuzzy (linguistic) variables based on Mamdani model. The rule is formed by several IF-THEN statements. For a function with two input values (here, velocity and displacement) and one output (here, active force), each rule has the following form:

$$R^i = \text{if } X_1 = A_i \text{ and } X_2 = B_i \text{ then } Y = C_i$$

(27)

where $i$ is number of control rules, $X_1$ and $X_2$ are variables of the antecedent part and $Y$ is a variable of the consequent part. $A_i$, $B_i$, and $C_i$ are linguistic values of the fuzzy variables. Fuzzification, Rule Base, Decision Making and Defuzzification are components of FLC. Fuzzification operations can transform mathematical input values into a linguistic value, and the opposite defuzzification operations can be used to transform a fuzzy output that is a linguistic value into a crisp output value. Then, the crisp output value can be used for decision or control purposes. Rule base consists of a collection of the expert control rules needed to achieve the control goal. Decision Making as a reasoning mechanism in the fuzzy controller, uses crisp data directly from the input and then, the data are transformed into linguistic values through the fuzzification process.

In this study, the fuzzy controller uses crisp data directly from the sensor (displacement and velocity of the top floor of the building) as the response of structures. Then through the fuzzification process, the data are transformed into linguistic values as fuzzy membership functions. Each input variable has five Gaussian membership functions (Fig. 5a), and the output variable (active external control force) has seven Gaussian membership functions (Fig. 5b). The fuzzy input and output variables' membership function abbreviations used to define the fuzzy space are: $LP = \text{Large and Positive}$; $P = \text{Positive}$; $Z = \text{Zero}$; $N = \text{Negative}$; $LN = \text{Large and Negative}$ (for input variable); and $PL = \text{Positive and Large}$; $PM = \text{Positive and Medium}$; $PS = \text{Positive and Small}$; $ZR = \text{Zero}$; $NL = \text{Negative and Large}$; $NM = \text{Negative and Small}$ (for output variable). The details of the inference rules employed in the present work are shown in Table 2. Mamdani method is used for combining the membership values for each rule, and the COA defuzzifier method is used for obtaining the crisp value. The crisp value is between $-1$ and $+1$ that must be multiplied in force coefficient for produce active force. The fuzzy logic control used in this study works separately in x and y directions; however, the algorithm is successful in producing proper active forces for reduction of 3D building responses.

5. Classical linear optimal controller (LQR)

In the classical linear optimal controller, the state-feedback law (active force) $U(t) = -GZ(t)$ minimizes the quadratic cost function

$$J = \int_0^{t_1} (Z^T Q Z + U^T R U) dt$$

(28)

where $t_1$ is the duration of an earthquake. $Q$ and $R$ are the weighting matrices with $(6n+17*6n+17)$ and $(2*2)$-dimensions, respectively. If it is requested to achieve a significant decline in structural response in the time domain, the values of the weighting matrix $Q$ should be greater than those of the weighting matrix $R$. The opposite is true when the elements of $R$ are larger in comparison with those of $Q$. The gain matrix $G$ is obtained by solving the following nonlinear Riccati matrix equation as below,

$$G = -R^{-1}B_a^T P + P B_a R^{-1} B_a^T P + Q = 0$$

(29)

(30)

6. Numerical examples

For numerical examples in the particular application studied in this research work, a 10-story steel building structure with different plans and eccentricities is considered. The building plan of the 1st example is assumed to be as shown in Fig. 6 where the story consists of eleven panel slab and one opening. Distributed load applied to the floor panels can be also observed in Fig. 6. Same plan is used in different stories. The structural systems of building in x and y directions are steel moment frame. Steel box with the outside dimensions of $200*200$ mm and the thickness 20 mm is used for all columns.

The length of column is 3200 mm. E is the steel modulus of elasticity that is equal to $2*10^5$ MPa. As a result, eccentricity between the

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**Fig. 6.** Plan of the 10-story building with applied distributed load.
Two different cases for the soil under building are considered. One of them is a rigid support for the structures. In this case, SSI effects will not be considered. In the other case, the structure is on soft soil and so SSI effects are considered. The properties of soft soils can be observed in Table 3.

As can be observed in Fig. 7, three near-fault earthquake records have been considered to be applied to the structures. All records have the data in both the normal (FN) and the parallel (FP) direction to the fault used for applying in the x-direction and in the y-direction, respectively. The records were corrected by Yanik et al. [23].

Different considered cases are collected in Table 4. For example, case 1 is the one in which the Erzincan earthquake is applied only in the x-direction to the rigidly supported structure and as another example, case 12 is the one in which the Northridge earthquake is applied in both x and y directions to the building located on the soft soil.

There are two ATMDs at the center of mass on the top floor such that they can move in orthogonal directions. Control fuzzy force coefficients are adjusted so that the control forces in the structure on rigid-base and soft soil are close to each other. For this reason, the coefficients for the structure on fixed-base and soft soil are considered $5\times 10^5$ and $7\times 10^5$, respectively. The crisp input data for displacement are between $-0.8$ and $0.8$ and for velocity are between $-2$ and $2$ that must be converted into linguistic data between $-1$ and $1$. The values have been obtained by trial /error procedure. The mass of each story is $m=33$ t then considering a proper mass ratio, mass of ATMD is selected $m_t=10$ t having the lowest mass with best performance. Since the first mode frequency is $\omega_1=2.85$ rad/s, so the stiffness and damping of ATMD are $k_t=m_t\omega_1^2=81,225$ N/m and $\xi=15\%$ (the damping has the best performance in this research), respectively.

Fig. 8 shows the displacement of the first and tenth stories of the case 1. In spite of applying earthquake force in x direction, there is also lateral displacement in y direction and rotation around z direction because of existing eccentricities in y direction. It is generally known that the seismic forces are applied to the center of mass of the building floors, and if there are eccentricities between the center of mass and the center of the stiffness, a torsional moment is produced around its z direction. The torsion causes an increase in lateral displacement in x direction and also causes to create lateral displacement in y direction.

According to Fig. 8, the reduction of displacement in the structure with ATMD with Fuzzy controller is higher than that for the structure with TMD and ATMD with LQR controller. Active force as expected induced a further reduction in displacement. But after a while, reducing structural responses in both cases are almost close to each other. Despite ATMD with LQR controller as a type of active control must be more efficient, but in this system where two ATMDs are placed at the center of mass on the top floor with orthogonal direction movements, active control scenarios including ATMD with fuzzy controller might not necessarily reduce response significantly at all peak values. However, ATMD with Fuzzy controller has been able to reduce the
maximum displacement significantly. According to Fig. 9, this induced to efficiency of ATMD in y direction producing force for reduction of lateral displacement in y direction and rotation around z direction due to existence of displacement in y direction and rotation around z direction.

In case 2 that the structure is on soft soil, the results of case 1 are obtained, however, soft soil induced to decrease structural response in both x and y directions and around z direction. The cases 3 and 4 are similar to cases 1 and 2, respectively. The only different is that the Erzincan earthquake was applied in two orthogonal directions. In Figs. 10 and 11, displacements of cases 3 and 4 where the structure is on the rigid-base and soft soil, respectively, are observable. In these two cases, maximum control forces are the same approximately, according to Table 4. As can be observed, the structure on soft soil experiences less displacement than the structure on rigid-base. This is induced to base shears of uncontrolled structure on soft soil (case 4) in x and y directions of 704 kN and 565 kN, according to Table 5. Existence of TMD or ATMD induced to reduce displacement in x and y directions and rotation around z direction, although the effect of ATMD is much more.

Base shears in cases 3 and 4, have reduced approximately 50% by adding ATMD with Fuzzy controller, while, by adding TMD, the response reduction is just less than 10%. The response reduction by adding ATMD with LQR controller is similar to the structure with TMD. Adding TMD and ATMD with LQR controller to the structure on soft soil has no significant effect. However, adding ATMD with Fuzzy controller to the structure on both rigid-base and soft soil, has enhanced effects on reducing displacement.

Despite similar reduction of base shear in these cases, reductions of relative displacements in the cases according to Tables 6–9 are not similar in different stories. In the Tables, the ratios of the maximum relative displacement of controlled states to the uncontrolled states are specified with RDCU. In Figs. 12 and 13, fuzzy control forces and LQR control forces for applying to ATMD in both x and y directions in case 3 are observed respectively.

According to Table 5, in cases 5–8 where the Loma Prieta earthquake is applied, base shear of the structure on rigid-base is much more (about twice) than that for the structure on soft soil. However, while the maximum control force in the structure with ATMD with fuzzy controller on soft soil is less than that in the structure on rigid-base, reduction of base shear in the structure on soft soil and the structure on

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**Fig. 8.** Displacement and rotation of the first and the tenth stories of the case 1 (example 1).

**Fig. 9.** Fuzzy control force applied to ATMD in x and y directions in case 1 (example 1).
fixed-base are 40% and 30%, respectively. Similar to the structures under the Erzincan earthquake, while by adding TMD, the displacement reduction especially in the structure on soft soil is far less, adding ATMD with Fuzzy controller to the structure on both rigid-base and soft soil severely reduces the lateral displacement.

In the cases 9–12, the Northridge earthquake is applied. For all cases, control forces are approximately equal to 34 kN. In the cases, soft soil reduces the base shear 1.8 times approximately. However, existing ATMD with Fuzzy controller induced to similar reduction of base shear in both cases on rigid-base and soft soil. The reduction in \( x \) direction is 30% while in \( y \) direction it is 60%, approximately. However, the reduction of relative displacement in different stories is not similar. Like the structures under the Erzincan and Loma Prieta earthquakes, by adding TMD and ATMD with LQR controller, the displacement reduction especially in the structure on soft soil is far less, but adding ATMD with Fuzzy controller in the structure on both rigid-base and soft soil,

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Fig. 10. All story responses of case 3 for the 10-story building (example 1) on rigid-base (stiff soil).

Fig. 11. All story responses of case 4 for the 10-story building (example 1) on soft soil.
### Table 5
Maximum base shears (kN) for all cases (example 1).

<table>
<thead>
<tr>
<th>Case</th>
<th>Story 1</th>
<th>Story 2</th>
<th>Story 3</th>
<th>Story 4</th>
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### Table 6
RDCU in the structures with ATMD for cases 1–6 (example 1).

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### Table 7
RDCU in the structures with ATMD cases 7–12 (example 1).

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### Table 8
RDCU in the structures with TMD for cases 1–6 (example 1).

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### Table 9
RDCU in the structures with TMD cases 7–12 (example 1).

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\* J is the ratio of the V Con. to V Uncon.
severely reduces the displacement. In Figs. 14 and 15, displacements of stories for cases 11 and 12 are shown.

Despite the lack of uniformity of relative displacement in different stories, it is certain that all story shears in different cases have reduced and two ATMDs with fuzzy controller have been successful in displacement reduction in all directions. While the structure on soft soil has less displacement and base shear, the effect of ATMD with fuzzy controller in the structure on soft soil has been very well and in some cases have better performance than the structure on rigid-base.

In Figs. 16 and 17, transfer functions of responses of first story in cases 11 and 12, respectively, are shown. The results show the peak values of all cases by adding ATMD with fuzzy controller have reduced severely. The reduction in the structure on rigid-base is more.

As another example, to validate the proposed method, a 15-story building with completely different plan (Fig. 18) is considered to be analyzed. Eccentricity between the center of mass and the center of stiffness of story in x and y directions is equal to 2.16 m and 1.47 m, respectively. According to Figs. 19 and 20, it can be concluded that the
The proposed method has also good performance in this example having different floor plan and eccentricities in two directions. Overall, the results are very similar to those obtained for the first building example. As shown in Fig. 20, since the impact of adding TMD or ATMD with \textit{LQR} to the building on soft soil is minor, no observable reduction is found for these two cases compared to the case without control but ATMD with fuzzy controller has been successful to significantly reduce response of the building even on the soft soil.

**Fig. 15.** All story responses of case 12 for the 10-story building (example 1) on soft soil.

**Fig. 16.** Transfer functions of case 11 in the first story of the 10-story building (example 1).
In this paper, a mathematical model was developed for seismic control of an irregular multi-story building using two ATMDs at center of mass on the top floor with fuzzy logic control algorithm considering SSI effect. Moreover, two TMDs were also used on center of top floor mass for result comparison purposes. When the earthquake excitations are applied in x direction only, because of existing eccentricities in y direction, in addition to the lateral displacement in x direction, there are both the lateral displacement in y direction and the rotation around z direction. Thus, ATMD in y direction becomes active producing force for reduction of lateral displacement in y direction and rotation around z direction. Soft soil induced to decrease structural responses in both x and y directions and rotation around z direction. This shows that base shears of uncontrolled structure on soft soil in x and y directions are less than those for the structure on rigid-base.

Using two ATMDs with fuzzy controller in 10 and 15-story asymmetric plan building examples led to reduction of displacement in x and y directions and rotation around z direction. While the fuzzy logic control used in this study worked separately in x and y directions, the algorithm was successful in producing proper active forces for reduction of 3D building responses in all x and y directions and around z direction. The results showed the reduction of base shear up to approximately 50% by adding ATMD with fuzzy controller in all cases under earthquake in two orthogonal directions, while the reduction is just less than 10% by adding TMD. Despite ATMD with LQR controller as a type of active control must be more efficient, but in this system where two ATMDs are placed at the center of mass on the top floor with orthogonal direction movements, active control scenarios including ATMD with fuzzy controller might not necessarily reduce response significantly at all peak values.

Although reduction of base shear in the cases studied in this paper seemed similar, relative displacement reductions in such cases were not similar in different stories. Despite the lack of uniformity of relative displacement in different stories, it is certain that all story shears in different cases have reduced and two ATMDs with fuzzy controller have been successful in displacement reduction in all directions. While the structure on soft soil has less displacement and base shear, the effect of ATMD with fuzzy controller on the case has been very well leading to better performance in some cases compared to the structure on rigid-

7. Conclusion

In this paper, a mathematical model was developed for seismic control of an irregular multi-story building using two ATMDs at center of mass on the top floor with fuzzy logic control algorithm considering SSI effect. Moreover, two TMDs were also used on center of top floor mass for result comparison purposes. When the earthquake excitations are applied in x direction only, because of existing eccentricities in y direction, in addition to the lateral displacement in x direction, there are both the lateral displacement in y direction and the rotation around z direction. Thus, ATMD in y direction becomes active producing force for reduction of lateral displacement in y direction and rotation around z direction. Soft soil induced to decrease structural responses in both x and y directions and rotation around z direction. This shows that base shears of uncontrolled structure on soft soil in x and y directions are less than those for the structure on rigid-base.

Using two ATMDs with fuzzy controller in 10 and 15-story asymmetric plan building examples led to reduction of displacement in x and y directions and rotation around z direction. While the fuzzy logic control used in this study worked separately in x and y directions, the algorithm was successful in producing proper active forces for reduction of 3D building responses in all x and y directions and around z direction. The results showed the reduction of base shear up to approximately 50% by adding ATMD with fuzzy controller in all cases under earthquake in two orthogonal directions, while the reduction is just less than 10% by adding TMD. Despite ATMD with LQR controller as a type of active control must be more efficient, but in this system where two ATMDs are placed at the center of mass on the top floor with orthogonal direction movements, active control scenarios including ATMD with fuzzy controller might not necessarily reduce response significantly at all peak values.

Although reduction of base shear in the cases studied in this paper seemed similar, relative displacement reductions in such cases were not similar in different stories. Despite the lack of uniformity of relative displacement in different stories, it is certain that all story shears in different cases have reduced and two ATMDs with fuzzy controller have been successful in displacement reduction in all directions. While the structure on soft soil has less displacement and base shear, the effect of ATMD with fuzzy controller on the case has been very well leading to better performance in some cases compared to the structure on rigid-
base. Finally, the displacement reduction especially in the structure on soft soil is far less in the case of using a TMD or ATMD with LQR controller.

References


