An Integration between Fuzzy PROMETHEE and Fuzzy Linear Program for Supplier Selection Problem: Case Study

Ahmad Jafarnejad*, Hossein safari, Ermia Aghasi

Faculty of Management, University of Tehran, Iran

*aghasi@gmail.com

Abstract: This paper proposes an integrated decision making model for supplier evaluation under fuzzy environment. First the fuzzy decision matrix is formed and using fuzzy PROMETHEE the pairwise comparison is performed. Then, by fuzzy linear programming (FLP) the weights of the attributes pairwise matrices are obtained. Finally, the fuzzy output is transformed to crisp one employing fuzzy analytic hierarchy process (FAHP). The applicability and the validity of the proposed method is evaluate in a case study. The results are helpful in business plan and road map of businesses for obtaining competitive advantage and profit maximization.

Keywords: Supplier selection; fuzzy PROMETHEE; full fuzzy linear program

1 INTRODUCTION

The aim of supply chain management and supplier selection is to fulfill customers’ needs through all the chain suppliers. Selection of appropriate supplier can significantly lessen the purchase cost of the companies and consequently enhance the enterprise competitiveness [1]. Different criteria need to be considered for the supplier selection process. [2] declared that main focuses of supplier selection were given to the price consideration. However single criterion policy in supplier selection was degraded the efficiency and performance of organization. In this regard an identification of 23 criteria has been done in a study by [2, 3]. Supplier selection is the process of finding the suitable suppliers who can provide the buyer with the right quality products and/or services at the right price, in the right quantities and at the right time [3-5]. Existing researches in the field of supplier selection can be divided into two major categories: those focusing on isolating different supply source selection criteria and assessing the degree of their importance from the purchasing firm’s point of view and second those aiming to identify different alternative suppliers by developing and applying specific methods [6]. In order to select the appropriate supplier recent problem solving method should be utilized. These problem solving method contribute to the problem definition, formulation of criteria, qualification, and finally choice of suitable supplier [7]. Evaluation process of appropriate supplier consists of several conflicting criteria which the MCDM method is implemented to manage this problem.

Supplier selection can be done using different supply chain strategies [8]. There are many frameworks for the aim of assessing suppliers’ performance [9]. These frameworks have different measures which lead in different scenarios [10]. Supplier selection measures may be both quantitative and qualitative. This can justify the need of integrating both qualitative and quantitative measures in a unique framework. In addition, companies’ desires are not the same regarding all criteria. They may increase the weight of one measure and decrease the other one. Multi-criteria decision making approaches (e.g. AHP, ANP and etc.) seem to be a proper tool for the aim of weighting these criteria. Finally, considering all
these issues can be a good contribution toward supplier selection.

Supplier selection (SS) is one of the most important activities of acquisition as its results have a great impact on the quality of goods and performance of organizations and supply chains [11–13]. Through SS it is also possible to anticipate evaluation of the potential of suppliers to establish a collaborative relationship [14]. Essentially, supplier selection is a decision process with the aim of reducing the initial set of potential suppliers to the final choices [15, 16]. Decisions are based on evaluation of suppliers on multiple quantitative as well as qualitative criteria. Depending on the situation at hand, selecting suppliers may require searching for new suppliers or choosing suppliers from the existing pool of suppliers. In any case there is a degree of uncertainty in the decision process, which is caused by subjective evaluation of qualitative or quantitative criteria, by multiple decision makers, with no previous data to rely on [12, 15, 17, 18].

Fuzzy set theory combined with multicriteria decision making (MCDM) methods has been extensively used to deal with uncertainty in the supplier selection decision process [19], since it provides a suitable language to handle imprecise criteria, being able to integrate the analysis of qualitative and quantitative factors. This is the case of Fuzzy AHP – Fuzzy Analytic Hierarchy Process [20–26], Fuzzy TOPSIS – Fuzzy Technique for Order Preference by Similarity to Ideal Solution [27–33], among others.

In this paper, we propose a methodology for supplier selection and ranking under uncertainty. The remainder of our work is organized as follows. Next, we develop the problem and its modelling. In Section 3 a case study is conducted to show the implementation of the methodology. We conclude in Section 4.

2 STATEMENT OF THE PROBLEM AND MODELLING

Due to strategic importance of supplier selection process, extensive research has been done on supplier evaluation and selection. Particularly, more recent researches reveal that the interest devoted to this topic is increasing. According to Nazeri et al. [34] supplier selection is one of the most significant processes of product and service management for many enterprises within supply chain. Especially, in manufacturing companies the raw materials and component parts can equal up to 70 % of the product cost. In such circumstances the purchasing unit can affect in cost reduction. Supplier evaluation is one of the most fundamental issues of purchasing management. They also emphasize that the process of supplier selection and evaluation is MCDM, that is, in supplier selection many criteria may be considered during this process. Therefore, supplier selection and evaluation is a MCDM problem which includes both tangible and intangible criteria, some of which may conflict. Fundamentally, supplier selection and evaluation can be divided into two categories, which are single sourcing and multiple sourcing. In single sourcing, there are constraints, which are not considered in the supplier selection process. In other words, all suppliers can satisfy the buyer’s requirements of demand, quality, delivery, and etc. The buyer only needs to make one decision, which supplier is the best. On the other hand, in multiple sourcing, there are some limitations such as supplier’s capacity, quality, and delivery, which are considered in the supplier selection process. In other words, no supplier can fulfill the buyer’s total requirements and the buyer needs to purchase some part of demand from one supplier and the other part of the demand from another supplier to compensate for the shortage of capacity or low quality of the first supplier. In these circumstances, buyers need to make two decisions: which suppliers are the best, and how much should be purchased from each selected supplier?

Agarwal et al. [35] review sixty-eight articles from 2000 to 2011 to find out the most prominent MCDM methodology followed by the researchers for supplier evaluation and selection. They report the distribution of MCDM methods used in these articles as follows: Data Envelopment Analysis (DEA): 30 %; mathematical programming models: 17 %; Analytic Hierarchy Process (AHP): 15 %; Case Based Reasoning (CBR): 11 %; Analytic Network Process (ANP): 5 %; Fuzzy Set Theory: 10 %; Simple Multi-Attribute Rating Technique (SMART): 3 %; Genetic Algorithm (GA): 2 %; and Criteria Based Decision Making Methods such as ELECTRE (ELimination Et Choix Traduisant la REalité-Elimination and choice expressing reality) and PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations): 7 %.

The use of AHP/ANP with fuzzy set theory is widely accepted for dealing with qualitative evaluation attributes. Chen et al. [36] used the fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method for supplier selection problem. In this study, the F-PROMETHEE technique is preferred because of the fuzzy nature of the supplier selection decision problem.
PROMETHEE is the abbreviation of Preference Ranking Organization METHod for Enrichment Evaluations, which is an outranking method that initial references are prepared by [37-39]. In PROMETHEE method, different preference functions can be defined for criteria [40]. It is a ranking method which is quite simple in conception and application compared to other methods for MCDM. It is well adapted to the problems where a finite set of alternatives are to be ranked according to several, sometimes conflicting criteria [41-43].

Ulengin et al. [44] listed the advantages of PROMETHEE as follows:

1) PROMETHEE is a user friendly outranking method,
2) It has been successfully applied to real life planning problems
3) Both PROMETHEE I and PROMETHEE II allow both partial and total ranking of the alternatives while still satisfying simplicity.

The evaluation is the starting point of PROMETHEE method. In this phase, alternatives are evaluated with respect to different criteria. These evaluations involve essentially numerical data. Macharis et al. [45] stated that the implementation of PROMETHEE requires two additional types of information, which are as follows:

- Information on the relative importance (i.e. the weights) of the criteria considered,
- Information on the decision-makers’ preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion.

In the literature, there are a few studies with respect to the fuzzy PROMETHEE (F-PROMETHEE) approach. Goumas and Lygerou [46] and Geldermann et al. [47] have used F-PROMETHEE previously. In the F-PROMETHEE, the main problem arises in comparing two fuzzy numbers and the index, which corresponds to a weighted average of the fuzzy numbers, is found a useful way to compare fuzzy numbers. It is determined by the center of weight of the surface representing its membership function [41, 46].

In this problem of supplier evaluation under uncertainty, first we form the decision matrix using fuzzy PROMETHE, as will be described later. All suppliers are compared using some specified attributes. Then, using fuzzy linear programming approach the matrices are weighed. Finally, using the obtained weights the suppliers are ranked. A summary of the research process is shown in Figure 1.

![Research process diagram](image-url)

**Fig. 1. Research process**

Here, we develop a new method to evaluate suppliers in a classical problem of supplier selection under uncertainty. Now, consider two fuzzy numbers below:

\[
\tilde{A} = (a_1, a_2, a_3) \\
\tilde{B} = (b_1, b_2, b_3)
\]

\[
\tilde{P}_{AB} = (PL_{AB}, PM_{AB}, Pu_{AB})
\]

Then P is the weight for comparing two fuzzy numbers. Like a triangular membership function, we have

\[
PL_{AB} = \begin{cases} 
0 & \frac{a_1 + a_2 + a_3}{3} \leq \frac{b_1 + b_2 + b_3}{3} \\
\frac{a_1 - b_3}{a_1 - b_3} & otherwise
\end{cases}
\]

\[
PM_{AB} = \begin{cases} 
0 & \frac{a_1 + a_2 + a_3}{3} \leq \frac{b_1 + b_2 + b_3}{3} \\
\frac{a_2 - b_2}{a_2 - b_2} & otherwise
\end{cases}
\]
PU\textsubscript{AB} = \begin{cases} 
0 & \frac{a_{3} + a_{2} + a_{3}}{3} \leq \frac{b_{1} + b_{2} + b_{3}}{3} \\
\frac{a_{3} - b_{2}}{3} & \text{otherwise}
\end{cases}

The steps of the proposed fuzzy PROMETHEE algorithm are given below.

### 2.1 Fuzzy PROMETHEE Algorithm

For each attribute a pairwise comparison matrix is in hand.

**Step 1: Decision Matrix**

<table>
<thead>
<tr>
<th>DM</th>
<th>C\textsubscript{1}</th>
<th>C\textsubscript{1}</th>
<th>...</th>
<th>C\textsubscript{n}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A\textsubscript{1}</td>
<td>(x_{11})</td>
<td>(x_{12})</td>
<td>...</td>
<td>(x_{1n})</td>
</tr>
<tr>
<td>A\textsubscript{2}</td>
<td>(x_{21})</td>
<td>(x_{22})</td>
<td>...</td>
<td>(x_{2n})</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>A\textsubscript{X}</td>
<td>(x_{m1})</td>
<td>(x_{m2})</td>
<td>...</td>
<td>(x_{mn})</td>
</tr>
</tbody>
</table>

**Step 2: pairwise matrix**

For any criterion the costs are compared.

\(p_{ij}^c = \left(\text{PL}_{ij}^c, \text{PM}_{ij}^c, \text{PU}_{ij}^c\right)\)

\(m: 1 \ldots j\)

\(n: 1 \ldots k\)

**Step 3: preference matrix**

<table>
<thead>
<tr>
<th>II</th>
<th>A\textsubscript{1}</th>
<th>A\textsubscript{2}</th>
<th>...</th>
<th>A\textsubscript{m}</th>
<th>(\phi^+)</th>
<th>(\phi^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A\textsubscript{1}</td>
<td>[\sum_{i,j} p_{ij}^c ]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>(\phi^+ = \sum_{j=1}^{n} \sum_{k=1}^{m} p_{ij}^c w_k)</td>
<td>(\phi^-<em>{A_1} = \phi^+</em>{A_1} - \bar{\phi}_{A_1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A\textsubscript{2}</td>
<td>[\sum_{i,j} p_{ij}^c ]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td></td>
</tr>
<tr>
<td>A\textsubscript{m}</td>
<td>[\sum_{i,j} p_{ij}^c ]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td>[\sum_{i,j} p_{ij}^c w_k]</td>
<td></td>
</tr>
</tbody>
</table>

\(\phi^+ = \sum_{j=1}^{n} \sum_{k=1}^{m} p_{ij}^c w_k\)

### 2.2 Fuzzy Linear Program

A fully fuzzy linear programming problem is used after fuzzy PROMETHEE. We make use of a special ranking function that is used in supplier selection. The uncertainty is modeled with the help of fuzzy triangular numbers. A fully fuzzy linear programming problem can be written as,
\[ \text{Max } \min \ z = c^T \tilde{x} \]
\[ s.t. \]
\[ \tilde{A} \tilde{x} \leq \tilde{b} \]
\[ \tilde{x} \geq 0 \]

Where
\[ z, z^* = (\tilde{c}_1, \ldots, \tilde{c}_m), \tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n), \]
\[ \tilde{A} = [\tilde{a}_{ij}]_{m \times n}, \quad \tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_m)^T, \]

denote the triangular fuzzy numbers for objective function, fuzzy objective function coefficients, fuzzy technical coefficients and fuzzy resource constraints of the linear programming.

At the end the ranks are fuzzy and should be transformed to crisp for suitable decision making. This way we make use of fuzzy AHP. The priority degree and the dignity degree are in the same scale. So, to obtain \( \sum \tilde{p}_{ij} \) the logic used in fuzzy AHP can be employed. But, the problem in FAHP is that the problem becomes crisp from the beginning which is not correct since fuzzy problem should be fuzzy till the final results are in hand.

### 3 NUMERICAL EXAMPLE

We consider a case study of supplier selection to implement our proposed methodology under uncertainty. The attributes to evaluate the suppliers are,

- Price,
- Product quality,
- On time delivery,
- Service quality,
- Brand,

and the suppliers are,

- LG,
- Samsung,
- Snowa,
- Bosch,
- Techno,
- Daewoo.

The population understudy is 150 and the decision matrix is completed by the experts in sale and customer relationship management departments.

As previously explained, first the fuzzy decision matrix is formed and using fuzzy PROMETHEE the pairwise comparison is performed. Then, by FFLP the weights of the attributes pairwise matrices are obtained. Finally, the fuzzy output is transformed to crisp one employing fuzzy analytic hierarchy process.

The decision matrix is shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Product quality</th>
<th>On time delivery</th>
<th>Service quality</th>
<th>Brand</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG</td>
<td>0.55</td>
<td>0.72</td>
<td>0.90</td>
<td>0.60</td>
<td>0.44</td>
</tr>
<tr>
<td>Samsung</td>
<td>0.27</td>
<td>0.40</td>
<td>0.57</td>
<td>0.42</td>
<td>0.60</td>
</tr>
<tr>
<td>Snowa</td>
<td>0.71</td>
<td>0.83</td>
<td>1.00</td>
<td>0.93</td>
<td>0.98</td>
</tr>
<tr>
<td>Techno</td>
<td>0.70</td>
<td>0.82</td>
<td>0.90</td>
<td>0.88</td>
<td>0.91</td>
</tr>
<tr>
<td>Bosch</td>
<td>0.50</td>
<td>0.65</td>
<td>0.80</td>
<td>0.64</td>
<td>0.82</td>
</tr>
<tr>
<td>Daewoo</td>
<td>0.63</td>
<td>0.81</td>
<td>0.91</td>
<td>0.24</td>
<td>0.39</td>
</tr>
<tr>
<td>Weight</td>
<td>0.21</td>
<td>0.39</td>
<td>0.54</td>
<td>0.56</td>
<td>0.67</td>
</tr>
</tbody>
</table>

The attributes pairwise comparison is shown in Tables 2 to 6.

<table>
<thead>
<tr>
<th></th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>LG</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Samsung</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Snowa</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Techno</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.6</td>
<td>0.0</td>
</tr>
</tbody>
</table>

To be continued.
Table 3. Pairwise comparison for product quality

<table>
<thead>
<tr>
<th></th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bosch</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Daewoo</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4. Pairwise comparison for on time delivery

<table>
<thead>
<tr>
<th></th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bosch</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Daewoo</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5. Pairwise comparison for service quality

<table>
<thead>
<tr>
<th></th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bosch</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Daewoo</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
To be continued.

Table 6. Pairwise comparison for brand

<table>
<thead>
<tr>
<th>Brand</th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Samsun g</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Snowa</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Techno</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Bosch</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Daewo o</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The resulted supplier/attribute matrix is shown in Table 7.

Table 7. Supplier/attribute matrix

<table>
<thead>
<tr>
<th></th>
<th>LG</th>
<th>Samsung</th>
<th>Snowa</th>
<th>Techno</th>
<th>Bosch</th>
<th>Daewoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Samsun g</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Snowa</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Techno</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Bosch</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Daewo o</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
The fuzzy linear program to obtain the weights is given below:

\[
\begin{align*}
\text{Min } Z &= (1-\lambda)^2 M \sum_{j} \sum_{i} y_{ij}^2 + z_{ij}^2 \\
\text{s.t. } & \\
& x_i - x_j - \lambda (\ln \frac{m_{ij}}{l_{ij}}) + y_{ij} \geq 0, \\
& -x_i + x_j - \lambda (\ln \frac{u_{ij}}{m_{ij}}) + z_{ij} \geq 0, \\
& w_i = \frac{\exp(x_i)}{\sum_{i} x_i}, \\
& x_i, y_{ij}, z_{ij}, \lambda \geq 0.
\end{align*}
\]

where \(M\) is a large number and the aim of the linear model is to find the weights.

Solving the model in lingo optimization software we have:

Local optimal solution found.
Objective value: 0.1232595E-27
Infeasibilities: 0.3178013E-14
Total solver iterations: 41

And finally, the crisp values and the full ranking of suppliers is:

The weights are obtained to be: 0.21, 0.14, 0.175, 0.12, 0.175, 0.18 for the suppliers, respectively, while LG and Daewoo are in the first and second ranks.

4 CONCLUSIONS
In this work a new integration between PROMETHEE and linear programming in fuzzy environment was proposed. The decision making problem is of supplier selection. First the fuzzy decision matrix was formed and using fuzzy PROMETHEE the pairwise comparison was performed. Then, by fuzzy linear programming the weights of the attributes pairwise matrices were obtained. Finally, the fuzzy output was transformed to crisp one employing fuzzy analytic hierarchy process. The applicability and the validity of the proposed method is evaluate in a case study.

References


