SYNCHRONIZING CHAOTIC SYSTEMS WITH PARAMETRIC UNCERTAINTY VIA A NOVEL ADAPTIVE IMPULSIVE OBSERVER

Moosa Ayati, Hamid Khaloozadeh, and Xinzhi Liu

ABSTRACT

This paper proposes a new class of observers, called adaptive impulsive observers. These observers are capable of estimating the states and unknown parameters of an uncertain system using the output of the system at discrete jump times only. Through a proposed theorem, the stability of the states estimation error system is proved and an upper bound on the maximum possible impulses (jumps) interval is given. Due to these advantages, the proposed adaptive impulsive observer is used in a chaotic systems synchronization scheme. The presented simulation results show the effectiveness of the proposed observer even when the coupling signal is scalar.

Key Words: Adaptive impulsive observer, impulsive synchronization, impulsive system, parametric uncertainty.

I. INTRODUCTION

Since synchronization phenomena are pervasive in nature, chaos synchronization has attracted considerable attention from scientists and engineers over the past two decades. In biological systems, e.g. chaotic neuron networks [1], a large number of studies have sought to unveil the mechanisms of synchronization from both physiological [2] and computational [3] viewpoints. Other applications of chaos synchronization are in networks [4], synchronizing robot manipulators [5], etc. (interested readers are referred to [6]). One of the interesting uses of chaotic systems is in chaotic secure communication systems. After the seminal work of Pecora and Carroll [7], several methods have been employed to theoretically and experimentally synchronize chaotic systems, such as sliding mode controllers and observers [8], adaptive control methods ([9–11]), impulsive control methods [12, 13], etc.

Chaotic communication systems [14] are communication systems where the transmitter (or drive system) modulates the message with a functional of the transmitter states or parameters and sends the modulated signal on the communication channel. In the receiver (or response system), utilizing a well-developed synchronization algorithm, the received signal is demodulated and the message is extracted. Generally, a synchronization algorithm is a method that estimates the drive system states or parameters using one or several coupling signals or control inputs.

Investigating different synchronization methods shows that impulsive synchronization has shown great efficiency in chaos communication applications as it maintains synchronization by rather small synchronization impulses. Other synchronization methods, called continuous synchronization methods, demand continuous synchronization signals. In impulsive synchronization, since the response system receives a sequence of synchronizing impulses at discrete time instants, the communication channel capacity will be reserved for
message transmission. Moreover, the redundancy of the synchronization information in the channel is reduced in comparison to continuous synchronization methods; therefore, the security of the chaotic communication system will increase. In addition, impulsive synchronization offers a direct method for the modulation of digital information onto a chaotic carrier signal for spread spectrum applications [14–16].

Many projects have investigated and developed different impulsive synchronization methods. For instance, [17–20] present algorithms for adaptive impulsive synchronization of chaotic systems but without consideration of the effect of drive system uncertainties. [21, 22] consider the effects of model and parameter uncertainties and propose robust impulsive synchronization schemes. These methods, however, demand that all of the states of the drive system be sent to the response system. [23, 24] propose impulsive synchronization algorithms for Lur’e systems and Chua’s oscillator when only the output signal of the drive is sent to the response. These algorithms, however, are suitable only for systems without uncertainties.

This work proposes a synchronization scheme where the receiver only needs the output of the drive system. In most of the impulsive synchronization methods, the response system needs to have access to all of the drive system states. This means that all of the drive system states must be transmitted to the response, which will decrease the security and capacity of the communication channel, along with increasing the complexity of the communication system. To solve these problems, an observer-based synchronization scheme, where the receiver uses an observer, called an adaptive impulsive observer (AIO), has been used to estimate the states of the drive. This will decrease the number of coupling signals between the drive and the response.

In this paper, the effects of the uncertainties are taken into account. The drive system has parametric uncertainties, and the proposed observer estimates the unknown parameters of the drive system. In the meantime, parameter estimation in the proposed adaptive impulsive observer is done by a scalar coupling signal. In the numerical simulations of the proposed adaptive impulsive observer, the uncertain drive and response systems are coupled with a scalar signal. The AIO is able to effectively estimate the states and unknown parameters of the drive system.

The remainder of the paper is organized as follows. Section II gives the basic concepts of impulsive systems and their stability. Section III introduces the proposed AIO observer and investigates its stability and convergence through a theorem. Section IV investigates the performance of the observer by numerical simulations of chaotic Lorenz system. Finally, the conclusions are given.

II. BASIC CONCEPTS OF IMPULSIVE CONTROL

Consider the nonlinear system

\[ \dot{x} = f(t, x) \] (1)

where \( x \in \mathbb{R}^n \) is the continuous states vector, \( t \) is the time variable, and \( f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n \). The jumps in the state variables happen at discrete times \( \tau_i \), i.e.,

\[ U(i, x) = \Delta x|_{t=\tau_i} = x(\tau_i^+) - x(\tau_i^-) \] (2)

where \( 0 < \tau_1 < \tau_2 < \cdots < \tau_i < \cdots \) and \( \lim_{i \to \infty} \tau_i = \infty \). By these definitions, an impulsive system could be described by

\[ \dot{x} = f(t, x) \quad t \neq \tau_i \]
\[ \Delta x = U(i, x) \quad t = \tau_i \]
\[ x(t_0^+) = x_0 \quad t_0 \geq 0, \quad i = 1, 2, \ldots \]

which is also called an impulsive differential equation [25]. Investigating the stability of impulsive systems demands specific definitions and theorems. For instance, for stable impulsive systems, the derivative of the Lyapunov function could be both positive and negative. For stable ordinary systems, however, the derivative of the Lyapunov function should be non-positive. Some of the basic theorems of impulsive control are presented in [26].

Definition 1. \( V : (\tau_{i-1}, \tau_i] \times \mathbb{R}^n \to \mathbb{R}^n \) is said to belong to class \( \mathcal{V}^0 \) if

a) \( V \) is continuous in \( (\tau_{i-1}, \tau_i] \times \mathbb{R}^n \to \mathbb{R}^n \) and for each \( x \in \mathbb{R}^n \)

\[ \lim_{(t,y) \to (\tau_i,x)} V(t, y) = V(\tau_i, x), \quad i = 1, 2, \ldots \] (4)

b) \( V \) is locally Lipschitz in \( x \).

Definition 2. For \((t, x) \in (\tau_{i-1}, \tau_i] \times \mathbb{R}^n\), the generalized derivative \( D^+ V(t, x) \) is defined as

\[ D^+ V(t, x) = \lim_{h \to 0} \sup \frac{1}{h} \left[ V(t+h, x+h f(t, x)) - V(t, x) \right] \] (5)
**Definition 3 (Companion system).** Let \( V \in \nu^0 \) and assume
\[
D^+ V(t, x) \leq g(t, V(t, x)), \quad t \neq \tau_i
\]
\[
V[t, x + U(i, x)] \leq \psi_i[V(t, x)], \quad t = \tau_i
\]
where \( g : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} \) is continuous and \( \psi_i : \mathbb{R}^+ \to \mathbb{R}^+ \) is non-decreasing. Then, the system
\[
\dot{w} = g(t, w) \quad t \neq \tau_i
\]
\[
w(\tau_i^+) = \psi_i[w(\tau_i^-)] \quad t = \tau_i
\]
\[
w(\tau_0^+) = w_0 \geq 0
\]
is called the companion system of (3).

**Definition 4.** A function \( \beta \) is said to belong to class \( \kappa \) if \( \beta \in C[\mathbb{R}^+, \mathbb{R}^+] \), \( \beta(0) = 0 \) and \( \beta(x) \) is strictly increasing in \( x \). Also, \( S_\rho \) is the ball
\[
S_\rho = \{ x \in \mathbb{R}^n \mid \| x \| \leq \rho \}
\]
where \( \| \cdot \| \) is the Euclidean norm.

**Assumption 1.** \( f(t, 0) = 0, U(t, 0) = 0, g(t, 0) = 0, \) and \( \psi_i = 0 \) for all \( i \). By these assumptions, the trivial solutions of (3) and (7) exist.

**Theorem 1 ([26]).** Assume the following conditions are satisfied

(a) \( V : \mathbb{R}^+ \times S_\rho \to \mathbb{R}^+ \), \( \rho > 0 \), \( V \in \nu^0 \), and \( D^+ V(t, x) \leq g(t, V(t, x)), t \neq \tau_i \)

(b) There exists a \( \rho_0 > 0 \) such that \( x \in S_{\rho_0} \) implies that \( x + U(i, x) \in S_{\rho_0} \) for all \( i \) and \( V[t, x + U(i, x)] \leq \psi_i[V(t, x)], t = \tau_i, x \in S_{\rho_0} \).

(c) \( \beta(\| x \|) \leq V(t, x) \leq \alpha(\| x \|) \) on \( \mathbb{R}^+ \times S_{\rho_0} \), where \( \alpha(\cdot), \beta(\cdot) \in \kappa \).

Then, the stability properties of the trivial solution of the companion system (7) implies the corresponding stability properties of the trivial solution of (3).

**Theorem 2 ([26]).** Let \( g(t, w) = \dot{\lambda}(t)w, \dot{\lambda} \in C^1[\mathbb{R}^+, \mathbb{R}^+] \), \( \psi_i(w) = d_iw, \) and \( d_i \geq 0 \) for all \( i \). Then, the origin of (3) is asymptotically stable if
\[
\dot{\lambda}(\tau_i) + \ln(\gamma d_i) \leq \lambda(\tau_i), \quad \gamma > 1, \quad i = 1, 2, \ldots
\]
and
\[
\dot{\lambda}(t) \geq 0
\]
are satisfied.

**III. PROPOSED ADAPTIVE IMPULSIVE OBSERVER**

In this section, a novel class of observers, called adaptive impulsive observers, which is used in a chaos synchronization scheme, is proposed. AIO can estimate the drive states using only the scalar output of the drive system at discrete jump times. Another advantage of the proposed observer is that it can overcome the effects of the parametric uncertainties, which are inevitable in practical applications, via an adaptive algorithm that estimates the unknown parameters of the drive system. In the following, the formulations of the AIO are presented.

Consider the drive system as:
\[
\begin{align*}
\dot{x} &= Ax + B_1u + f(x) + Bh(x)\theta \\
y &= Cx
\end{align*}
\]
where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) is the control input, \( y \in \mathbb{R}^p \) is the output vector, and \( \theta \in \mathbb{R}^q \) is the unknown parameters vector. \( A, B_1, B, \) and \( C \) are known constant matrices with appropriate dimensions.

**Assumption 2.** \( f \) and \( h \) satisfy the following Lipschitz condition
\[
\begin{align*}
\| f(x_1) - f(x_2) \| &\leq K_f \| x_1 - x_2 \|; \quad \forall x_1, x_2 \in \mathbb{R}^n \\
\| h(x_1) - h(x_2) \| &\leq K_h \| x_1 - x_2 \|; \quad \forall x_1, x_2 \in \mathbb{R}^n
\end{align*}
\]
where \( K_f \in \mathbb{R}^+ \) and \( K_h \in \mathbb{R}^+ \) are the Lipschitz constants.

The formulation of the proposed adaptive impulsive observer (or response system) is:
\[
\begin{align*}
\dot{x} &= A\hat{x} + B_1u + f(\hat{x}) + Bh(\hat{x})\hat{\theta} \\
t &\in (\tau_i, \tau_{i+1}]
\end{align*}
\]
\[
\hat{y} = C\hat{x}
\]
\[
\Delta \hat{x} = -F_1(\hat{y} - \hat{y}) \quad t = \tau_{i+1}^+
\]
where \( F_1 : \mathbb{R}^{n+p} \) is the state impulses gain matrix, \( \hat{x} \in \mathbb{R}^n \) is the states vector, \( u \in \mathbb{R}^m \) is the control input vector, \( \hat{y} \in \mathbb{R}^p \) is the estimated output vector, and \( \hat{\theta} \in \mathbb{R}^q \) is the estimated parameters vector. \( \tau_i, i = 1, 2, 3, \ldots \) are the jump times where impulses of the drive output signal are
if the following conditions hold. There exist positive $H$, $Q$ from the following estimation law:

$$\hat{\theta} = \Omega(\hat{x})Ce \quad t \in (\tau_i, \tau_{i+1}]$$

$$\Omega(\hat{x}) = -\phi^{-1}H^T(\hat{x})H$$

$$\Delta \hat{\theta} = F_2(y - \hat{y}) \quad t = \tau_{i+1}^-$$

$P$ is a symmetric matrix that will be introduced later in this section. $F_2 \in \mathbb{R}^{q \times p}$ is the parameter impulses gain matrix, $e = x - \hat{x}$ is the state estimation error, $\phi > 0$ is a symmetric matrix, and $H$ is a constant matrix of appropriate dimensions.

**Theorem 3 (Adaptive impulsive observer).** Let Assumption 2 hold for System (10). The adaptive impulsive observer (13) with the parameters estimation law (14) estimates the states and unknown parameters of (10) if the following conditions hold. There exist positive scalar $\gamma > 1$, matrices $F$, $H$, and symmetric positive definite matrices $\phi > 0$ and $P > 0$ such that

$$\hat{\lambda}(I + E) \leq 1, \quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \quad E = [FC \ 0]$$

$$B^T P = HC$$

$$\hat{\lambda}(P^{-1}Q) + \frac{2 \hat{\lambda}(\Psi)(K_f + K_B K_h K_\theta)}{\hat{\lambda}(\Psi)} \geq 0,$$ (17)

where $Q = A^TP + P A$, $\Psi = \begin{bmatrix} P & 0 \\ 0 & \phi \end{bmatrix}$

$$\hat{\lambda}(\tau_{i+1}) = \hat{\lambda}(\tau_i) + \ln(\gamma d_i) \leq 0,$$

$$d_i = \hat{\lambda}(\Psi^{-1}) \{(I + E)^T \Psi (I + E)\}$$

where $I$ is the identity matrix with appropriate dimensions and $\hat{\lambda}(\cdot)$ is introduced in Theorem 2. $\hat{\lambda}(A)$ and $\hat{\lambda}(A)$ represent the largest and the smallest eigenvalues of a symmetric matrix $A$, respectively.

**Proof.** Using (10) and (13), the state estimation system is

$$\dot{e} = Ae + f(x) - f(\hat{x}) + B(h(x)\hat{\theta} - h(\hat{x})\hat{\theta}), \quad t \in (\tau_i, \tau_{i+1}]$$

(19)

$$\Delta e = F_1(y - \hat{y}) \quad t = \tau_{i+1}^-$$

By defining $\hat{\theta} = \theta - \hat{\theta}$ as the parameter estimation error, $\tilde{f} = f(x) - f(\hat{x})$, $\tilde{h} = h(x) - h(\hat{x})$, and $X^T = [e \tilde{\theta}]$, the augmented impulsive system combined of (14) and (19) is:

$$\dot{X} = A \tilde{X} + f \quad t \in (\tau_i, \tau_{i+1}]$$

$$\Delta \tilde{X} = F \tilde{X} \quad t = \tau_{i+1}^-$$

(20)

where

$$A = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} f + B\hat{\theta} + Bh(\hat{x})\hat{\theta} \\ -\Omega(\hat{x})Ce \end{bmatrix}$$

Consider the Lyapunov function candidate:

$$V(X) = X^T \Psi X$$

(21)

In the first step, for $t \in (\tau_i, \tau_{i+1}]$, the time-derivative of the Lyapunov function along solutions of (20) is

$$\dot{V}(e, \hat{\theta}) = e^T Pe + \hat{\theta}^T \phi \dot{\hat{\theta}} + e^T \dot{P} \tilde{f} \phi$$

(22)

Substituting (14) and (19) into (22) yields:

$$\dot{V}(e, \hat{\theta}) = e^T (PA + A^T P)e + e^T PBh(\hat{x})\hat{\theta} + \tilde{f}^T Pe + \tilde{f}^T e \tilde{f} + e^T Pb\hat{\theta}$$

(23)

Substituting (14) into (23), and using (16) and Assumption 2 gives

$$\dot{V}(e, \hat{\theta}) = e^T Qe + \tilde{f}^T Pe + e^T P \tilde{f} + e^T Pb\hat{\theta} + \tilde{f}^T e \tilde{f} + e^T Pb\hat{\theta} + \tilde{f}^T e \tilde{f} + e^T Pb\hat{\theta} + \tilde{f}^T e \tilde{f}$$

(24)

According to (24) and Theorem 1, $g(t, w)$ is:

$$g(t, w) = \left(\hat{\lambda}(\Psi^{-1}Q) + \frac{2 \hat{\lambda}(\Psi)(K_f + K_B K_h K_\theta)}{\hat{\lambda}(\Psi)}\right)w$$

$$= \hat{\lambda}(w)$$

(25)

Also, since Condition (17) holds, we have $\hat{\lambda}(t) \geq 0$. In the second step, it is true that for $t = \tau_{i+1}^+$, for any $\rho_0 > 0$ there exists a ball $S_{\rho_0}$ where $X(\tau_0) \in S_{\rho_0}$. Using (20) and (15) we get

$$X(\tau_{i+1}^-) = (X(\tau_{i+1}) + F \tilde{X}(\tau_{i+1})) \in S_{\rho_0}$$

and the Lyapunov function for $t = \tau_{i+1}^+$ is:

$$V(\tau_{i+1}^+) = X^T(\tau_{i+1})(I + F)^T \Psi (I + F)X(\tau_{i+1})$$

$$\leq \hat{\lambda}(\Psi^{-1} \{(I + F)^T \Psi (I + F)\})V(\tau_{i+1})$$

(26)
Considering the fact that $\Psi > 0$ makes $\lambda^{-1}(I + E^T \Psi(I + E))$ always positive and using Inequality (26) and Theorem 1, we obtain:
\[
\psi_t(w) = \lambda^{-1}(I + E^T \Psi(I + E)) w = d_i w \quad (27)
\]
where $d_i \geq 0$. Therefore, all the conditions of Theorem 1 are satisfied and the stability properties of the companion system
\[
\begin{align*}
\dot{w} &= g(t, w) \quad t \in (\tau_i, \tau_{i+1}] \\
w(\tau_i^+) &= \psi_t(w(\tau_i)) \quad t = \tau_i^+ \\
w(\tau_i^+) &= w_0 \geq 0
\end{align*}
\]
imply the same stability properties for (20). On the other hand, since Conditions (15) to (18) hold, according to Theorem 2, the origin of (28) is asymptotically stable and the same results are true for (20). This means that both the state and parameter estimation errors converge to zero.

**Remark 1.** Using (18), the maximum distance of the impulses is
\[
\tilde{\Lambda} = \max_i [\tau_{i+1} - \tau_i] = \frac{-\ln(\lambda^{-1}(I + E^T \Psi(I + E)))}{\lambda(P^{-1}Q + \frac{2\lambda^{-1}(K_f + K_B K_h K_l)\psi_t}{\lambda(\Psi)})} \quad (29)
\]
and if a constant impulse period $\Delta$ is considered, it should satisfy $\Delta \leq \tilde{\Lambda}$.

**IV. SIMULATIONS**

In this section, the proposed adaptive impulsive observer is used to estimate the states of the Lorenz system:
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-\sigma_1 & \sigma_2 & 0 \\
r & -1 & 0 \\
0 & 0 & -b
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
-x_1 x_3 \\
x_1 x_2
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} x_2 \Delta \sigma_2
\]
\[
y = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]
where $r = 28$, $\sigma_1 = 10$, $\sigma_2 = \bar{\sigma}_2 + \Delta \sigma_2 = 10.3$, $\Delta \sigma_2 = 0.3$, and $b = 1.25$ are the parameters. In the simulations, the following initial conditions for the chaotic system (30) and adaptive impulsive observer (13) are considered:
\[
x_1(0) = 8, \quad x_2(0) = -10, \quad x_3(0) = 12
\]
\[
\hat{x}_1(0) = 0, \quad \hat{x}_2(0) = 0, \quad \hat{x}_3(0) = 0
\]

Also, according to the proposed theorem, the following matrices for the observer are calculated:
\[
P = 2 \times I_3, \quad \phi = 0.3, \quad F_1 = [-0.5 \quad -1 \quad 1]^T, \quad F_2 = [-0.001]
\]
and $\Delta = 5 \times 10^{-3}$. It is assumed that the parameter $\sigma_2$ is known for the drive but it is unknown to the AIO. In the AIO, the initial value of $\sigma_2$ in the parameter adaptation law is set to $\bar{\sigma}_2 = 10$ and the AIO should estimate the unknown part i.e. $\Delta \sigma_2$. Figure 1 depicts the states of the system and AIO. It shows that the estimated states converge to the drive system states and the estimation error is small (in a time window from the fourth second to the fifth second, the mean square error of the estimated states is $[1.47 \times 10^{-5} \quad 1.39 \times 10^{-3} \quad 4.71 \times 10^{-5}]^T$).

Figure 2 illustrates the state trajectory of the drive and response. The figure shows that the states of the drive and AIO start from different points but converge to each other as time increases. Figure 3 illustrates the impulses applied to each state of the AIO at jump times.

In the early times of the simulation, since state estimation errors are large, the magnitude of the synchronization impulses is larger. As the time increases, the magnitude of the impulses will decrease. It is worth mentioning that the drive system and AIO are coupled with a scalar signal at discrete jump times. Figure 4 shows that the estimated parameter by the proposed adaptive impulsive observer converges to its true value as time goes to infinity. This will boost the synchronization scheme performance in states estimation.

Figure 5 depicts the Lyapunov function and the time derivative of the Lyapunov function. According to the context, the time derivative of the Lyapunov function of the impulsive system has both negative and positive values.

**V. CONCLUSIONS**

Impulsive synchronization shows better efficiency than continuous synchronization and is very suitable for
chaotic secure communication systems. Impulsive synchronization reduces the redundancy of the synchronization information in the channel and increases the security of the channel. This work presents a synchronization scheme where the receiver is a novel adaptive impulsive observer called AIO. The proposed
observer synchronizes drive and response systems only using discrete impulses of the output of the uncertain drive system. This is in contrast to most of the other impulsive synchronization methods, which need all of the states of the drive at the receiver. Moreover, the proposed adaptive impulsive observer can deal with parametric uncertainties and gives an estimate for the uncertain parameters of the drive. The mentioned properties of the AIO would improve the security and would reduce the complexity of the chaotic communication system.

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Moosa Ayati received his B.Sc. degree from Isfahan University of Technology, Isfahan, Iran, in 2004 and his M.Sc. degree in 2006 from K. N. Toosi University of Technology, Tehran, Iran, both in Electrical Engineering with first rank honors. He is currently a Ph.D. student in Control Engineering.
at K. N. Toosi University of Technology, Tehran, Iran. Also, He serves as a TA, RA, and Lecturer at this university. His areas of interest include: chaotic systems theory and applications, stochastic estimation and control, system identification, adaptive control.

Mr. Ayati is a member of the Iranian Society of Instrumentation and Control, and the National Society of Mechatronics.

Hamid Khaloozadeh received his B.S. degree in Control Engineering from Sharif University of Technology (Tehran, Iran), in 1990, M.Sc. degree in Control Engineering from K. N. Toosi University of Technology Tehran, Iran, in 1993, and Ph.D. degree in control engineering from Tarbiat Modarres University (Tehran, Iran), in 1998. He is currently an Associate Professor teaching in the Department of Control Engineering in K. N. Toosi University of Technology (Tehran, Iran). His interest areas are system identification, optimal control, adaptive control, stochastic estimation and control, neural networks, digital control, nonlinear modeling, and time series analysis.

Xinzhi Liu received his Ph.D. degree in Applied Mathematics from the University of Texas, Arlington, U.S.A., in 1988. He joined the Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada, as an Assistant Professor in 1990, where he became an Associate Professor and a Full Professor in 1994 and 1997, respectively. His research areas include systems analysis, stability theory, hybrid dynamical systems, impulsive control, chaos synchronization, artificial neural networks, and communication security. He is the author or co-author of over 200 research articles and two research monographs and 15 edited books.