Optimal design of distributed energy system in a neighborhood under uncertainty

Kaveh Akbari*, Fariborz Jolai, Seyed Farid Ghaderi

Integrated Systems Engineering, The Ohio State University, 210 Baker Systems, 1971 Neil Avenue, Columbus, OH, 43210, USA

Abstract

Distributed energy systems (DES) are widely accepted as the future generation of the energy systems. The number of studies in all related fields corroborates the assertion that these systems are in their infancy and need to develop more in terms of efficiency and economizing. Admittedly, these systems are hardly lucrative and poor planning is one of many hurdles standing in the way of their profitability. Disregarding uncertainty as an innate characteristic of the real world seems one of the improper simplifications of this planning. To cover this gap, the paper is mainly focused on designing an energy system in a neighborhood including its pipeline network under demand uncertainty concerning data insufficiency. Therefore, a new model for planning in a neighborhood is presented and then reformulated to its robust counterpart. Various technologies like PV array, chillers, boiler, storage tank, and CHPs are considered in order to meet the cooling, heating and electrical demands. The probable consequences of the demand uncertainty are studied to the length. The outcomes reveal that the unit sizes and pipeline network are highly dependent on the decision maker’s level of conservatism.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

An energy system could be regarded as a supply chain consisting of production, conversion, and transmission to the end-users [1]. Conventionally, energy is produced in large power plants operating in central locations and all the produced electricity is transmitted through distribution networks to the long distances. About a decade ago, distributed energy system (DES) was introduced as a new concept on energy generation as the opposite of centralized energy system. A distributed generation network refers to small-scale producers which are located near to end-users.

One of the main challenges for designing an energy system is the degree of decentralization. Studies have discussed energy systems in different degrees of decentralization. Some of them bounded the scope of the study to a building while others discussed the energy system in a district.

Undeniably, despite all advantages of distributed energy systems, there are some barriers which make an applicatory plan a necessity. All the mentioned studies in the literature shared a common assumption. They all proposed the mathematical model in a deterministic environment and it is highly probable that the outcomes are distorted by the ignored uncertainties which it may jeopardize the profitability of the whole project. As it is obvious from Table 1, some studies considered energy system planning of a single building under inherent uncertainty while to the best of our knowledge; this planning is never conducted in any studies in the scale of a neighborhood.

This paper is mainly focused on designing an energy system in a neighborhood under demand uncertainty. Literature review reveals the lack of attention to the field of energy system planning in a neighborhood while the quality of studies in a single building seems quite satisfying. For filling this gap, a mathematical model for investment planning of the energy system in a neighborhood is proposed and then for overcoming the uncertain environment, the robust counterpart of proposed model is obtained.

In the proposed deterministic model, a wide range of technologies is considered. The buildings could also exchange heat via pipeline network which its existence depends on the model’s preferences. A combined cooling, heat and power (CCHP) unit by assisting a back-up boiler, solar thermal cells and a heat storage tank could supply both heating and electricity demand and meanwhile, both electric and absorption chillers are considered for meeting the cooling demand. Moreover, some environmentally friendly technologies like Photovoltaic arrays (PV) are considered. It
should be noted that both CHP and boiler use natural gas. The objective function also consisted of multiple criteria namely investment costs, operation and maintenance costs, Carbon emission costs and revenues.

The paper is assembled in eight sections. In Section 1, an introduction to distributed energy systems (DES) are presented and then concerns of this paper is highlighted. A generic mathematical model is presented at Section 3 and later in Section 2, a comprehensive review on literature is provided. In Section 4, the mathematical model consisted of the objective function and constraints discussed thoroughly. In Section 5, the robust model is explained and in Section 6, the test problems are described. Consequently, the outcomes are discussed to the length in Section 7. Finally, the remarks are concluded in Section 8.

2. Literature review

In this paper, the previous studies in the context of energy system planning are classified into two main groups. The first group is focused on researches that aimed to plan the energy system in a sole building while in the second group the studies are concentrated on planning the energy system several buildings.

2.1. Energy system planning in a building

There have been numerous efforts focused on the planning of the energy system in a building like Ref. [2]. The remarkable point is a majority of the studies in this group are conducted in a hospital or commercial buildings which could afford to finance in their energy sector. For instance, Ref. [3] did the feasibility study of a hospital in Parma, the case study of [4] was a hospital in Athens, Ref. [5] studied on a hospital in Iran. As it is obvious, there is a focus on hospitals. Ref. [6] concluded that due to simultaneous and flat loads of hospitals, distributed generation systems are more efficient in the hospitals.

To begin with, Ref. [7] proposed a new method consisted of a genetic algorithm for the purpose of planning an energy system even if the problem is non-linear. The outcomes suggested that the model could be applied to the complex energy systems. In Ref. [8] a mixed-integer linear programming (MILP) model is proposed for unit sizing of candidate technologies in a hypothetic hotel in China. The results suggested that an optimal configuration of DES with various technologies is more efficient than the conventional centralized energy systems as well as distributed combined cooling, heating and power systems. The proposed mathematical model of [9] intended to manage the daily operation of smart poly-generation microgrid (SPM) in the university of Genoa and results demonstrated the efficiency of the model.

It is probable that a minor perturbation in coefficients distort the results and adversely affect some of the determinant factors like Return of Investment (ROI).

In Ref. [10], a two-stage stochastic programming model is implemented in a hospital for the purpose of planning under both demand and supply uncertainty. A decomposition-based solution strategy and a Monte Carlo method are applied for first and second stage respectively. The outcomes illustrated the slight difference between stochastic and deterministic design and due to the computational efficiency of the deterministic one, this model is suggested to implement in the energy system planning. In Ref. [4], a fuzzy programming model is proposed for the purpose of investment planning under demand uncertainty in a hospital in Athens. The outcomes illustrated that the uncertainty of objective function could be effectively tackled by implementing fuzzy programming approach. In Ref. [11], an integrated framework consisted of mathematical programming and Monte Carlo simulation is implemented in order to manage the risk of some volatile parameters in the objective function such as fuel costs and interest rate. This study is also conducted in a hospital in Athens. In Ref. [12] the demand’s uncertainty is modeled using probabilistic theory and stochastic programming. A simulation-based optimization is implemented and the outcomes illustrated the better cost performance of FTL (following thermal load) strategy in comparison with other alternative configurations. Also, by increasing demand uncertainty, the amount of CHP and absorption chiller decreased while the capacity of the boiler and electric chiller increased. In Ref. [13], for tackling the demand’s volatility a robust optimization method is implemented. Also in Akbari et al. [5], for facing multiple uncertainties (for instance carbon emission costs, electricity prices and demand) a robust optimization method is applied.

2.2. Energy system planning in a neighborhood

This section is devoted to energy planning studies that are conducted in the scope of several buildings or a neighborhood.

### Table 1

<table>
<thead>
<tr>
<th>Author</th>
<th>Uncertainty</th>
<th>Solution approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>[13]</td>
<td></td>
<td>Robust optimization</td>
</tr>
<tr>
<td>[12]</td>
<td></td>
<td>Probabilistic theory (simulation-based optimization)</td>
</tr>
<tr>
<td>[10]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
<tr>
<td>[4]</td>
<td></td>
<td>Fuzzy programming</td>
</tr>
<tr>
<td>[14]</td>
<td></td>
<td>Fuzzy programming</td>
</tr>
<tr>
<td>[15]</td>
<td></td>
<td>Sensitivity analysis</td>
</tr>
<tr>
<td>[16]</td>
<td></td>
<td>Monte Carlo simulation and optimization</td>
</tr>
<tr>
<td>[17]</td>
<td></td>
<td>Robust optimization (Based on maximum regret rate)</td>
</tr>
<tr>
<td>[18]</td>
<td></td>
<td>Interval programming</td>
</tr>
<tr>
<td>[19]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
<tr>
<td>[20]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
<tr>
<td>[21]</td>
<td></td>
<td>Stochastic optimization and interval programming</td>
</tr>
<tr>
<td>[22]</td>
<td></td>
<td>Probabilistic programming</td>
</tr>
<tr>
<td>[23]</td>
<td></td>
<td>Fuzzy programming</td>
</tr>
<tr>
<td>[24]</td>
<td></td>
<td>Robust optimization</td>
</tr>
<tr>
<td>[25]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
<tr>
<td>[26]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
<tr>
<td>[27]</td>
<td></td>
<td>Stochastic optimization</td>
</tr>
</tbody>
</table>
To begin with, Ref. [3] did a feasibility study on the trigeneration plant of a 714-bed hospital including six complex buildings. The main results suggested that the unit sizing of technologies depend strictly on the load patterns. Mourmouis and Potolias [28] aimed to develop a multi-criteria decision-making structure for the purpose of energy planning at a regional level. The optimal amount of renewable energy resources are discovered for an island of Thassos in Greece. The results recognized wind, wind-biomass and wind-bio-PV as the most efficient combination of RESs (renewable energy resources) in the region. Moreover, the outcomes corroborated the ability of RES for satisfying the increasing power demands through an environmentally-friendly combination of the wind, biomass, and PV. In Ref. [29], some valuable advice is presented on the topic of the cooling system (for instance unit sizes, heat recovery exchanges, pipelines) for enterprises and industrial plants. Weber and Shah [30] presented a DESDOP tool based on linear programming techniques which help decision makers to find the optimal mix of technologies in an eco-town. The outcomes illustrates that the reduction of Carbon emissions are achievable up to 20% at no extra costs. In Ref. [31], a Mixed Integer Linear Programming (MILP) model was developed for integrated planning and evaluation of the DER systems and conducted in an eco-campus in Kitakyushu, Japan. The model minimizes the energy costs in a one-year window and determines which technologies should be installed and their operating schedules. Mehleri et al. [32] proposed MILP models for a neighborhood which consisted of pipeline network between nodes. In Ref. [33], a bi-level programming model is presented at a regional level for the purpose of DES network planning. Also, optimal capacity and locations of the energy suppliers as well as the transmission network are the outcomes of the model. The results of a real case study illustrated that the proposed method is effective for solving the energy network planning problems.

Although there is some research on energy system planning in a sole building (Table 1), to the best of our knowledge, there is no research in the field of regional energy system planning that consider the uncertainty parameters in several buildings. Therefore, in this paper it is decided to develop a proper mathematical modeling and then, it is tried to study the effects of inherent uncertainty of parameters on the outputs.

3. A generic mathematical model

For the purpose of optimal design, a number of mathematical models have been developed. All these models share a common structure and in what follows, it is presented [34].

Minimize $f(b, d, x_t)$

s.t. $\phi^{BC}(b, d) = 0$

$\psi^{HC}(b, d) \leq 0$

$\phi^{NC}(b, d, x_t) = 0$

$\psi^{NC}(b, d, x_t) \leq 0$

$b \in \{0, 1\}^m, d \in R^1, x_t \in R^n$

where, $f$ stands for the objective function which could consisted of costs, energy saving and etc. Binary and continuous design variables $b$ and $d$ would decide on the capacity of equipment that should be installed. There is a vector of continuous variables $x_t$ which represents quantitative decisions. Also, quality and inequality design and operational constraints are represented by $\psi$ and $\phi$ respectively.

4. Mathematical model

Various mathematical models have been proposed to address the issues related to a building’s energy system. A common superstructure and systematic relations of these systems are represented in Fig. 1. This superstructure is also used in this paper too. The inputs of these models consist of energy loads, market tariffs, and resource constraints. Typically, the unit sizes are the common outputs of these mathematical models.

The proposed mathematical model is inspired from a single building’s model which is presented in Refs. [13,5]. Then, by scrutinizing previous studies like [35,32] a new mathematical model for a neighborhood is adopted and presented here.

4.1. Objective function

The objective function of the model is to minimize present value of all costs for a regional energy system. The function is partitioned into four criteria to find the optimal investment for all buildings of the neighborhood as an integrated system. The constituent criteria of objective function (Eq. (2)) are investment costs ($C_{INV}$), operation and maintenance costs ($C_{NPV}$), Carbon emission costs ($C_{EM}$) and incomes ($INC$).

$$C_{total} = C_{INV} + C_{NPV} + C_{EM} - INC + \sum_h \left( Pen^C \times UM^C_h + Pen^H \times UM^H_h \right)$$

(2)

The last term is related to the penalty costs which are poses to...
the system when any unit of energy remains unsatisfied.

The first term in Eq. (2) is total investment costs \(C_{INV}\) and it includes all initial costs of running the system namely the cost of installing considered technologies and piping costs.

\[
C_{INV} = \sum_{D} \left( \sum_{l} c_i^l \times CC^l + A_D^PV \times CC^PV + C_{PPIP,ST}^P \times CC^ST \right. \\
\times CP_{PPIP,ST}^P + \sum_{Y} Y^C_D \times CC_{CHP} + \left. C_{PI,DD,ST}^P \times \sum_{D} \sum_{D'} \left( \text{DIST}_{D,D'} \right) \right) \\
(3)
\]

As it is stated, the system is designed for a neighborhood area and heat exchange is allowed. For this purpose, a design for the pipeline has to be devised. In the last term of Eq. (3), the \(C_{PI,DD,ST}^P\) is unit cost of a pipeline which is multiplied by the distant of two nodes. It should be noted that the calculations depend on installed pipelines. A binary variable namely \(PI_{DD,ST}^P\) would show whether there is any pipeline installed between nodes \(D\) and \(D'\).

The operation and maintenance cost of the system consists of the fixed costs of technologies, cost of consumed gas by boiler and CHPs and at last, the cost of purchasing electricity from the grid.

\[
C_{NPV} = \sum_{y} \sum_{D} \frac{1}{(1 + r)^y} \left[ \sum_{l} c_i^l \times FC^l + A_D^PV \times FC^PV + C_{PPIP,ST}^P \times FC^ST \right. \\
\times CP_{PPIP,ST}^P + \sum_{Y} Y^C_D \times FC_{CHP} + \left. \sum_{h} \sum_{m} \left( \frac{OP^H_{h,m,D}}{GE} \times CG + \frac{1 + HR_i}{HR_i} \times \frac{OP^C_{h,m,D}}{GE} \times CG + PE_{h,m,D} \times CE_{h,m} \right) \right] \\
(4)
\]

Due to the eminent enactment of emission taxes, \(C_{EM}\) is devised in the objective function to optimize the penalty of emitted Carbon to the atmosphere. Therefore, different ways of emitting Carbon should be identified and calculated. As it is shown in Eq. (5), the monetary equivalent of the released Carbon from the electrical utility, all installed the boilers and CHPs in all nodes are calculated and presented as \(C_{EM}\).

\[
C_{EM} = \sum_{y} \sum_{h} \sum_{m} \sum_{D} \left( \frac{PE_{h,m,D}}{E^P_{h,m,D}} + \frac{OP^H_{h,m,D}}{GE} + \sum_{l} \frac{1 + HR_i}{HR_i} \times \frac{OP^C_{h,m,D}}{E^C_{h,m,D}} \right) \times C \times EC \\
(1 + r)^y \\
(5)
\]

The devised energy system could also earn some money by selling the excessive electricity to the grid. Also, it could be assumed that there are different prices for the generated electricity from PV cells and CHPs.

Therefore, the endowed incentives on PV cells could be considered.

So, legislators could scheme for incentives.

\[
INC = \sum_{y} \sum_{h} \sum_{m} \sum_{D} \frac{ES^l_{h,m,D} \times Pr_{h,m}^{CHP} + ES^{PV}_{h,m,D} \times Pr_{h,m}^{PV}}{(1 + r)^y} \\
(6)
\]

4.2. Constraints

4.2.1. Energy balances

The key constraint in modeling an energy system is balancing all supply-demand relationships in all nodes. Eq. (7) guarantees that in each node, the aggregated supply of electricity should meet the demands in either form of transferring from other nodes or purchasing from utilities.

\[
OP^P_{h,m,D} + \sum_{l} \frac{OPCHP^P_{h,m,D}}{GE} + PE_{h,m,D} - \left( ES^l_{h,m,D} + ES^{PV}_{h,m,D} \right) \\
\geq Dem^E_{h,m,D} + \frac{OP^EC_{h,m,D}}{COP^EC} + A \times I_{h,m,D} \\
(7)
\]

The balance for heating flows in nodes is shown in Eq. (8). As a pipeline system and heat exchange is devised, the net transferred heat from node \(D\) to the other nodes has to be excluded from energy equilibrium of the node. Due to the distance between nodes, a part of heat may lose through the pipelines and it is assumed to be proportionate to the distance of two involved node.

\[
\beta_{D,D'} = 1 - 4 \times 10^{-5} \times \text{DIST}_{D,D'} \\
OP^B_{h,m,D} + OP^H_{h,m,D} + \sum_{l} \frac{OPCHP^H_{h,m,D}}{HR_i} + UM^H_{h,m,D} + HO_{h,m,D} \\
\geq \sum_{D} \left( \beta_{D,D'} \times TH_{h,m,D,D} - TH_{h,m,D,D'} \right) \\
\geq Dem^H_{h,m,D} + \frac{OP^HC_{h,m,D}}{COP^HC} + HI_{h,m,D} \\
\quad \geq Dem^C_{h,m,D} \\
(8)
\]

The cooling load could be satisfied by either absorption or electric chiller and Eq. (9) would assure that.

4.2.2. Equipment’s capacity

Eq. (10) guarantee the balance of heat storage tank. Theoretically, the storage of tank in every period should be same as the storage of previous period plus net input of heat to the storage tank. Also, a percentage of previous heat storages would decay in every period.

\[
HS_{h,m,D} = HS_{h-1,m,D} + SC \times HI_{h,m,D} - \frac{HO_{h,m,D}}{SD} - SE \times HS_{h-1,m,D} \\
(10)
\]

A fraction of storage capacity could be charged or discharged which is shown in Eq. (11).
Photovoltaic and solar thermal cells. In boilers, there needs to be a limitation on minimum installed capacity ($Cap^B_{MIN}$). Therefore, a binary variable ($Y_B$) would decide whether to install the boiler in the building $D$ or not and if it was chosen for the optimum configuration, it’s capacity has to exceed $Cap^B_{MIN}$.

$$Cap^B_{MIN} \leq Y_B \leq \bar{C}^B_B$$

The operation level for every equipment should not exceed the installed capacity (Eqs. (13) and (14)) Eq. (13) defines the upper bound on CHP units, boilers, chillers, and storage tank.

$$OPCHP^B_{h,m,D} \leq Y_C^D \times C_{CHP}$$

$$OPD^AC_{h,m,D} \leq C^D_{AC} \times CF$$

$$OPD^EC_{h,m,D} \leq C^D_{EC} \times CF$$

$$HS_{h,m,D} \leq C_{HBT}$$

$$OPB_{h,m,D} \leq C_{B}^B$$

Eq. (14) illustrates the admissible bound of operation for both Photovoltaic and solar thermal cells.

$$OPPV^D_{h,m,D} \leq A^D_{PV} \times I_t^h \times E^{PV}$$

$$OPST^D_{h,m,D} \leq A^D_{ST} \times I_t^h \times E^{ST}$$

Eq. (15) assures to install at most one CHP unit in each node.

$$\sum_i Y_C^D \leq 1$$

4.2.3. Electricity selling restrictions

There are interactions between nodes and grid. The variable $ES$ stands for electricity sold to the grid and meanwhile, the variable $ET$ represents transferred electricity to the other nodes. Eq. (16) guarantees that the amount of sold and transferred electricity is not allowed to surpass the produced amount.

$$ES^{PV}_{h,m,D} + \sum_{D} ET^{PV}_{h,m,D,D} \leq OPV^D_{h,m,D}$$

$$ES^{ST}_{h,m,D} + \sum_{D} ET^{ST}_{h,m,D,D} \leq OPCHP^D_{h,m,D}$$

A majority of electricity markets in developed countries started to be restructured and there need to be some private investors to change this oligopolistic market. Legislators endowed incentives in order to support this process and made the market more appealing for the private sector. For instance, enacting different prices for not only each hour in a day in each season, but also for each technology may mutually benefit both government and blossoming business of solar panels. Unfortunately, the disparity between purchasing and selling the electricity may deflect the model to buy from the grid and sell it back. Therefore, a variable $X_{h,m,D}$ devised to know whether a node traded any electricity or not.

$$ES^P_{h,m,D} + ES^V_{h,m,D} \leq M \times X_{h,m,D}$$

$$PE_{h,m,D} \leq Dem^F_{h,m,D} \times (1 - X_{h,m,D})$$

4.2.4. Heating pipeline network

The proposed model of an energy system in the neighborhood consists of several buildings that can transfer both electricity and heat. Therefore, there need to be some logical constraints on network. The installing of pipelines are prior to any heat transfers. According to Eq. (18), the heat could be transferred unless any pipeline installed between nodes $D$ and $D'$. Also, Eq. (19) depicts that the heat has to be transferred in all installed pipelines only in one direction. Therefore, the network is a directed graph.

$$TH_{h,m,D,D'} \leq N \times PIP_{D,D'}$$

$$\sum D PIP_{D,D} \leq 1$$

The main challenge of network design for heat exchange scheme is to control the heat circulation in the pipelines. The heat circulation may be caused by various reasons but it all ends to the erroneous results. For instance, it may stimulate the system to produce and sell more electricity and circulate the produced heat in the endless loops until the heat either be consumed or gradually diminished. These everlasting heat circulations would increase the pipelines’ rate of depreciation. Therefore, eliminating subtours seems inevitable.

For the purpose of subtour elimination, there is two sets of constraints. The first set is known as DFJ (Dantzig, Fulkerson and Johnson-1954) while the second set is known as MTZ (Miller, Tucker and Zelman). The following Eq. (21) is motivated by the Traveling Salesman Problem (TSP) and is known as MTZ subtour elimination constraint.

$$U_{D} \geq U_{D} + 1 - |D| \times (1 - PIP_{D,D}) \quad \forall D, D' \neq D$$

In Eq. (21), the positive variable $U$ stands for visiting order of nodes while binary variable $PIP_{D,D}$ illustrates whether to install pipeline between $D$ and $D$ or not. Eq. (21) assures that there is no subtour in the designed network.

5. Robust optimization model

Planning of energy systems by means of mathematical modeling have had a long history. Since the high sensitivity of results to a minor data perturbation is illustrated in Ref. [36], ignoring these perturbations make the outcomes unreliable. These innate uncertainties stem from imprecisions in the process of data gathering, measurement errors, forecasting and so on. In the planning of building’s energy system, the resource allocation for an unpredictable long-run window looks risky. Besides, data insufficiency would also adversely affect the results and consequently, misinform the decision makers. Numerous approaches have been presented for tackling the consequences of these uncertainties namely fuzzy programming, stochastic programming, robust optimization and sensitivity analysis. The first two approaches are mostly used in the literature of building’s energy planning.

The classification of fuzzy programming can commonly be conducted to main subgroups: flexible programming and possibilistic programming [37]. Flexible programming usually deals with decision-making problem under the flexibility of the target values of objective functions and the elasticity of constraints; while the latter one (i.e., possibilistic programming) deals with ambiguous coefficients of objective functions and constraints that are usually
modeled considering available objective data and subjective knowledge of the decision maker (for further information regarding the classification see Refs. [38,39]). In the literature of building’s energy system, some studies like [4,11,14] used fuzzy programming approach for the purpose of facing the innate uncertainties of the parameters.

Robust optimization is one of the most important methodologies for tackling the optimization problems resulting from data uncertainty. These methods consist of two stages. In the first stage a set of input data is defined within its uncertain space. Then, in the second stage an optimal yet always feasible solution (for any realization of the uncertain data in the given set) is obtained. In the literature, the second stage is also known as robust counterpart optimization[40]. First, consider the following model is subjected to the data uncertainties.

\[
\text{Max } c x \\
\text{Subject to: } \sum_{j} \tilde{a}_{ij} x_j \leq b_i, \quad \forall i
\]

In the problem 22, \( \tilde{a}_{ij} \) stands for true value of parameters in the left hand side. Assume that left-hand side (LHS) parameters in the constraints are affected by uncertain environment and all the constraints are affected independently. The uncertainty of each parameter could be defined as follows:

\[
\tilde{a}_{ij} = a_{ij} + \xi_{ij} \bar{a}_{ij}, \quad \forall j \in J_i
\]

In Eq. (23), the nominal value of LHSs are represented by \( a_{ij} \) while \( \bar{a}_{ij} \) which is positive parameter stands for constant perturbation. Also, \( J_i \) is set of LHS coefficients \( (a_{ij}) \) in row \( i \) that are subject to the parameter uncertainty. The independent uncertain parameters \( \xi_{ij} \) are also distributed in the interval \([-1,1]\). With regarding to Eq. (23), the constraint of problem 22 could be reformulated as follows:

\[
\sum_{j \in J_i} a_{ij} x_j + \sum_{j \in J_i} \tilde{a}_{ij} x_j \leq b_i
\]

The Eq. (24) could be rewritten as:

\[
\sum_{j} a_{ij} x_j + \left\lbrack \sum_{j \in J_i} \xi_{ij} \bar{a}_{ij} x_j \right\rbrack \leq b_i
\]

In set-induced robust optimization method, it is aimed to find feasible solutions for any \( \xi \) in the predefined uncertainty set \( U \).

\[
\sum_{j} a_{ij} x_j + \max \{ \bar{a} x_j \} \leq b_i
\]

By replacing Eq. (26) in the given problem 22 robust counterpart model, which is immunized with very high probability to infeasibility, can be obtained.

For elimination of inner optimization problem in Eq. (26), the optimization part is transformed to its conic dual problem and then by incorporation of the dual problem into the original constraint, the equivalent robust formulation is obtained. For the sake of integrity, all the proofs and procedures are presented in the [40].

Several uncertainty sets are introduced in the literature namely interval set, combined interval-ellipsoidal set and so on. In this study, the implemented uncertainty set is the same as [41,42] which is basically the intersection of the polyhedral and interval uncertainty sets.

First, the intersection of the box and polyhedral should be introduced by their norms.

\[
U_{\cap \infty} = \left\{ \xi \middle| \sum_{j \in J_i} |\xi_j| \leq \Gamma, \quad \xi \leq \Psi, \quad \forall j \in J_i \right\}
\]

With regarding the set \( U \) as a box-polyhedral uncertainty set, the corresponding constraints of obtained robust counterpart model would be the presented constraints in Eq. (28).

\[
\sum_{j \in J_i} a_{ij} x_j + \Psi \sum_{j \in J_i} w_{ij} + \Gamma z_i \leq b_i \\
z_i + w_{ij} \geq a_{ij} x_j, \quad \forall j \in J_i \\
z_i \geq 0, \quad w_{ij} \geq 0
\]

When \( \Psi = 1 \), the uncertainty set is known as interval-polyhedral set and the corresponding robust counterpart becomes the original model proposed by Refs. [41,42] (constraints 29).

It should be noted that \( |x_j| \) is replaced by an auxiliary variable \( u_j \).

\[
\sum_{j \in J_i} a_{ij} x_j + \sum_{j \in J_i} w_{ij} + \Gamma z_i \leq b_i \\
z_i + w_{ij} \geq \tilde{a}_{ij} u_j, \quad \forall j \in J_i \\
u_j \leq x_j \leq u_j, \quad \forall j \in J_i \\
z_i \geq 0, \quad w_{ij} \geq 0
\]

The over-conservativeness of Soyster’s model in Ref. [43] motivated the researchers to study more on a formulation that allows a trade-off between robustness and performance of the model, for instance, [41]. The budget of uncertainty (\( \Gamma \)) is a parameter that supervises the conservatism level in each constraint. This means based on decision makers preference, various degrees of robustness could be achieved. The parameter \( \Gamma \) could also vary in the range \([0,|J_i|]\). The case \( \Gamma = 0 \) corresponds to the deterministic model while by \( \Gamma = |J_i| \), the Soyster’s model could be achieved.

According to the described procedure, the resultant robust counterpart of the proposed model for energy system planning is as follows:

\[
\text{Minimize } C_{total} = C_{INV} + C_{NPV} + C_{EM} - \text{INC} + \sum_{h} \left( Pen^C \times U_{MC}^C + \text{Pen}^H \times U_{MC}^H \right)
\]

\[
0 \leq h \leq 31
\]

Subject to :

\[
\begin{align*}
\text{OPPV}_{h,m,D} & + \sum_{h} \text{OPCHP}_{h,m,D} + P_{h,m,D} - \left( E_{h,m,D}^I + E_{h,m,D}^P \right) - \sum_{D} \left( \left( E_{h,m,D,D}^I + E_{h,m,D,D}^P \right) - \left( E_{h,m,D,D}^I + E_{h,m,D,D}^P \right) \right) \\
\text{Dem}_{h,m,D} - \frac{\text{OPPE}_{h,m,D}}{\text{COP}_{EC}} & - A \times t_{h,m,D} - Z_{1,h,m,D} \times \Gamma_1 - W_{1,h,m,D} \geq 0
\end{align*}
\]
Z_{1,h,m,D} \times \Gamma_1 + W_{1,h,m,D} \geq D\text{em}^E_{h,m,D} \tag{32}

\begin{align*}
\text{Op}^E_{h,m,D} + \text{Op}^ST_{h,m,D} + \sum_{i} \left( \frac{\text{OpCHP}_i^E_{h,m,D}}{\text{HR}_i} + \text{Um}^E_{h,m,D} + \text{Ho}_{h,m,D} \right) \\
+ \sum_{D} \left( \delta_{D,D} \times \text{Th}_{h,m,D,D} - \text{Th}_{h,m,D,D} \right) \\
\geq D\text{em}^H_{h,m,D} + \frac{\text{OpAC}^E_{h,m,D}}{\text{COP}^E} + \text{Hi}_{h,m,D} - Z_2,h,m,D \times \Gamma_2 - W_2,h,m,D
\end{align*} \tag{33}

\begin{align*}
\text{Op}_{h,m,D} + \text{Op}^EC_{h,m,D} + \text{Um}^C_{h,m,D} - \text{Dem}^C_{h,m,D} - Z_3,h,m,D \times \Gamma_3 \\
- W_3,h,m,D \\
\geq 0
\end{align*} \tag{35}

Z_{3,h,m,D} \times \Gamma_3 + W_{3,h,m,D} \geq D\text{em}^C_{h,m,D} \tag{36}

and Eq. (3)–(6) and (10)–(21).

In this study, the uncertainty level of parameters are considered to be 20% which it means \( \alpha_{ij} = 0.2 \times \bar{\alpha}_{ij} \).

### 6. Computational results

The presented study is focused on a neighborhood consisted of 4 and 5 houses. In these buildings, different DER technologies like gas turbines, Stirling engines, solar thermal collectors, Photovoltaic cells, chillers, boiler and heat storage are considered in order to satisfy the electrical, heating and cooling demand of a neighborhood. Also, heating and electricity transmission lines are devised. All the energy demands, tariffs and technical characteristics of the DER technologies are described in further sections.

#### 6.1. Energy loads

The loads are derived from the example proposed in Ref. [35] and they are attached to every node in the neighborhood. To define these profiles, three seasons are considered in a year namely winter, summer and mid-season. As well as seasons, each day is also divided into 6 periods. The heat loads are consisted of space heating demand so in the summer, it is assumed to be zero. Also, cooling demand is also considered to be zero in winter and mid-season.

#### 6.2. Electricity and gas tariffs

The other class of inputs which are considered as important as the energy loads are market data such as fuel tariff rate. It seems any changes in these rates as policy making would substantially alter the profitability of these systems. In the following research, the cost of gas and emission tax are considered 0.072$/m^3$ and 0.01$/kgCO_2$ respectively [5,13].

#### 6.3. DER technology characteristics

The characteristics of DER technologies are listed in Table 2. Meanwhile, the discount rate is assumed to be 10% while the planning horizon is considered to be 16 years. The candidate CHP

---

### Table 2 - DER characteristics.

<table>
<thead>
<tr>
<th>Technologies</th>
<th>Cost item</th>
<th>Value</th>
<th>Technologies</th>
<th>Cost item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV units</td>
<td>Capital cost (CC^PV) ($/kWp)</td>
<td>3000</td>
<td>Absorption chiller</td>
<td>Capital cost (CC^AC) ($/kWp)</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>Op &amp; M cost (FC^PV) ($/kWp/year)</td>
<td>16.47</td>
<td></td>
<td>COP (AC)</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Efficiency (E^{PV})</td>
<td>0.12</td>
<td></td>
<td>Electricity required for AC (A) (kW/RT)</td>
<td>0.2</td>
</tr>
<tr>
<td>Boiler</td>
<td>Rated capacity (CP_{rz}) (kWp/m²)</td>
<td>0.15</td>
<td>Heat storage tank</td>
<td>Capital cost (CC^HP) ($/kWp)</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Capital cost (CC^B) ($/kW)</td>
<td>100</td>
<td></td>
<td>Op &amp; M cost (FC^HP) ($/kWp)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Efficiency (E^{B})</td>
<td>0.8</td>
<td></td>
<td>Charging efficiency</td>
<td>0.9</td>
</tr>
<tr>
<td>Solar thermal</td>
<td>Capital cost (CC^ST) ($/kWp)</td>
<td>800</td>
<td></td>
<td>Discharging efficiency</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Op &amp; M cost (FC^ST) ($/kWp/year)</td>
<td>6</td>
<td></td>
<td>Efficiency decay</td>
<td>0.01</td>
</tr>
<tr>
<td>Electric chiller</td>
<td>Capital cost (CC^E) ($/kWp)</td>
<td>700</td>
<td></td>
<td>Maximum and minimum charging and discharging</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>COP</td>
<td>3.5</td>
<td></td>
<td>CO₂ emitted</td>
<td>0.781</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Gas</td>
<td>0.072</td>
</tr>
</tbody>
</table>

---

![Fig. 2. Location of five nodes.](image-url)
technologies are also presented in Table 3. As it is discussed in Section 4.2.2, the boilers need a lower bound which is set to 5 kWh and the upper bound of PV arrays and solar thermal collectors are set at 100 m².

7. Results and discussion

All the deterministic models and their robust counterparts are solved by CPLEX solver of the GAMS to their optimality. In Sections 7.1 and 7.2, two numerical experiment of the proposed model in a neighborhood are presented and the effects of uncertainties are discussed thoroughly.

It should be noted that in Section 5 the conservatism level is considered to be in the interval of $\Gamma \in [0, |\gamma|]$. In this study, the $\Gamma$ is divided into four value and every value of $\Gamma$ would represents a group. The "Deter" stands for $\Gamma = 0$ which is introduced before. The $\Gamma = 0.2 \times |\gamma|$ known as low-conservativeness while $\Gamma = 0.5 \times |\gamma|$ represent medium-conservativeness. The $\Gamma = |\gamma|$ equalize the robust counterpart to the Soyster’s model which is known for its over-conservativeness.

7.1. The application of model for 4 buildings

The proposed mathematical model and its robust counterpart is implemented in a neighborhood consisted of 4 dwellings (nodes 1 to 4 in Fig. 2).
7.1. Unit sizes of technologies

All the proposed deterministic and its robust counterpart are solved under nominal data and the technologies’ unit sizes are presented in this section. To begin with, in Figs. 3–6 unit sizes of every technology are presented in all dwellings under different values of conservatism level. Fig. 3 depicts installed technologies in node 1. By rising levels of conservativeness, there is an increasing yet smooth trend in implementation of chillers because cooling energy is assumed to be untransferable. In medium-conservativeness and low-conservativeness, application of both chillers are suggested for the building. In the heating sector, boilers are the chosen technology for the satisfaction of heating demand and meanwhile the PV cells, transferred and purchased electricity are devised for meeting the electricity demand. In building one, CHPs remains non-optimal.

In buildings two to four (Figs. 4–6), by increasing amount of conservativeness level the trend of chillers remains the same but due to the heating exchange system, the size of installed boilers fluctuate arbitrarily. Moreover, in full-conservativeness level, the heat exchange system supplies the heating demand of the fourth building. It should be noted that in second and fourth building, application of a Stirling engine is recommended.

7.1.2. Validation test

To assess the performance of obtained solutions from both deterministic and robust models, ten random realizations are generated uniformly in the demand’s uncertainty set (i.e., $\sim [Dem_{m,D}^C - Dem_{m,D}^C, Dem_{m,D}^C + Dem_{m,D}^C]$). Changing the tactical decisions like the operation levels are permissible while changing strategic decisions like installed levels of candidate
technologies are not allowed in the models and then, the models are solved. For the purpose of performance evaluation, some measures are used. First two measures are mean and standard deviation of the objective function under random realizations. Besides, the mean percentage of the unmet demands and purchased electricity are also calculated by Eq. (37) and later, the outcomes are presented in Table 4 as other measures.

Unmet heating demand = \[
\sum_{h} \sum_{m} \sum_{D} \frac{UM_{h,m,D}^{H}}{Dem_{h,m,D}^{H} + \left(\frac{OP_{h,m,D}^{AC}}{cop_{h,m,D}^{EC}}\right) + AC \times t_{h,m,D}}
\]  
(38)

Purchased electricity = \[
\sum_{h} \sum_{m} \sum_{D} \frac{PE_{h,m,D}}{Dem_{h,m,D}^{E} + \left(\frac{OP_{h,m,D}^{EC}}{cop_{h,m,D}^{EC}}\right) + AC \times t_{h,m,D}}
\]  
(37)
Unmet cooling demand = \sum_{h} \sum_{m} \sum_{D} \frac{UMC_{h,m,D}}{D_{h,m,D}} \quad (39)

As it was expected, the mean value of the objective function in the deterministic model gains the best value in comparison to different conservatism level of the robust model. Also, the

Table 4  
Summary of test results under demand uncertainty in application of four houses.

<table>
<thead>
<tr>
<th>Conservatism level</th>
<th>Mean objective function value under realizations ($)</th>
<th>Standard deviation of objective function value under realizations ($)</th>
<th>Mean percentage of unmet demands (heat – cool) (%)</th>
<th>Mean percentage of purchased electricity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deterministic</td>
<td>Robust</td>
<td>Deterministic</td>
<td>Robust</td>
</tr>
<tr>
<td>0.2</td>
<td>14,729.09</td>
<td>15,518.52 (5.36%)</td>
<td>314.775</td>
<td>177.051 (5.36%)</td>
</tr>
<tr>
<td>0.5</td>
<td>15,656.83 (6.3%)</td>
<td>100.539 (68.1%)</td>
<td>1.965 (99.38%)</td>
<td>(0–0) (-100% – -100%)</td>
</tr>
<tr>
<td>1</td>
<td>17,857.86 (21.24%)</td>
<td>1.965 (99.38%)</td>
<td>(0–0) (-100% – -100%)</td>
<td>15.05</td>
</tr>
</tbody>
</table>
The difference of objective functions in robust and deterministic models are presented in the parenthesis. The increasing trend in the objective functions’ values are obvious. The standard deviation of objective function in realizations are also demonstrated in Table 4. The deterministic model acquired the worst value in comparison with robust models. In the full-conservativeness mode, the standard deviation of objective functions is improved up to 99.34% in compared to its deterministic model.

The other important measure in energy systems is the mean percentage of unmet demands which is calculated by Eqs. (38) and (39) for unmet heating and cooling demand respectively. The robust models reduced the percentage of unmet demands and in the full-conservativeness mode, the proportion of these unmet demands is eliminated thoroughly.

In the proposed energy system, purchasing electricity from the national grid is allowed. Therefore, it seems it is a good measure for comparing the level of dependency of different energy systems to the grid. The percentile of purchased electricity from the grid is calculated by Eq. (37). The results demonstrated an increase of purchased electricity in low and medium conservativeness while in full-conservativeness, it seems electricity production is preferred to purchasing. In robust models with increasing level of conservativeness, there is a decrease in dependency on electricity of the grid.

7.1.3. Heating transmission network

The placement of all dwellings in the neighborhood are presented in Fig. 2 (nodes 1 to 4 for the first test).

In Figs. 7 and 8, the proposed heating pipeline of the deterministic and robust model are presented respectively. As it is obvious from Fig. 8, in low and medium conservativeness, there is a tendency to implement pipeline between dwellings while in full-conservativeness the level of this tendency decreased and the pipeline is not proposed in the optimum solution.

Also, the amount of transferred heat is as important as the shape of the network. Due to the importance of the transferred heat by the network, in Fig. 8 the amount of transferred heat in a year is illustrated. The outcomes suggest that in an uncertain environment with increasing level of conservativeness, there is a decreasing tendency toward implementing the pipeline network in the neighborhood.

A sensitivity analysis is also done on the value of carbon emission cost in the robust model with the low level of conservativeness. By increasing this value, expectedly the proposed amount of Photovoltaic arrays and solar-thermal cells are increased. In the cooling sector, the higher amount of absorption chiller and lower amount of electric chiller are suggested.

7.2. The application of model for 5 buildings

In this section, the proposed robust model is implemented in a neighborhood consisted of 5 buildings (Fig. 2).

7.2.1. Unit sizes of technologies

The proposed unit sizes of the robust model are presented under different level of conservatism. Due to the resemblance of analysis in the both neighborhoods, the unit sizes of just two buildings (out of five) are presented in Figs. 9 and 10.

7.2.2. Validation test

The performance assessments are also the same like the previous application of the model for four buildings. Ten random realizations are generated based on demands' uncertainty set. Then, the strategic decisions of both deterministic and robust solutions are compared under demands uncertainty. Also, the percentage of purchased electricity, unmet heating, and cooling demand are calculated by Eqs. (37)–(39) respectively. All the outcomes are presented in Table 5.

While the mean value of the deterministic model’s objective function gains the best mean in comparison with other mean values, its standard deviation is improved in robust models up to 51%. The mean percentage of the unmet demands are also decreased up to 79% and 95% for heating and cooling demand respectively. The other beneficial measure is the mean percentage of purchased electricity which it seems under low and medium conservativeness level, the proposed energy system relies more on the grid while in high-conservativeness level it is reduced about 50% in comparison with the deterministic model.

It should be noted that level of conservatives is a parameter which should be made by the decision makers. Therefore, they
need to have an accurate trade-off between the proposed measures.

7.2.3. Heating transmission network

All the five dwellings are presented in Fig. 2. In the deterministic model, the heating transmission network is not preferred and all the demands are met by the buildings’ own energy production. Same as the highlighted trends in Section 7.1.3, under low level of conservativeness, the system relies more on implementing pipeline while by increasing the conservativeness level, this dependency is decreased (see Fig. 11).

The dependency of the system on the pipeline network is decreased in the robust model. The pipeline network and the transmitted heat is represented in Fig. 12.

8. Conclusion

This research is mainly focused on developing an energy system for a neighborhood while the inputs (demands) are affected by uncertainty. First, a mixed-integer optimization model is developed for investment planning of a neighborhood. Then, through reformulating the proposed deterministic model to its tractable robust counterpart, the effects of uncertainty on the outcomes are studied.

The results on unit sizes in both applications revealed that by increasing conservativeness of decision makers, an increase in unit sizes are suggested. Due to the energy exchange in heating and electricity sector, this increase in unit sizes is not as smooth as the cooling sector. Therefore, when heating and electricity demands
Fig. 10. Unit sizes of installed technologies in the last building under different level of conservatism - application of 5 buildings.

Fig. 11. The obtained heating transmission network from the robust model under low (left hand) and medium (right hand) levels of conservativeness.
increases, decision makers could not guess the unit sizes linearly and an optimization problem is needed to be solved.

The outcomes of the validation tests are analyzed and compared by few measures namely mean and standard deviation of the objective function, unmet demands and percentage of the purchased electricity from the national grid. The results of validation test in Tables 4 and 5 reveal an increase in the mean of objective function value while an improvement is observed in the value of standard deviation. Moreover, the percentage of unmet demands decreases. In the application of four houses (Table 4) the value of unmet demands is reduced almost 79% and 95% for heating and cooling demand respectively.

The heat exchange is also schemed in the model. As a result, the nodes could easily produce and transfer heat to other nodes via a network. The outcomes on the network design reveal that the nodes could easily produce and transfer heat to other nodes via a network. The obtained heating transmission network from the robust model under high levels of conservativeness.

![Diagram of heating transmission network](image)

**Fig. 12.** The obtained heating transmission network from the robust model under high levels of conservativeness.

![Diagram of heating transmission network](image)

**Table 5** Summary of test results under demand uncertainty in application of five houses.

<table>
<thead>
<tr>
<th>Conservatism level</th>
<th>Mean objective function value under realizations ($)</th>
<th>Standard deviation of objective function value under realizations ($)</th>
<th>Mean percentage of unmet demands (heat−cool) (%)</th>
<th>Mean percentage of purchased electricity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deterministic</td>
<td>Robust</td>
<td>Deterministic</td>
<td>Robust</td>
</tr>
<tr>
<td>0.2</td>
<td>13,063</td>
<td>17,346 (32.8%)</td>
<td>283,916</td>
<td>214,704 (−24.4%)</td>
</tr>
<tr>
<td>0.5</td>
<td>18,658</td>
<td>208,838 (−26.4%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20,012</td>
<td>137,72 (−51.5%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Nomenclature**

**Parameters**

- $A$: Electrical load for absorption chiller, kW/RT
- $C$: CO$_2$ emission per kWh of gas consumption, Ton/kWh
- $C^i_{CHP}$: Capital investment cost for technology type $i$, $/kW$
- $CC^i$: Capital investment cost for CHP type $i$, $/kW$
- $CC^P$: Capital investment cost for PV arrays, $/kW$
- $C_{PV}$: Rated capacity for PV arrays, kWp/m2
- $CP_{CHP}$: Rated capacity for solar thermal cells, kWp/m2
- $CP_{ST}$: Capital investment cost for solar thermal cells, $/kW$
- $Cap^B_{MIN}$: Minimum capacity for boiler
- $CE_{h}$: Utility charge for electricity in hour $h$ of season $m$, $/kWh$
- $CF$: Conversion factor for RT to kW, kW/RT
- $CG$: Unit cost of gas, $/m3$
- $COP_{AC}$: COP for absorption chiller
- $COP_{FC}$: COP for electric chiller
- $Dem_{h,m}$: Nominal electrical demand for hour $h$ of season $m$ in building $D$, kWh
- $Dem_{h,m,1}$: Nominal heating demand, kWh
- $Dem_{h,m,2}$: Nominal cooling demand, kWh
- $E$: Efficiency
- $EC$: Cost of carbon emission per kg, $/kg$
- $FC$: Fixed cost of technologies
- $GE$: Energy of gas in cubic meter of gas, kWh/m3
- $HR_{i}$: Heat to power ratio for CHP type $i$
- $I_{T}$: Irradiance at each hour
- $M$: Big M
- $Pr_{CHP}$: Selling price of produced electricity by the installed CHP to the grid
- $Pr_{PV}$: Selling price of produced electricity by PV to the grid
- $R$: Yearly discount rate, percent
- $SC$: fraction of heat sent to thermal storage that is not lost (charging efficiency), (%)
- $SD$: fraction of discharged heat that is not lost (discharging efficiency), (%)
- $SE$: fraction of stored heat that is lost in each hour (efficiency decay), (%)
- $SMC$: maximum fraction of thermal storage capacity that can be charged in each hour, (%)
- $SMD$: maximum fraction of thermal storage capacity that can be discharged in each hour, (%)
- $Pen^H$, $Pen^C$: Penalty cost for unmet heating and cooling demands, $$/kWh$
- $\beta_{D,r}$: Heat loss due to transmission between nodes

**Binary variables**

- $y_{D,i}^B$: 1 if boiler is selected for the optimal configuration of building $D$
\[ X_{\text{h,m,D}} \]

1 in period \( h \) of season \( m \) if any electricity sold to the grid from building \( D \)

\[ Y_{\text{C,D}} \]

1 if CHP type \( i \) should be installed in the building \( D \)

\[ P_{\text{P,D}} \]

1 if any pipes should be installed between node \( D \) and \( D' \)

**Continuous variables**

\[ A_{\text{PV,m}} \]

Installed surface of PV panel, m²

\[ A_{\text{ST,m}} \]

Installed surface of ST panel, m²

\[ C_{\text{t}} \]

Installed capacity of technology \( L \)

\[ C_{\text{Total}} \]

Objective function, $

\[ C_{\text{INV}} \]

Total investment cost, $

\[ C_{\text{NPV}} \]

Net present value of costs, $

\[ C_{\text{EM}} \]

Total cost of emission, $

\[ E_{\text{S,h,m,D}} \]

Electricity sold to the grid, kWh

\[ E_{\text{T,h,m,D}} \]

Transferred electricity from node \( D \) to \( D' \)

\[ H_{\text{S,h,m,D}} \]

Thermal storage input, kWh

\[ H_{\text{O,h,m,D}} \]

Thermal storage output, kWh

\[ H_{\text{S,h,m,D}} \]

Heat stored in thermal storage, kWh

\[ H_{\text{O,h,m,D}} \]

Heat stored in thermal storage, kWh

\[ \text{OP}_{\text{C,H,m,D}} \]

Operation of CHP type \( i \), kWh

\[ \text{OP}_{\text{P,V,m,D}} \]

Operation of PV, kWh

\[ \text{OP}_{\text{S,T,m,D}} \]

Operation of ST, kWh

\[ P_{\text{E,h,m,D}} \]

Purchased electricity from national grid (kWh)

\[ U_{\text{M,h,m,D}} \]

Unmet demands for cooling and heating kWh

\[ U_{\text{M,h,m,D}} \]

Unmet demands for cooling and heating kWh

\[ h \]

Hours

\( m \)

Seasons

\( H \)

Horizons

\( D \)

Nodes

\( L \)

Technologies (Absorption Chiller, Electric Chiller, Boiler, Heat Buffer Tank)

\( i \)

Different kind of CHPs

**References**


