Fresnel diffraction from a step in the general case

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Recently, Fresnel diffraction from phase steps with parallel plates has been studied in detail, and the subject has led to many interesting metrological applications. In this report we formulate Fresnel diffraction from a physical step with arbitrarily oriented plates in reflection mode. We simulate the diffraction patterns for different orientations of the plates and develop the required procedure for determining the involved angles by analysis of the diffraction pattern. In the experimental part of the report we arrange a setup to form diffraction patterns in different orientations of the step plates and test the derived formulations. Also, we briefly review the application potentials of the subject. © 2017 Optical Society of America

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1. INTRODUCTION

The familiar Fresnel diffraction occurs as the passage of a coherent beam of light is partially obstructed by an opaque object. In this process the amplitude of the optical wave experiences a sharp change at the boundary between the light field and the object. However, it is also observed that a sharp change in the phase of a part of an optical field leads to significant Fresnel diffraction. A sharp change in phase occurs as a coherent beam of light reflects from a physical step or passes through the boundary region of two transparent media with different refractive indices. Recently, this subject has been studied in some detail [1–11]. Since the phase difference between the waves diffracted from the two parts of the phase step can be varied very smoothly by changing the light incident angle, and the visibility of the diffraction fringes varies sharply by the change of phase difference, the effect has led to many interesting applications, namely, in the precise measurements of film thickness [12–14], refractive indices of solids and liquids [15–22], nanometer displacements [23,24], wavelength [25], diffusion coefficient [8,26], coherence length and spectral line shape [27,28], etching rate [29], and focal lengths of lenses [30]. The step used in the cited reports mainly consisted of two parallel rectangular plates. In this report we formulate Fresnel diffraction from a step with two arbitrarily oriented plates. By “arbitrarily oriented,” we mean one of the plates is rotated by small angles around two axes located in the plane of the other plate, one perpendicular and the other parallel to the step edge, as shown in Fig. 1(d). We simulate the intensity distributions on diffraction patterns for different orientations of the step’s plates. Also we develop the procedure for specifying the involved angles by analysis of the diffraction pattern. In the experimental part, using a step with a changeable position of the plates, we produce the diffraction patterns for different orientations of the plates and test the derived formulations. Also, we discuss the application potentials of the subject.

2. THEORETICAL APPROACH

In Fig. 1 the experimental diffraction patterns for different orientations of the step’s plates are illustrated. The diffraction pattern in Fig. 1(a) belongs to a step with parallel plates. The pattern in Fig. 1(b) is produced by plates making a wedge. The orientation of the plates in Fig. 1(c) resembles the Fresnel’s double mirror, and the pattern in Fig. 1(d) associates to a step with an arbitrary orientation of the plates.

In Fig. 2 the light emitted from a point source S, located in the x–z plane of the coordinate system, strikes the plates of the step. We want to calculate the complex amplitude of the wave diffracted from the plates to an arbitrary point P on the ray reflected from point M. Considering Fig. 2, the amplitude \( U(P) \) at point \( P \) resulting from light diffraction over the plates can be given by the following integral:

\[
U(P) = KD \left[ r_L \int_{-\infty}^{0} dx \int_{-\infty}^{\infty} \frac{e^{ik(r_1 + r')}}{r_1 r'} dy \right. \\
+ r_R \int_{0}^{\infty} dx \int_{-\infty}^{\infty} \frac{e^{ik(r_1 + r')}}{r_2 r'} dy \right],
\]

where \( K, D, r_L, r_R, k \) stand for the propagation factor, incident amplitude, reflection coefficients of the left and the
Appendix A and using the Fresnel approximation, we arrive at the step equation:

\[
\begin{align*}
X_1 &= x_1 - x_0 \\
Y_1 &= y_1 - y_0
\end{align*}
\]

\[
\begin{align*}
X_2 &= (x_2 - x_0) - X_0 \\
Y_2 &= (y_2 - y_0) - Y_0
\end{align*}
\]

\[
\varphi = k\Delta = 2k\frac{R_0}{v_0} \cos \theta + (\tan \beta \cos \theta)x_0 + (\tan \alpha \cos \theta)y_0 + (\tan \beta \cos \theta)^2/2\gamma + (\tan \alpha \cos \theta)^2/2\gamma
\]

We should mention that in Eq. (2) it is assumed that the amplitude does not change appreciably with the changes in \(r_1\), \(r_1'\), \(r_2\), and \(r_2'\). Also, for parallel illumination of the step we have \(\gamma = 1/(2R')\).

Now, we change the variables as follows:

\[
\begin{align*}
kyX_1^2 &= \frac{\pi x_1}{2} \\
kyY_1^2 &= \frac{\pi y_1}{2}
\end{align*}
\]

\[
\begin{align*}
kyX_2^2 &= \frac{\pi x_2}{2} \\
kyY_2^2 &= \frac{\pi y_2}{2}
\end{align*}
\]

and represent

\[
\begin{align*}
V &= -\sqrt{\frac{2k}{\pi}}x_0 \\
V_0 &= \sqrt{\frac{2k}{\pi}}X_0 = \sqrt{\frac{2k}{\pi}}\tan \beta \cos \theta
\end{align*}
\]

We substitute the new variables in Eq. (2) to get

\[
\begin{align*}
U(P) &= KD \frac{e^{i\Delta(r+R')}}{RR'} \int_{-\pi}^{\pi} r_L \pi V e^{i\frac{\pi}{2}r_1} dV_1 \int_{-\infty}^{\infty} e^{i\frac{\pi}{2}r_2} dV_2 \\
&+ r_L e^{-i\varphi} \int_{-\pi}^{\pi} r_L \pi V e^{i\frac{\pi}{2}r_1} dV_1 \int_{-\infty}^{\infty} e^{i\frac{\pi}{2}r_2} dV_2
\end{align*}
\]

or

\[
U(P) = KD \frac{e^{i\Delta(r+R')}}{RR'} \frac{\pi}{2} (1 + i) \\
\times \left[ r_L \int_{-\pi}^{\pi} e^{i\frac{\pi}{2}r_1} dV_1 + r_L e^{-i\varphi} \int_{-\pi}^{\pi} e^{i\frac{\pi}{2}r_1} dV_2 \right]
\]

For \(r_L = r_R\) we represent the Fresnel integral as follows:

\[
\int_{0}^{V} e^{i\frac{\pi}{2}r_1} dV = C(V) + iS(V)
\]

and recalling that

\[
\begin{align*}
C(-\infty) &= -\frac{1}{2} \\
C(0) &= 0 \\
C(\infty) &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
S(-\infty) &= -\frac{1}{2} \\
S(0) &= 0 \\
S(\infty) &= \frac{1}{2}
\end{align*}
\]

we can express the complex amplitude as

\[
U(P) = \left(1 + i\frac{1}{2}\right) U_0\{A + iB\}
\]

where

\[
U_0 = KD \frac{e^{i\Delta(r+R')}}{R' \pi r_L}
\]

\[
= K D e^{i\Delta(r+R')} R' \pi r_L
\]
The corresponding intensity is

\[ I(P) = \frac{1}{2}I_0(A^2 + B^2), \]  

(17)

where

\[ I_0 = U_0U_0^*. \]  

(18)

Using the following abbreviations,

\[ W = -\sqrt{2k\gamma/\pi y_0}, \quad W_0 = \sqrt{2k\gamma/\pi}, \]  

(19)

we separate the phases that depend on \( h_0 \), angle \( \beta \), and angle \( \alpha \):

\[ \varphi = (4\pi/\lambda)h_0 \cos \theta - \pi(VV_0 - V_0^2/2) \]  

(20)

Substituting \( A \) and \( B \) from Eq. (16) into Eq. (17) the normalized intensity, \( I_\alpha = I/I_0 \), can be expressed as follows:

\[ I_\alpha = I_1 + I_2 \cos \varphi + I_3 \sin \varphi, \]  

(21)

\[ \begin{aligned}
I_1 &= \frac{1}{2} \left\{ \left[ \frac{1}{2} + C(V) \right]^2 + \left[ \frac{1}{2} + S(V) \right]^2 + \left[ \frac{1}{2} - C(V - V_0) \right]^2 + \left[ \frac{1}{2} - S(V - V_0) \right]^2 \right\} \\
I_2 &= \frac{1}{2} \left\{ \left[ \frac{1}{2} + C(V) \right] \left[ \frac{1}{2} - C(V - V_0) \right] + \left[ \frac{1}{2} + S(V) \right] \left[ \frac{1}{2} - S(V - V_0) \right] \right\} \\
I_3 &= \frac{1}{2} \left\{ \left[ \frac{1}{2} + C(V) \right] \left[ \frac{1}{2} - S(V - V_0) \right] - \left[ \frac{1}{2} + S(V) \right] \left[ \frac{1}{2} - C(V - V_0) \right] \right\}.
\end{aligned} \]  

(22)

A. Special Cases

1. Step with Parallel Plates

For this case \( W_0 \) and \( V_0 \) are zero and Eq. (20) reduces to

\[ \varphi = (4\pi/\lambda)h_0 \cos \theta. \]  

(23)

Substituting from Eq. (22) into Eq. (21), we get

\[ I_\alpha = \cos^2(\varphi/2) + 2[C^2(V) + S^2(V)]\sin^2(\varphi/2) + [C(V) - S(V)] \sin \varphi, \]  

(24)

which is already reported in [5]. The simulation of the corresponding diffraction pattern for \( (4\pi/\lambda)h_0 \cos \theta = (2m+1)\pi \), \( m = 0, \pm 1, \pm 2, \ldots \) is shown in Fig. 3(a). For this case the intensity profiles in the directions normal and parallel to the step edge are shown in Figs. 3(a’1) and 3(a’2), respectively. The visibility of the fringes depends on the phase difference \( \varphi \) [25].

2. Wedge Shape Step

In this case \( \beta = 0, \alpha \neq 0 \), and Eq. (20) leads to

\[ \varphi = (4\pi/\lambda)h_0 \cos \theta - \pi(WW_0 - W_0^2/2). \]  

(25)

According to Eq. (19), the phase changes along the \( y \) axis, the step edge. Also in this case the intensity is represented by Eq. (24). The corresponding simulation of the diffraction pattern is illustrated in Fig. 3(b). In this case the intensity profile along the \( x \) axis, in a direction perpendicular to the step edge, Fig. 3(b’), is similar to the profile of previous case, Fig. 3(a’), but intensity varies periodically in the \( y \) direction, the direction parallel to the step edge, as in Fig. 3(b’).

3. Step Resembling Fresnel’s Double Mirror

In this case \( \beta \neq 0, \alpha = 0 \), and Eq. (20) becomes

\[ \varphi = (4\pi/\lambda)h_0 \cos \theta - \pi(VV_0 - V_0^2/2). \]  

(26)

Considering Eq. (9), we see \( \varphi \) changes versus \( x_0 \); that is, \( \varphi \) varies in the direction perpendicular to the step edge. Also the amplitude of normalized intensity, Eqs. (21) and (22), varies along the direction normal to the step edge. Therefore, the intensity does not change periodically along the \( x \) axis [9]. The simulation of the diffraction pattern for this case is shown in Fig. 3(c) and the corresponding normalized intensity profiles versus the \( x \) and \( y \) axes are plotted in Figs. 3(c’1) and 3(c’2), respectively.

4. General Case

In this case \( \beta \neq 0, \alpha \neq 0 \), and the phase difference is given by Eq. (20). The phase changes in the directions parallel and perpendicular to the step edge. The simulation of the diffraction pattern for this case is shown in Fig. 3(d). The corresponding normalized intensity profiles along the \( x \) and \( y \) axes are shown in Figs. 3(d’1) and 3(d’2), respectively. These are similar to the intensity profiles shown in Figs. 3(c’1) and 3(c’2).

B. Procedure to Determine the Plates’ Angles, \( \alpha \) and \( \beta \)

In order to analyze the diffraction fringes of a step in the general case, it is necessary to specify the direction of the step edge on the diffraction pattern. For this purpose we consider the intensity change in the \( y \) direction. For \( V = V_0/2 \), Eq. (22) leads to

\[ I_1 = I_2 \text{ and } I_3 = 0, \]  

and Eq. (21) reduces to

\[ I_\alpha = I_1(1 + \cos \varphi), \]  

(27)

where \( \varphi \) is given by Eq. (25). In this case, the intensity varies as a function of \( y_0 \); see Eq. (19). However, for \( \varphi = (2m+1)\pi \), \( m = 0, \pm 1, \pm 2, \ldots \), we have \( I_\alpha = 0 \). Thus, as \( y_0 \) varies we arrive at points with zero intensity in the direction of the \( y \) axis, which is parallel to the step edge. For \( 2\pi \) change in phase in the direction of the \( y \) axis, we can write

\[ \pi W_0\sqrt{2k\gamma/\pi \delta y_0} = 2\pi, \]  

(28)

which leads to

\[ W_0 = 2/\left(\sqrt{2k\gamma/\pi \delta y_0}\right). \]  

(29)

Equating Eq. (29) by \( W_0 \) given by Eq. (19) we get

\[ \tan \alpha = \lambda/(2\delta y_0 \cos \theta). \]  

(30)

Thus, for the known angle of incidence, \( \theta \), \( \alpha \) is specified.
In order to specify angle $\beta$ we fit Eq. (21) on a diffraction pattern with the given $I_1$, $I_2$, and $I_3$ in Eq. (22) in a direction normal to the step edge, $y$ axis, that passes through a point with $I_y = 0$. The fitting provides the correct values of $x_0$ and its coefficient, $\sqrt{2k/V_0/\pi}$, see Eq. (9). The first term at $V = V_0/2$ provides the distance from the step edge to the line passing through the points with zero intensity. The second term provides the distance between the step and the observation screen, $\gamma$, for $R = \infty$. Thus, by locating the points with zero intensity on the diffraction pattern and fitting the intensity functions in directions parallel and perpendicular to the direction passing through the latter points, we specify the location of the step edge, angle $\alpha$, angle $\beta$, and the distance between the step and the observation screen.

3. EXPERIMENTS AND RESULTS

The schemes of the experimental setup are shown in Figs. 4(a) and 4(b). The scheme in Fig. 4(a) is suitable for zero angle of incidence, while with the scheme in Fig. 4(b) one can change the angle of incidence at will. The He–Ne laser beam which is expanded by the beam expander, BE, after passing through the beam splitter, BS, illuminates the step. The reflected beam strikes a CCD which is connected to a PC. The step consists of two plates located side by side, almost in contact. In order to change the height of the step, $h_0$, one of the plates can be displaced in the direction normal to the plate. The three screws installed in the support of the other plate permit changing the orientation of the plate with respect to the other plate. Thus, by these screws one can fix the angles $\alpha$ and $\beta$, defined in Section 2.

In Fig. 5 the experimental diffraction patterns are shown for different orientations of the step’s plates. The distance between the step and CCD was 120 mm. The patterns in Fig. 5 belong to (a) a step with parallel plates, (b) a wedge-shaped step, (c) a step resembling the Fresnel’s double mirror, and (d) a step with general orientations of the plates. The corresponding experimental intensity profiles (dotted curves) in the directions perpendicular and parallel to the step edge are provided in Figs. 5(a′)–5(d′) and 5(a′′)–5(d′′), respectively. The intensity profiles along the directions parallel to the step edge are passed through the zero intensity points. The solid lines are corresponding fitted normalized intensity functions (by normalized intensity, we mean the intensity at each point divided by the average of the intensities at points away from the step edge, when the step is illuminated uniformly). Comparing the illustrations in Figs. 3 and 5, we see that the agreement between the theoretical formulations and experimental results are quite satisfactory.

In another experiment we adjusted the screws of the step in such a way that the plates made angles $\alpha = 0.075^\circ \pm 0.001^\circ$ and $\beta = 0.125^\circ \pm 0.001^\circ$ with each other. Then, we used the step to produce the diffraction pattern shown in Fig. 6(a). We specified the spots with zero intensity on the pattern and plotted the axes parallel and perpendicular to the step edge, $y$ and $x$ axes, in Fig. 6(a). Fitting the intensity functions on experimental profiles in the latter directions, Figs. 6(b) and 6(c), we deduced $\alpha = 0.0759^\circ \pm 0.0005^\circ$ and $\beta = 0.1243^\circ \pm 0.0005^\circ$, which are in good agreement with the arranged values.

This study shows that Fresnel diffraction from a step can be used to measure small angles between two plates very accurately.

![Fig. 3. Simulations of the diffraction patterns for a step with different orientations of its plates: (a) for a step with parallel plates, (b) for a wedge shape step, (c) for a step resembling Fresnel’s double mirror, and (d) for a step with arbitrary orientations of its plates. The plots (a′), (b′), (c′), (d′) and (a′′), (b′′), (c′′), (d′′) represent the normalized intensity profiles along directions normal and parallel to the step edge, the x and y directions, respectively.](image1)

![Fig. 4. Schemes of the experimental setup (a) for zero angle of incidence, (b) for changeable angle of incidence.](image2)
even in small scales. This permits determining the curvature and profile of a surface. Superimposing a flat plate on a surface with the plate edge in a plane normal to the surface, a wedge is formed. By illuminating the wedge by a coherent monochromatic beam of light, the discussed fringes of the wedge are formed. By analyzing the fringes, the angles between the plate and surface at different points are deduced, which lead to the determination of the surface curvature. This technique is indispensable in measuring very small curvatures of spherical and aspherical surfaces. Besides, since we can determine the angles $\alpha$ and $\beta$ in an area of small size, we can determine the profile of a surface by scanning a reference plate over a test surface. This opens a new window in surface topography.

**4. CONCLUSIONS**

Fresnel diffraction from a step in the general case is formulated, which is a significant topic in physical optics. The subject can be utilized in the measurements of the surface curvatures in a wide range of sizes. Also, physical effects that affect the curvatures of surfaces can be studied by the introduced technique. By superimposing a reference flat plate on a test surface and recording the diffraction patterns as the plate scans the surface, it is possible to specify the profile of the surface. Thus, the technique is highly valuable in topography of surfaces with rather smooth profiles in a wide range of sizes.

**APPENDIX A**

Considering Fig. 2 and Eq. (1) in the text, we can write

\[
\begin{aligned}
    \tilde{r}_1 &= \tilde{R} + (\tilde{T}_1 - \tilde{r}_0) \\
    \tilde{r}_1' &= \tilde{R}' - (\tilde{T}_1 - \tilde{r}_0)',
\end{aligned}
\]  

(A1)

\[
\begin{aligned}
    \tilde{r}_2 &= \tilde{R} + (\tilde{T}_2 - \tilde{r}_0) + \tilde{h}_0 = \tilde{R} + (\tilde{T}_{2xy} - \tilde{r}_0) + (\tilde{T}_{2z} + \tilde{h}_0) \\
    \tilde{r}_2' &= \tilde{R}' - (\tilde{T}_2 - \tilde{r}_0) - \tilde{h}_0 = \tilde{R}' + (\tilde{T}_{2xy} - \tilde{r}_0) + (\tilde{T}_{2z} + \tilde{h}_0)',
\end{aligned}
\]  

(A2)

where $\tilde{T}_1$, $\tilde{T}_2$, and $\tilde{r}_0$ are defined as

\[
\begin{aligned}
    \tilde{T}_1 &= x_1\hat{x} + y_1\hat{y} \\
    \tilde{T}_2 &= x_2\hat{x} + y_2\hat{y} + z_2\hat{z} \\
    \tilde{r}_0 &= x_0\hat{x} + y_0\hat{y}.
\end{aligned}
\]  

(A3)

Using the Fresnel–Kirchhoff approximation, we can write

\[
\begin{aligned}
    r_1 + r_1' &\geq (R + R') + \frac{1}{2} (x_1 - x_0)^2 \left( \frac{1}{R} + \frac{1}{R'} \right) \\
    &\quad + \frac{1}{2} (y_1 - y_0)^2 \left( \frac{1}{R} + \frac{1}{R'} \right).
\end{aligned}
\]  

(A4)

Denoting $\gamma = \frac{1}{2} (\frac{x_1}{R} + \frac{y_1}{R'})$, Eq. (A4) reduces to

\[
\begin{aligned}
    r_1 + r_1' &\geq (R + R') + \gamma (x_1 - x_0)^2 + \gamma (y_1 - y_0)^2.
\end{aligned}
\]  

(A5)

Also, ignoring $\gamma (h_0 + z_2)^2$ in the Fresnel diffraction approximation, we can write

\[
\begin{aligned}
    r_2 + r_2' &\geq (R + R') + \gamma [(x_2 - x_0)^2 + (y_2 - y_0)^2] \\
    &\quad - 2z_2(x_2, y_2) \cos \theta - 2h_0 \cos \theta.
\end{aligned}
\]  

(A6)

To express $z_2(x_2, y_2)$ explicitly, we make use of the following relations:
\[ \hat{n}' \cdot \hat{T}_2 = [\hat{x}' + j\hat{y}' + \hat{z}'] \cdot [\hat{x}_2 + j\hat{y}_2 + \hat{z}_2] = 0, \quad (A7) \]

where \( \hat{n}' \) is the unit vector normal to the plate on the right side and \( \hat{T}_2 \) is located in the latter plate, Fig. 2. If we rotate the plate on the right side around axes \( x \) and \( y \), by angles \( \alpha \) and \( \beta \), respectively, \( \hat{n}' \) will be expressed as follows:

\[ \hat{n}' = -\hat{x} \cos \alpha \sin \beta - \hat{y} \sin \alpha + \hat{z} \cos \alpha \cos \beta. \quad (A8) \]

Substituting from Eq. (A8) in Eq. (A7), we get

\[ (-\cos \alpha \sin \beta x_2 + (-\sin \alpha) y_2 + (\cos \alpha \cos \beta) z_2) = 0, \quad (A9) \]

or

\[ z_2 = \tan \alpha x_2 + \tan \beta y_2. \quad (A10) \]

For small angles we use \( \cos \alpha \approx \cos \beta \approx 1 \) and arrive at

\[ z_2 = \tan \beta x_2 + \tan \alpha y_2. \quad (A11) \]

Using Eq. (A11) in Eq. (A6), we get

\[ r_2 + r'_2 = (R + R') + \gamma \left( x_2 - x_0 \right) + (y_2 - y_0)^2 \]

\[ -2(b_0 + \tan \beta x_2 + \tan \alpha y_2) \cos \theta. \quad (A12) \]

Now, if we separate the terms involving \( x_2 \) and \( y_2 \) and express them in quadratic form, we get

\[ r_2 + r'_2 = (R + R') \]

\[ + \gamma \left( x_2 - x_0 \right) - \tan \beta \cos \theta / \gamma^2 \]

\[ + \gamma \left( y_2 - y_0 \right) - \tan \alpha \cos \theta / \gamma^2 \]

\[ -2b_0 \cos \theta + (\tan \beta \cos \theta) x_0 + (\tan \alpha \cos \theta) y_0 \]

\[ + (\tan \beta \cos \theta)^2 / 2\gamma + (\tan \alpha \cos \theta)^2 / 2\gamma. \]

Using the following abbreviations,

\[ \Delta = 2b_0 \cos \theta + (\tan \beta \cos \theta) x_0 + (\tan \alpha \cos \theta) y_0 \]

\[ + (\tan \beta \cos \theta)^2 / 2\gamma + (\tan \alpha \cos \theta)^2 / 2\gamma, \]

\[ X_0 = \tan \beta \cos \theta / \gamma, \quad Y_0 = \tan \alpha \cos \theta / \gamma, \]

\[ \left\{ \begin{array}{l}
X_2 = x_2 - x_0 \\
Y_2 = y_2 - y_0
\end{array} \right. \quad (A14) \]

\[ \left\{ \begin{array}{l}
X_1 = x_1 - x_0 \\
Y_1 = y_1 - y_0
\end{array} \right. \quad (A15) \]

Eq. (A5) and Eq. (A12) are expressed as follows:

\[ \left\{ \begin{array}{l}
r_1 + r'_1 = (R + R') + \gamma X_1^2 + \gamma Y_1^2 \\
r_2 + r'_2 = (R + R') + \gamma X_2^2 + \gamma Y_2^2 - \Delta.
\end{array} \right. \quad (A16) \]

Substituting from Eq. (A18) into Eq. (1) and using the defined variables, we get

\[ U(P) = \frac{KD}{RR'} e^{i(k(R+R'))} \int_{-\infty}^{\infty} e^{i\xi X_1^2} dX_1 \int_{-\infty}^{\infty} e^{i\eta Y_1^2} dY_1 \]

\[ + \rho e^{i\phi} \int_{-\infty}^{\infty} e^{i\xi X_2^2} dX_2 \int_{-\infty}^{\infty} e^{i\eta Y_2^2} dY_2, \quad (A19) \]

where \( \phi = k\Delta. \)