Parametric assessment and multi-objective optimization of an internal auto-cascade refrigeration cycle based on advanced exergy and exergoeconomic concepts

Sahar Asgaria, A.R. Noorpoor, Fateme Ahmadi Boyaghchi

Graduate Faculty of Environment, University of Tehran, Iran
Department of Mechanical Engineering, Faculty of Engineering & Technology, Alzahra University, Deh-Vanak, Tehran, Iran

ARTICLE INFO

Article history:
Received 21 September 2016
Received in revised form 26 February 2017
Accepted 28 February 2017

Keywords:
Internal auto-cascade refrigeration
Avoidable exergy destruction rate
Avoidable cost rates
Sensitivity study
Optimization
NSGA-II

ABSTRACT

This research deals with the advanced exergy and exergoeconomic analyses and multi-objective optimization of an internal auto-cascade refrigeration cycle. Butane is used as the refrigerant and all heat exchangers are modeled by considering pressure drops. Sensitivity study is carried out to assess the variation of exergetic and economic improvement potentials; namely, total avoidable exergy destruction, total avoidable exergy destruction cost and total avoidable investment cost rates to the compressor mass flow rate, condenser, refrigerator evaporator and freezer evaporator inlet temperatures. Parametric study indicates that the condenser inlet temperature growth improves the total avoidable exergy destruction within 88.19%, the total avoidable investment cost rate increases by about 126.92% and 3.68% as compressor inlet mass and refrigerator evaporator inlet temperature rise, respectively and the increment of refrigerator evaporator inlet temperature shows a positive effect on the total avoidable exergy destruction cost rate. In addition, improvement potentials are maximized by applying Non-dominated Sort Genetic Algorithm-II. The multi-objective optimization indicates 76.78%, 38.66% and 103.38% improvements in total avoidable exergy destruction rate, total avoidable investment and total avoidable exergy destruction cost rates, respectively relative to the base design point.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

More recently, studies on the auto-cascade refrigeration cycles (ACRCs) have been carried out in various aspects, such as the cycle characteristics of the system, exergy analysis of the system and the system modifications. ACRCs show more advantages in comparison with traditional cascade refrigeration cycles (CRCs); as a result, more investigations have been focused on ACRC. It is suggested that adding a fractionation device in the phase separator or reducing separation stages may sharply cut down the cost of system [1].

Kim and Kim [2] studied the performance of an ACRC using zeotropic refrigerant mixtures of R744-R134a and R744-R290 as working fluid. It was found that ACRC had a merit of low operating pressure as low as that of a conventional vapor compression refrigeration cycle. Yu et al. [3] proposed a novel ACRC with an ejector with refrigerant mixture of R23/R134a to recover available work and to investigate the effects of major design parameters on the desired system performance. The results showed the decrement in the pressure ratio of compressor as well as the increment in the coefficient of performance (COP). Nayak and Venkataramnam [4] studied an ACRC operating with optimized R22/R404A mixtures and various cascade heat exchangers. In this research, the optimum stages of cascade heat exchangers were suggested for different operating temperatures. Wang et al. [5] assessed the performance of ACRC operating with two vapor-liquid separators and six binary refrigerants, i.e. R23-R134a, R23-R227ea, R23-R236fa, R170-R290, R170-R600a and R170-R600, with a new approach at the temperature level of –60 °C. Sivakumar and Somasundaram [6] performed an exergetic analysis of three stage ACRC using two combinations of R290/R23/R14 and R1270/R170/R14. COP, exergy loss, exergic efficiency, efficiency defect and the evaporating temperature were evaluated for various mass fractions.

Tan et al. [7] proposed and analyzed thermodynamically an ejector enhanced ACRC to obtain lower refrigeration temperature.
The R32/R236fa zeotropic mixture was applied as a refrigerant. The results showed that refrigerant mixture composition, condenser outlet temperature and evaporation pressure had considerable effects on the performance of ACRC. Yan et al. [8] designed and investigated a new ejector enhanced ACRC applying R134a/R23 refrigerant mixture to recover the work loss in the throttling process. Results showed that refrigerant mixture composition, condenser outlet temperature and evaporation pressure had considerable effects on the performance of ACRC. Yan et al. [8] designed and investigated a new ejector enhanced ACRC applying R134a/R23 refrigerant mixture to recover the work loss in the throttling process. The performance comparison of the new proposed system indicated that COP, volumetric refrigeration capacity and pressure ratio of compressor were further improved. The performance characteristics of the IARC showed its promise in domestic refrigerator-freezers applications.

The conventional exergy and exergoeconomic analyses are the powerful tools to estimate the location, magnitudes and causes of inefficiencies and costs related to these irreversibilities in the energy systems. However, this is not always effective because the conventional analyses are not able to estimate the interaction among the components or to reveal the real improvement potential of the system. Therefore, optimization strategies for energy systems can be misguided. Advanced analyses attempt to address this weakness. Splitting the exergy destruction rate and cost rates into unavoidable (the part of the exergy destruction that cannot be reduced with the improvement of the component itself) and unavoidable (the part of the exergy destruction that cannot be reduced with improvement of the component itself due to the technological limitations) subdivisions provides a realistic measure of improvement potential of each component. Moreover, splitting the exergy investment cost rate of components ($/year) into exogenous (the part of the exergy destruction that is not associated with the system itself) and endogenous (the part of the exergy destruction that is associated with the system itself) subdivisions provides a realistic measure of improvement potential of each component.
In the literature, there are few articles on advanced exergy and exergoeconomic analyses on the refrigeration systems. Morosuk et al. [15], discussed the Voorhees’ compression process in refrigeration as an alternative process to the two-stage vapor-compression refrigeration machines by applying the conventional and advanced exergetic analyses. The conventional exergetic analysis identified the condenser as the most important component. Thus, improvement efforts should focus on this component. However, the results of the advanced exergetic analysis showed that the evaporator is by far the most important component.

Chen et al. [16] conducted the conventional and advanced exergy analyses on an ejector refrigeration system. Conventional analysis indicated that about half of the total exergy destruction rate was caused by the ejector and the advanced analysis implied that the ejector had the highest priority to be improved. Moreover, the temperature difference inside the condenser had the highest influence on the exergy destruction rate. Bai et al. [17] analyzed an ejector expansion transcritical CO₂ refrigeration system using advanced exergy analysis. The outcomes revealed that the compressor with largest avoidable endogenous exergy destruction rate dominated the highest improvement priority. Additionally, the effect of several substantial design parameters were evaluated on the exergetic performance of the desired system. Bai et al. [18] presented and modeled an ejector ACRC with conventional and advanced exergy analyses. The results indicated that the compressor with the largest avoidable endogenous exergy destruction rate possessed the highest priority of improvement differing from the results obtained applying the conventional analysis. Moreover, the evaporator/condenser had a drastic effect on the exogenous exergy destruction rate within the system components.

Gullo et al. [19], conducted the advanced exergy analysis to an R744 booster refrigeration system with parallel compression. According to the results obtained from the advanced exergy evaluation, the avoidable irreversibilities of the high stage compressor and those of the medium temperature evaporator were mainly and completely endogenous, respectively.

Mehroopya and Ansarinab [20] evaluated two single mixed refrigerant processes by applying advanced exergy and exergoeconomic analyses. According to the avoidable exergy destruction cost rate in Linde process, E – I heat exchanger and in Air Product Chemical Inc. process, C-I compressor, because of their higher improving potential, should be considered for modification.

In the open literature, no advanced and sensitivity analysis of major parameter on advanced exergy destruction and cost rates of IARC is reported. In this paper, an IARC is modeled with R600 as working fluid. Advanced exergy and exergoeconomic analyses are applied to evaluate the realistic exergetic and economic improvement potentials of overall system. The effect of major parameters namely, the compressor mass flow rate, m₁, condenser inlet temperature, T₃, refrigeration evaporator inlet temperature, T₆, and freezer evaporator inlet temperature, T₁₁, on the total avoidable exergy destruction, Exₐ, and cost rates, i.e. Zₐₘ and Cₐₗ, are evaluated. Finally, Non-dominated Sort Genetic Algorithm-II (NSGA-II) technic is used to find the optimum value of improvement potentials of overall system.

2. System description

Fig. 1 illustrates the typical domestic IARC considered consisting of a compressor, a condenser, three capillary tubes, a subcooler, a phase separator, a cascade heat exchanger and two evaporators, i.e. refrigerator and freezer evaporators, operating at different evaporation pressures. The hot compressed R600 stream flows into the cooler, the condenser and the subcooler to reject heat from the hot refrigerant. The subcooling refrigerant leaving the subcooler is led to the capillary tube I to be expanded to two-phase fluid. The expanded two-phase refrigerant passing through the refrigerator evaporator absorbs heat from the refrigerator compartment. The more two phase vapor/liquid flow leaving the refrigerator, evaporator enters the separator where the liquid and vapor phases are separated. The liquid phase flows into the capillary tube II to the cascade heat exchanger where it is completely vaporized and the vapor phase flows into the cascade heat exchanger and then is totally condensed; the condensed refrigerant is expanded through capillary tube III into the freezer evaporator at a lower evaporation pressure to cool the freezer component. The flow of vapor from both cascade heat exchanger and freezer evaporator is mixed and then is returned to the compressor via the subcooler.

In the current research, Butane (R600) hydrocarbon is selected as a convenient refrigerant. Hydrocarbons as alternatives to conventional refrigerants, have zero ozone depletion potential (ODP) and negligible global warming potential (GWP) [21–23]. They also have favorable advantages such as availability, non-toxicity, high miscibility with mineral or synthetic oil, etc. [24–26]. Furthermore, the performance simulation of some hydrocarbon refrigerants in refrigeration systems indicates the higher energetic and exergetic COPs for R600 [27,28].

3. Methodology

A mathematical model is developed to analyze the desired IARC. The following assumptions are also made for the simulation [13,28]:

1) All components are assumed to reach a steady state process;
2) Kinetic and potential energy changes and heat losses to or from the environment are negligible;
3) The compression process through the compressor is irreversible and the isentropic efficiency according to its pressure ratio is taken into account;
4) The throttling processes within capillary tubes are considered to be isenthalpic;
5) The vapor leaving the freezer evaporator and liquid leaving the condenser are both saturated;
6) The leaving vapor and liquid from the phase separator are saturated;
7) The streams passing through channels are fully developed.

3.1. Conventional analyses

By considering the steady-state operation for each component, the mass and energy balances are defined as follows [13]:

\[
\sum_{\text{in}} m = \sum_{\text{out}} m \quad (1)
\]
Heat exchangers are the major components of refrigeration system affecting the cost of the system. So, the thermal design of a heat exchanger is required. In this work, all heat exchangers are considered the plate type due to its high efficiency and compactness [29]. The heat transfer processes for single-phase flow, i.e. cooler and subcooler, and two-phase flow, i.e. condenser, refrigerator evaporator, cascade heat exchanger and freezer evaporator, are modeled applying the logarithmic mean temperature difference ($\Delta T_{\text{LMTD}}$), the heat transfer area ($A$), and the overall heat transfer coefficient ($U$) [30] as:

$$Q = UA\Delta T_{\text{LMTD}}$$  \hspace{1cm} (3)

By neglecting the heat losses and fouling effects in the channel with fully developed flow, the overall heat transfer coefficient is calculated by:

$$\frac{1}{U} = \frac{1}{h_H} + \frac{\delta_{\text{plate}}}{\lambda_{\text{plate}}} + \frac{1}{h_C}$$  \hspace{1cm} (4)

In Eq. (4), $h_H$ and $h_C$ refer to the convection heat transfer coefficients for the hot and cold sides, respectively, $\delta_{\text{plate}}$ and $\lambda_{\text{plate}}$ represent, respectively, the thickness and thermal conductivity of the plate.

$$h = \frac{\lambda Nu}{D_H}$$  \hspace{1cm} (5)

Here, $D_H$ and $Nu$ denote, respectively, the hydraulic diameter of flow channel and Nusselt number [31].

For the single-phase flow, the Chisholm and Wanniarachchi correlation is used to calculate the Nusselt number for both hot and cold sides [29]:

$$Nu = 0.724 \left(\frac{6\beta}{\pi}\right)^{0.646} \text{Re}^{0.583} \text{Pr}^{1/3}$$  \hspace{1cm} (6)

Here, $Re$ and $Pr$ are Reynolds number and Prandtl number, respectively and $\beta$ refers to the chevron angle of the plates [29].

For the two-phase flow, the heat transfer process can be split into small parts so that in each part the properties can vary slightly. The Nusselt number relations are different for vaporization or condensation processes [29]. The Nusselt numbers on the cold and hot sides for each section, $m$, are calculated by Eqs. (7) and (8), respectively.

$$Nu_{m,C} = 1.926Pr_H^{1/3}Bo_m^{0.3}Re_m^{0.5} \left[1 - X_m + X_m \left(\frac{p_l}{p_v}\right)^{0.5}\right]$$  \hspace{1cm} (7)

$$Nu_{m,H} = 4.118Re_m^{0.4}Pr_1^{1/3}$$  \hspace{1cm} (8)
In Eq. (7), \( B_0 \) stands for boiling number, being expressed by Ref. [29].

By considering the pressure drop in the heat exchangers, their performance approach the real conditions. The pressure loss is the function of thermophysical properties, the stream velocity and the heat exchanger geometry [29] or a plate heat exchanger with specific roughness and single-phase flow, total pressure drop \( (dP_{\text{tot}}) \) could be calculated by adding the frictional pressure drop \( (dP_F) \), the gravitational pressure drop \( (dP_G) \), and the pressure loss at the entrance and exit of the test unit \( (dP_M) \) as in the formula below [29]:

\[
dP_{\text{tot}} = dP_F \pm dP_G + dP_M
\] (9)

In Eq. (9), the gravitational pressure drop can be calculated as

\[
dP_G = \pm \frac{gL}{\nu}
\] (10)

where \( L \) refers to the length between the centers of entry and exit tubes, \( \nu \) is the specific volume of liquid and \( g \) is the gravitational acceleration. The plus sign \((+\) is used in the case of vertical downward flow while the minus sign \((-\) is used in the case of vertical upward flow.

Pressure loss at the entrance and exit of the heat exchanger can be calculated from the correlation of Shah and Focke [32].

\[
dP_M = 1.5 \left( \frac{V^2}{2\nu} \right)
\] (11)

where \( V \) is the velocity of liquid. Frictional pressure drop can be calculated from the following equation:

\[
dP_F = 4f \left( \frac{L}{D_h} \right) \left( \frac{V^2}{2\nu} \right)
\] (12)

For the two-phase flow, Eq. (13) is added to Eq. (9).

\[
dP_X = G^2 \mu_g \Delta X
\] (13)

where \( G \) is the mass flux, \( \mu_g \) is the difference of specific volume between vapor and liquid phases, and \( \Delta X \) is the total quality change in the heat exchanger.

The exergy destruction rate within each component, \( \dot{Ex}_{D,k} \) can be calculated as [33]:

\[
\dot{Ex}_{D,k} = \dot{Ex}_{F,k} - \dot{Ex}_{P,k}
\] (14)

In Eq. (14), \( \dot{Ex}_{F,k} \) and \( \dot{Ex}_{P,k} \) indicate the exergy of product and fuel for the \( k \)th component, respectively.

The cost balance for the \( k \)th component can be written as follows [34]:

\[
\dot{C}_{P,k} = \dot{C}_{F,k} + \dot{Z}_{\text{tot}}
\] (15)

Here, \( \dot{C}_{P,k} \) and \( \dot{C}_{F,k} \) indicate the product and fuel cost rates within the \( k \)th component and \( \dot{Z}_{\text{tot}} \) shows the cost rate associated with the operating and maintenance as well as the capital investment.

3.2. Advanced exergy and exergoeconomic analyses

According to the advanced exergy concept, the cost and technical constraints determine a minimum value of the exergy destruction rate. This unavoidable subdivision of exergy destruction rate \( (\dot{Ex}_{D,k}^{\text{UN}}) \) is calculated considering the \( k \)th component in isolation, separated from the overall system. The ratio of \( (\dot{Ex}_{D,k}^{\text{UN}} / \dot{Ex}_{F,k}^{\text{UN}}) \) is estimated assuming operation with high efficiency and low losses called unavoidable conditions listed in Table 1. For the \( k \)th component, the unavoidable exergy destruction rate can be calculated as [30]:

\[
\dot{Ex}_{D,k}^{\text{UN}} = \dot{Ex}_{F,k}^{\text{UN}} \left( \frac{\dot{Ex}_{D,k}}{\dot{Ex}_{F,k}} \right)
\] (16)

The avoidable exergy destruction rate \( (\dot{Ex}_{D,k}^{\text{AV}}) \), that can be reduced by technological improvement of the \( k \)th component, can be obtained with Eq. (17) [30].

\[
\dot{Ex}_{D,k}^{\text{AV}} = \dot{Ex}_{D,k} - \dot{Ex}_{D,k}^{\text{UN}}
\] (17)

The exergy destruction rate within the \( k \)th component can be further split into endogenous and exogenous exergy destruction rates. The endogenous part \( (\dot{Ex}_{D,k}^{\text{EN}}) \) is caused due to the irreversibilities of the \( k \)th component itself when it operates under real conditions while the remaining components operate theoretically and the exogenous one \( (\dot{Ex}_{D,k}^{\text{EX}}) \) is the part of exergy destruction destroyed in the \( k \)th component, due to the irreversibilities that occur in the remaining components. In all cases, the output of the overall system is kept constant and equal to the real case. The real and theoretical conditions are listed in Table 1. By subtracting \( \dot{Ex}_{D,k}^{\text{EN}} \) from the exergy destruction rate, the exogenous part of the exergy destruction rate within the \( k \)th component can be calculated as [15]:

\[
\dot{Ex}_{D,k}^{\text{EX}} = \dot{Ex}_{D,k} - \dot{Ex}_{D,k}^{\text{EN}}
\] (18)

To provide a deeper insight, the avoidable and unavoidable parts of exergy destruction rate are split into endogenous/exogenous parts. The avoidable endogenous exergy destruction rate \( (\dot{Ex}_{D,k}^{\text{EN,AV}}) \) as well as the avoidable exogenous part \( (\dot{Ex}_{D,k}^{\text{EX,AV}}) \) can be improved by optimization of the component and can be estimated by using Eqs. (19) and (20):

\[
\dot{Ex}_{D,k}^{\text{EX,AV}} = \dot{Ex}_{D,k}^{\text{EN}} - \dot{Ex}_{D,k}^{\text{EN,AV}}
\] (19)

\[
\dot{Ex}_{D,k}^{\text{EX,AV}} = \dot{Ex}_{D,k}^{\text{EX}} - \dot{Ex}_{D,k}^{\text{EX,UN}}
\] (20)

The unavoidable endogenous exergy destruction rate \( (\dot{Ex}_{D,k}^{\text{EN,UN}}) \) and the unavoidable exogenous parts \( (\dot{Ex}_{D,k}^{\text{EX,UN}}) \) cannot be lowered, due to the technical constraints of the \( k \)th component, and can be calculated by using Eqs. (21) and (22):

\[
\dot{Ex}_{D,k}^{\text{EN,UN}} = \dot{Ex}_{F,k}^{\text{UN}} \left( \frac{\dot{Ex}_{D,k}}{\dot{Ex}_{F,k}} \right)
\] (21)

\[
\dot{Ex}_{D,k}^{\text{EX,UN}} = \dot{Ex}_{D,k}^{\text{UN}} - \dot{Ex}_{D,k}^{\text{EN,UN}}
\] (22)

Similar to the advanced exergy analysis, the investment cost and the cost of exergy destruction rates related to the internal operating conditions and the component interactions, respectively [15], can be divided into various parts.

The unavoidable/avoidable, endogenous/exogenous parts of the cost rate associated with exergy destruction rate of the \( k \)th component \( (\dot{C}_{D,k}) \) and their combinations are calculated by applying the results of the conventional exergoeconomic and
The four categorized segments of the investment cost rates are obtained from Eqs. (31) and (32): 

\[ Z_{k}^{UN} = \frac{Z_{k}}{Z_{k}^{UN}} \]  

(31)

\[ Z_{k}^{AV} = Z_{k}^{AV} - Z_{k}^{UN} \]  

(32)

Similarly, the endogenous part within the kth component \( Z_{k}^{EN} \) can be calculated by using the assumptions listed in Table 1 as follows:

\[ Z_{k}^{EN} = \frac{Z_{k}^{real}}{Z_{k}^{real}} \]  

(33)

And the exogenous part can be calculated by subtracting the endogenous one from the real investment cost rate:

\[ Z_{k}^{EX} = Z_{k}^{real} - Z_{k}^{EN} \]  

(34)

The mentioned parts and the four categorized segments of the investment cost flow rates can be calculated by using the following equations:

\[ \frac{c_{EN}^{EN}}{c_{EN}^{EX}} = c_{EN}^{EN} - c_{EN}^{EX} \]  

(35)

\[ \frac{c_{AV}^{EN}}{c_{AV}^{EX}} = c_{AV}^{EN} - c_{AV}^{EX} \]  

(36)

\[ \frac{c_{AV}^{UN}}{c_{AV}^{EX}} = c_{AV}^{UN} - c_{AV}^{EX} \]  

(37)

\[ \frac{c_{EN}^{UN}}{c_{EN}^{EX}} = c_{EN}^{UN} - c_{EN}^{EX} \]  

(38)

4. Results and discussion

Conventional and advanced analyses of the desired IARC are carried out by applying EES (Engineering Equation Solver) software. The standard operating conditions of simulation are summarized in Table 2. Table 3 represents the conventional and advanced exergy analyses results of the desired system. It is clearly revealed that cascade heat exchanger has the maximum exergy destruction rate (35.17% of total exergy destruction rate) among all components followed by the freezer evaporator, compressor and other components with 19.11%, 18.30% and 27.42% of the total exergy destruction rate, respectively. In cascade heat exchanger, 65.30% of the exergy destruction rate is unavoidable and 89.19% of that is endogenous.

According to Table 3, the value of avoidable parts for all components are positive and the total avoidable exergy destruction rate contains 35.95% of the total exergy destruction rate which indicates the high improvement potential of the system. In addition, 34.43% and 33.83% of the total avoidable exergy destruction belong to the compressor and auto-cascade heat exchanger, respectively. Therefore, a study on these components seems to be of value to improve the system. Due to the importance of the avoidable parts of the exergy destruction rate, the components with high \( \dot{E}_{EN,comp}^{EN} \) and \( \dot{E}_{EX,comp}^{EX} \) are firstly evaluated. Compressor with the highest value of \( \dot{E}_{EN,comp}^{EN} \) is in priority for improvement. On the other hand, the great value of \( \dot{E}_{D,comp}^{EN} \) relative to \( \dot{E}_{D,comp}^{EX} \) indicates that the

### Table 1: Assumptions for calculating the real, theoretical and the unavoidable exergy destruction rate [36].

<table>
<thead>
<tr>
<th>Component, k</th>
<th>Parameter</th>
<th>Real conditions</th>
<th>Theoretical conditions</th>
<th>Unavoidable conditions</th>
<th>Unavoidable conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>condenser</td>
<td>( \Delta T_{cond} (\degree C) )</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Evaporators</td>
<td>( \Delta T_{evaps} (\degree C) )</td>
<td>4.7</td>
<td>0</td>
<td>0.2</td>
<td>14</td>
</tr>
<tr>
<td>Heat exchangers</td>
<td>( \Delta T_{HE} (\degree C) )</td>
<td>5.4</td>
<td>0</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Compressor</td>
<td>( \eta_{m} (%) )</td>
<td>85</td>
<td>100</td>
<td>95</td>
<td>83.44</td>
</tr>
</tbody>
</table>

Notes:

- Exergy destruction.
- Investment cost rate.

### Table 2: Conditions of simulation for the IARC.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead state temperature, ( T_{a} (\degree C) )</td>
<td>32</td>
</tr>
<tr>
<td>Dead state pressure, ( P_{0} (kPa) )</td>
<td>101.3</td>
</tr>
<tr>
<td>Compressor mass flow rate, ( m_{1} (kg/s) )</td>
<td>0.08</td>
</tr>
<tr>
<td>Condenser inlet temperature, ( T_{1} (\degree C) )</td>
<td>40</td>
</tr>
<tr>
<td>Refrigerator evaporator inlet temperature, ( T_{a} (\degree C) )</td>
<td>5</td>
</tr>
<tr>
<td>Freezer evaporator inlet temperature, ( T_{11} (\degree C) )</td>
<td>-58</td>
</tr>
<tr>
<td>Isentropic efficiency of compressor, ( \eta_{m} (%) )</td>
<td>86</td>
</tr>
<tr>
<td>Refrigeration capacity rate, ( Q_{e} (kW) )</td>
<td>10.406</td>
</tr>
</tbody>
</table>
improvement of compressor itself is more effective than the other components. The second component which can be improved is cascade heat exchanger with 69.40% of the total avoidable exogenous exergy destruction rate. As seen from Table 3, for this component $\tilde{Z}_{\text{CD}}^{\text{UN}} < \tilde{Z}_{\text{EN}}^{\text{AV}}$ indicates that the focus should be on improving other components. The cooler, sub-cooler, cascade heat exchanger and freezer evaporator have the negative values of $\tilde{Z}_{\text{CD}}^{\text{UN}}$ meaning that the irreversibilities of the kth component are greater than its avoidable exergy destruction rate. The negative values of $\tilde{Z}_{\text{CD}}^{\text{UN}}$ represent that the increment of exergy destruction rate in other components reduces the exergy destruction rate of the kth component. Results show that the amount of the system irreversibilities is high because the unavoidable endogenous exergy destruction rate of the overall system is within 10.679 kW and due to its value of the avoidable exogenous part, i.e. 6.017 kW, the overall system can be improved.

The conventional and advanced exergoeconomic analyses are presented in Table 4. Results show that the maximum and minimum values of the investment cost rates belong to the compressor and freezer evaporator with values of 3599.992 $/year and 24,959 $/year (i.e. 71.45% and 0.50% of $Z_{\text{tot}}$), respectively. It is clearly observed that in all components, except the condenser, the value of the unavoidable investment cost rate ($Z_{\text{UN}}^{\text{AV}}$) is higher than the avoidable one ($Z_{\text{UN}}^{\text{UN}}$). The compressor with 71.19% of $Z_{\text{tot}}$ and 73.23% of $Z_{\text{AV}}$ has the maximum values of unavoidable and avoidable investment cost rates, respectively. Due to the importance of the avoidable part of the investment cost to improve the system, the focus should be on the compressor. As clearly revealed, the maximum values of $Z_{\text{UN}}^{\text{EN}}$ are related to the compressor and cascade heat exchanger, respectively (3456.165 $/year and 1622.997 $/year which are 51.64% and 25.15% of $Z_{\text{tot}}$). It is observed that the endogenous investment cost rates of all components are higher than their exogenous parts ($Z_{\text{EN}}^{\text{EN}} < Z_{\text{EN}}^{\text{UN}}$). It is included that the focus should mostly be on decreasing the irreversibilities of the components themselves. Evaluating the overall system indicates that the total investment cost rate of the system is 5038.33 $/year and 87.04% of this is unavoidable, while the remaining part is avoidable.

Table 5 indicates the results of conventional and advanced exergoeconomic analyses. According to Eqs. (23)–(30), the values of exergy destruction cost rates are related to the exergy destruction rates and the percentage of their increment or decrement are equal in the economic conditions.

It is observed that the freezer evaporator with 53.88% of $C_{\text{D,tot}}$ is dominant and the cascade heat exchanger, the compressor and other components with values of 14.34%, 9.35% and 22.43%, respectively, are in the next ranking. According to Table 5, the values of $C_{\text{D,k}}$ for all components are greater than the avoidable parts, so that the freezer evaporator with 47.22% of $C_{\text{D,tot}}$ can be dominant among the components. Results indicate that the most values of exergy destruction cost rates within the components are endogenous. Therefore, the exergy destruction cost rates are not affected by the components interactions. In this case, the efficiency of the desired component can be increased or can be substituted by a more efficient one.

Comparing the results indicate that from the conventional exergoeconomic perspective, the freezer evaporator with the maximum exergy destruction cost rate is the most effective component, while 108.35% of its exergy destruction cost rate is unavoidable and the condenser with the maximum value of $C_{\text{D,EN,AV}}$ is in priority of the improvement. The negative value of $C_{\text{D,tot}}$ represents the low improvement potential of the overall system.

### 5. Sensitivity study

#### 5.1. The effects of key parameters on various parts of $\tilde{Z}_{\text{D,tot}}$

Fig. 2 illustrates the effects of compressor mass flow rate variations from 0.08 to 0.3 kg/s on the different parts of the total exergy destruction rate ($\tilde{Z}_{\text{D,tot}}$) of the system. Sensitivity analysis indicates that the amount of pressure loss and consequently $\tilde{Z}_{\text{EN,AV}}$ in the heat exchangers increase as the mass flow rate rises. Since the amount of $\tilde{Z}_{\text{UN,AV}}$ is higher than that of $\tilde{Z}_{\text{D,tot}}$, the value of $\tilde{Z}_{\text{D,UN}}$
becomes negative meaning that the increment in mass flow rate cannot reduce the irreversibilities.

It is clearly observed that the $\dot{E}_{\text{EX}}^{\text{EN,Un}}$ increases within 290.21%, owing to the mass flow rate increment, indicating the weak interaction between other components. Moreover, the high contribution of $\dot{E}_{\text{EX}}^{\text{EN,Un}}$ is unavoidable indicating the high irreversibilities in the system. The exogenous part of the total exergy destruction rate rises slightly as the mass flow rate increases because the value of $\dot{E}_{\text{EX}}^{\text{EN,Un}}$ is almost equal to $\dot{E}_{\text{EX}}^{\text{D,Un}}$, and $\dot{E}_{\text{EX}}^{\text{AV}}$ increases slightly. Furthermore, the significant contribution of $\dot{E}_{\text{EX}}^{\text{D,Un}}$ is avoidable showing the amount of improvement potential of the system. According to Fig. 2, the $\dot{E}_{\text{EX}}^{\text{UN}}$ increases within 543.89% while $\dot{E}_{\text{EX}}^{\text{EN,Un}}$ and $\dot{E}_{\text{EX}}^{\text{AV}}$ decrease strongly as the mass flow rate increases. Under unavoidable conditions, an increment in compressor mass flow rate leads to an increment in $\frac{\dot{E}_{\text{EX}}^{\text{EN}}}{\dot{E}_{\text{EX}}^{\text{D}}}$ of each component. Therefore, $\dot{E}_{\text{EX}}^{\text{D,Un}}$ and $\dot{E}_{\text{EX}}^{\text{D,AV}}$ increase according to Eqs. (16) and (21), respectively.

The effects of condenser inlet temperature on various parts of the total exergy destruction rate are shown in Fig. 3. It is revealed that the total exergy destruction rate rises 14.86% as the condenser inlet temperature is supposed to change from 32 °C to 40 °C. This indicates that the total irreversibilities of the overall system is growing. Outcomes indicate that the pressure loss decreases strongly as $T_3$ increases leading to the decrement of unavoidable exergy destruction rate of the system. On the other hand, the value of $\dot{E}_{\text{EX}}^{\text{D,Un}}$ is higher than $\dot{E}_{\text{EX}}^{\text{D,AV}}$ causing the negative value for $\dot{E}_{\text{EX}}^{\text{D,Un}}$. Results show that $\dot{E}_{\text{EX}}^{\text{UN}}$ increases slightly as $T_3$ grows. Due to the increase of $T_3$, the temperature and the pressure of the streams rise leading to the increments of the fuel and product exergies of the components but under the theoretical operating of the remaining components and the real conditions of the desired component, the fuel exergy value is greater than the product one. Moreover, another reason can be due to the increment of the steam leaving the separator compared to the liquid phase which increases the values of $\dot{E}_{\text{EX}}^{\text{EN}}$ in the freezer evaporator and the cascade heat exchanger. As observed in Fig. 3, most of the $\dot{E}_{\text{EX}}^{\text{D,Un}}$ value is unavoidable, which increases significantly 37.99% as $T_3$ rises. On the other hand, the unavoidable part of $\dot{E}_{\text{EX}}^{\text{D,Un}}$ grows with the same slope.
which indicates the high potential improvement of components themselves at higher condenser inlet temperature. It is clearly seen that \( \dot{E}_{\text{Dtot}}^{\text{EX}} \) is slightly affected by variation of \( T_3 \) and its increment is imperceptible. Additionally, the avoidable part of \( \dot{E}_{\text{Dtot}}^{\text{EX}} \) decreases from 12.222 kW to 7.497 kW and the \( \dot{E}_{\text{Dtot}}^{\text{EN,UN}} \) increases with the same slope.

The effects of inlet temperature of the refrigerator evaporator \( (T_6) \) on the total exergy destruction rate subdivisions are plotted in Fig. 4. It is revealed that the increment of \( T_6 \) from \(-5^\circ C \) to \(0^\circ C \) does not have any significant impact on \( \dot{E}_{\text{Dtot}}^{\text{EN}} \) (within 12.73% increment) because, under the real conditions, the loss pressure inside the heat exchangers does not experience sensible variations and the value of the streams exergy rates remain almost constant as \( T_6 \) increases. Results show that the most of the total exergy destruction rate is unavoidable, while the little remaining part is avoidable and the value of \( \dot{E}_{\text{Dtot}}^{\text{AV}} \) is negative due to \( \dot{E}_{\text{Dtot}}^{\text{EX}} < \dot{E}_{\text{Dtot}}^{\text{EN}} \).

In addition, the value of \( \dot{E}_{\text{Dtot}}^{\text{UN}} \) is almost fixed and high which indicates the low improvement potential of the entire system. This is, because \( \frac{\dot{E}_{\text{Dtot}}^{\text{EX}}}{\dot{E}_{\text{Dtot}}^{\text{EN}}} \) for each component is great and remains constant as \( T_6 \) grows. On the other hand, the value of \( \dot{E}_{\text{Dtot}}^{\text{EX}} \) is lower than \( \dot{E}_{\text{Dtot}}^{\text{EN}} \), indicating the weak interactions among the components and to improve the system, focus should be on decreasing the irreversibilities within the component. Increasing \( T_6 \) causes the decrement of the total avoidable endogenous part within 47.52% and the increment of avoidable exogenous part by about 71.42%. It is concluded that the high percentage of \( \dot{E}_{\text{Dtot}}^{\text{AV}} \) is based on the performance of the various components as \( T_6 \) grows.

Fig. 5 indicates the behavior of entire system exergy destruction rates as the inlet temperature of the freezer evaporator increases \( (T_{11}) \) from \(-58^\circ C \) to \(-22^\circ C \). The values of mass flow rate, enthalpy and entropy of system do not significantly vary under the both real and theoretical conditions as \( T_{11} \) increases. Therefore, the irreversibilities generation within the system is not affected by \( T_{11} \) and \( \dot{E}_{\text{Dtot}}^{\text{EN}} \) is close to zero, i.e. almost all values of exergy destruction rates are endogenous (95.80% of \( \dot{E}_{\text{Dtot}}^{\text{EN}} \)), obviously from Fig. 5, the great contribution of that is \( \dot{E}_{\text{Dtot}}^{\text{EN,UN}} \). In addition, the high amount of exogenous exergy destruction rate is avoidable. As \( T_{11} \) increases, all parts of the exergy destruction rates except the exogenous one decrease slightly. The exogenous part increases from 0.304 kW to 0.3944 kW. This means that the increment of \( T_{11} \) strengthens the interactions of the components.

5.2. The effects of key parameters on various parts of \( Z_{\text{tot}} \)

The results of the conventional and advanced exergoeconomic analyses are obtained applying Eqs. (23)–(38). By dividing the investment cost and exergy destruction cost rates into the endogenous/exogenous and avoidable/unavoidable parts, the improvement potential of exergy destruction cost rates of the system can be identified correctly while the conventional exergoeconomic analysis cannot present this valuable result.

Fig. 6 illustrates the variations of total investment cost rate parts when \( m_1 \) is supposed to change from 0.08 kg/s to 0.3 kg/s. As mentioned in section 5.1, when \( m_1 \) increases, the pressure loss in the heat exchangers rises causing the increment of \( Z_{\text{tot}} \) by about 97.41% whose large portion is unavoidable. Moreover, the values of \( \dot{E}_{\text{Dtot}}^{\text{EN}} \) and \( Z_{\text{tot}} \) rise within 126.92% and 88.17%, respectively with \( m_1 \) increasing. The increment of avoidable/unavoidable parts of investment cost rates are owing to the \( m_1 \) increment. For all values of \( m_1 \), the total avoidable part is positive and lower than the total unavoidable part \( (Z_{\text{tot}}^{\text{EN}} < Z_{\text{tot}}^{\text{UN}}) \). Thus, it is concluded that the improvement potential to reduce the investment cost rates is high.

Among the various investment cost parts, \( Z_{\text{tot}}^{\text{EN}} \) has the maximum value and increases strongly (223.05%) in comparison with the other parts as \( m_1 \) rises. This increment is due to the significant increase of endogenous product exergy rate of the component that, according to Eq. (31), influences the endogenous investment cost rates. In addition, the total exogenous part is lower than the total endogenous part for all values of \( m_1 \) which indicates weak interactions among the components.

Outcomes clarify that \( Z_{\text{tot}} < Z_{\text{tot}}^{\text{EN}} \) leads to the negative values for exogenous investment cost rate. Since \( Z_{\text{tot}}^{\text{EN}} \) increases with relatively sharp slope, the improvement potential of endogenous investment cost rate is considerable and to decrease the investment cost rates, a special care should be taken on this part.

Fig. 7 represents the variations of the conventional and advanced total investment cost rates of the system when \( T_3 \) is supposed to vary from 32 °C to 40 °C. Sensitivity analysis indicates that the irreversibilities in most components, especially in the compressor, increase and the value of \( Z_{\text{tot}} \) rises within 0.74% as \( T_3 \)
grows. Moreover, a large portion of $Z_{tot}$ is related to the compressor and the enthalpy difference between the inlet and outlet of compressor increases strongly causing the high value for $Z_{tot}$ based on investment cost equation for compressor [16]. By dividing the total investment cost to endogenous/exogenous, it is revealed that a large amount of it is $Z_{tot}^{EN}$, among other parts of the investment cost rate. Under the endogenous conditions, a large contribution of $Z_{tot}^{EN}$ belongs to the compressor as well as the cascade heat exchanger. In this case, the required heat exchanger area decreases as $T_3$ increases. Hence, the value of $Z_{tot}^{EN}$ is lowered according to the relation between the investment cost rate and the heat exchanger area [16]. Similar to Fig. 6, the high value of $Z_{tot}^{EN}$ in comparison with $Z_{tot}$ leads to a negative value for $Z_{tot}^{EX}$ indicating a weak interaction among the system components because, for all values of $T_3$, the endogenous part is higher than the exogenous one ($Z_{tot}^{EN} > Z_{tot}^{EX}$). On the other hand, splitting the total investment cost rate into the avoidable/unavoidable parts indicates that the high portion of $Z_{tot}$ belongs to unavoidable part which increases by about 4.3% with $T_3$ increasing and a little portion of $Z_{tot}$ is avoidable which decreases within 10.05% when $T_3$ grows. Therefore, the increment of $T_3$ has a negative effect on improvement potential of $Z_{tot}$. It is clearly observed that $Z_{tot}^{EN,AV}$ is more important than $Z_{tot}^{EX,AV}$ owing to its high positive value relative to $Z_{tot}^{EX,AV}$ and grows with $T_3$ increment. Moreover, it is revealed that $Z_{tot}^{EN,AV}$ decreases within 72.1% as $T_3$ increases.

Fig. 10. Effects of compressor mass flow rate on the total exergy destruction cost rate.

Fig. 8. Effects of refrigerator evaporator inlet temperature on the total investment cost rate.

Outcomes indicate that the behavior of the investment cost rate as $T_{11}$ increases is similar to that of Fig. 5 and the various parts of investment cost rates experiment no drastic variations because with $T_{11}$ increment, the mass flow rate of
streams does not change significantly and the values of enthalpies and entropies remain almost constant. Among the various parts of investment cost rate parts, the endogenous part has the maximum value (similar to the reason represented for Fig. 7) and its value decreases within 4.38% with $T_{11}$ growth. It is observed that the avoidable portion of endogenous part has a positive value and changes slightly. Therefore, the increment in $T_{11}$ leads to the increment of that part which can be reduced.

5.3. Effect of key parameters on various parts of $C_{D, tot}$

The results of the conventional and advanced exergy destruction cost rates variations as the mass flow rate of compressor ($m_1$) changes from 0.08 kg/s to 0.3 kg/s are plotted in Fig. 10. Obviously, the increment of $m_1$ causes an increase in the irreversibilities and thus $C_{D, tot}$ rises from 13820.35 $/year to 45895.20 $/year. A large portion of the exergy destruction rates is endogenous (103.78% of $C_{D, tot}$). Since $C_{D, tot} < C_{EN, tot}$, the values of $C_{EN, tot}$ become negative and their amounts are low compared to other exergy destruction cost rates. It is revealed that $C_{EN, tot}$ increases as $m_1$ rises and for all values of $m_1$, the endogenous part is larger than the exogenous one ($C_{EX, tot} < C_{EN, tot}$). Therefore, to decrease the exergy destruction cost rate, the efficiency of the components should be increased.

On the other hand, the $C_{UN, tot}$ value increases 241.53% as $m_1$ grows and $C_{AV, tot} < C_{D, tot}$ for all values of $m_1$. Since the value of $C_{UN, tot}$

---

Table 6

<table>
<thead>
<tr>
<th>Decision variable</th>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor mass flow rate, $m_1$ (kg/s)</td>
<td>0.08</td>
<td>0.3</td>
</tr>
<tr>
<td>Condenser inlet temperature, $T_3$ (°C)</td>
<td>32</td>
<td>40</td>
</tr>
<tr>
<td>Refrigeration evaporator inlet temperature, $T_6$ (°C)</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Freezer evaporator inlet temperature, $T_{11}$ (°C)</td>
<td>58</td>
<td>22</td>
</tr>
</tbody>
</table>

---

Fig. 11. Effects of condenser inlet temperature on the total exergy destruction cost rate.

Fig. 12. Effects of refrigerator evaporator inlet temperature on the total exergy destruction cost rate.

Fig. 13. Effects of freezer evaporator inlet temperature on the total exergy destruction cost rate.

Fig. 14. The flow chart of NSGA-II for the multi-objective optimization process.
is greater than \( C_{D_{\text{tot}}} \), the avoidable part is negative and decreases as \( m_1 \) increases. It is concluded that the increment in \( m_1 \) leads to the reduction of the improvement potential of \( C_{D_{\text{tot}}} \). A large contribution of \( C_{D_{\text{tot}}} \) is endogenous that increases drastically as \( m_1 \) increases and for all values of \( m_1 \), is the maximum value. 

The effects of \( T_3 \) on various parts of the total exergy destruction cost rate are plotted in Fig. 11. The amount of irreversibilities inside the system increases as \( T_3 \) grows. Therefore the value of \( C_{D_{\text{tot}}} \) increases within 15.68%. Under the unavoidable conditions, the pressure loss and \( \left( \frac{E_{x,1}}{E_{x,k}} \right)_{\text{UN}} \) values within the components rise leading to the increment of \( E_{x,\text{DUN}} \). According to Eq. (26), when the unavoidable exergy destruction cost rate rises, the avoidable one decreases. As observed from Fig. 11, \( C_{D_{\text{tot}}} \) increases by about 11.80% as \( T_3 \) increases. This indicates that substituting the new components or improving their efficiencies can reduce \( C_{D_{\text{tot}}} \). A large contribution of the total endogenous exergy destruction cost rate is unavoidable that decreases up to 11.21% as \( T_3 \) increases. According to Fig. 11, \( C_{D_{\text{tot}}} \) value is larger than other parts of the total exergy destruction cost rates. Moreover, the values of \( C_{D_{\text{tot}}} \) increase up to 19.99% with \( T_3 \) growth. A large portion of exogenous part is unavoidable which increases within 36.04% as \( T_3 \) increases while the avoidable part decreases with the same value.

The variations of the exergy destruction cost rate parts with varying \( T_6 \) from -5 °C to 0 °C are plotted in Fig. 12. As observed, \( C_{D_{\text{tot}}} \) decreases slightly by 14.70% when \( T_6 \) increases because under the real conditions, the temperature difference decreases in most components as \( T_6 \) rises. Therefore, the irreversibilities, the large portions of which are unavoidable parts, decrease. Under the unavoidable conditions, the increment of \( T_6 \) leads to the growth of the steam production by the separator causing a slight reduction of \( C_{D_{\text{tot}}} \). Similar to Fig. 11, due to \( C_{D_{\text{tot}}} > C_{D_{\text{tot}}} \), the values of \( C_{D_{\text{tot}}} \) are negative and almost fixed. As seen from Fig. 11, the significant portion of the total exergy destruction cost rate is endogenous and \( C_{D_{\text{tot}}} \) value is a bit more than \( C_{D_{\text{tot}}} \) leading to the negative values for \( C_{D_{\text{tot}}} \). It is concluded that the portion of improvement potential of the exergy destruction cost rate within each component which is due to the remaining components increases.

Fig. 13 demonstrates the effects of \( T_{11} \) on the various parts of the
total exergy destruction cost rate. When $T_1$ increases from $-59 \degree C$ to $-22 \degree C$, the cost rates associated with the fuel within the components rise which cause the increment in the total exergy destruction cost rate by 55.55% with the high percentage of unavoidable part. Under the unavoidable conditions, the irreversibilities and the ratio of $\frac{C_{UN}}{C_{EX}}$ for most components, especially for freezer evaporator and cascade heat exchanger, rise. Since a large portion of $C_{UN_D}^{tot}$ is related to the freezer evaporator and cascade heat exchanger and their $C_{UN_D}^{tot}$ values increase significantly by an increase in $T_1$, the values of $C_{UN_D}^{tot}$ rise and show a strong slope close to $-26 \degree C$ (because the irreversibilities of freezer evaporator and cascade heat exchanger reach their maximum values at this temperature). Moreover, as $T_1$ increases, the value of $C_{AV_D}^{tot}$ drops and has a drastic reduction near $-26 \degree C$ and its value is negative for all values of $T_1$, so the improvement potential of the system decreases by increasing $T_1$. Under the theoretical conditions all components except freezer evaporator and cascade heat exchanger remain fixed while the exergy destruction cost rates of these components increase causing the increment of the endogenous exergy destruction cost rate. As revealed from Fig. 13, the values of $C_{EX_D}^{AV_D}$ increase within 164.49% as $T_1$ rises. Thus, increasing the efficiency of other components can reduce the endogenous exergy destruction cost rates. On the other hand, $C_{EN_D}^{AV_D}$ drops by increasing $T_1$ and has the negative values because the unavoidable part of the endogenous cost rates are higher than the endogenous parts.

Table 7
GA parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>60</td>
</tr>
<tr>
<td>Individual number</td>
<td>16</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.2625</td>
</tr>
<tr>
<td>Maximum generation</td>
<td>64</td>
</tr>
</tbody>
</table>

Fig. 15. Pareto optimal frontier from multi-objective optimization for IARC.

Fig. 16. Pareto optimal (total avoidable exergy destruction cost and total avoidable exergy destruction) frontier for IARC.

Fig. 17. Pareto optimal (total avoidable investment cost and total avoidable exergy destruction) frontier for IARC.

Fig. 18. Pareto optimal (total avoidable exergy destruction cost and total avoidable exergy destruction cost) frontier for IARC.
6. Optimization

To assess the thermodynamic and thermoeconomic performance of the energy systems effectively, focus should be on the avoidable parts in advanced exergy and exergoeconomic analyses. As mentioned before, the avoidable parts of exergy and cost rates express the improvement potential of a system. Indeed, their high values indicate the great percentage of exergy as well as cost rates improvement in the system, so that by increasing the components efficiencies or substituting them with more effective components a significant amount of total exergy destruction and cost rates can be improved. According to Tables 3–5, the avoidable parts of exergy destruction and costs rates related to the most of the components are significant. Therefore, the study and optimization of these parts are valuable. Moreover, the sensitivity analyses in section 5 indicate that the behaviors of avoidable parts are affected by variations of some input variables. Thus, it is concluded that selecting the convenient values for input parameters in thermodynamic modelling leads to the increment in the improvement potential of the system and decrement of the unavoidable parts.

In this way, \( E_{AV, D} \), \( Z_{AV} \), \( C_{AV} \) are selected as three objective functions which should be maximized and the compressor mass flow rate, the condenser outlet temperature, the refrigeration evaporator inlet pressure and the freezer evaporator inlet pressure are selected as decision variables. Table 6 lists the range of decision variables.

The NSGA-II is used to find the optimum objectives and design parameters. Fig. 14 illustrates the flow chart of optimization which leads to the optimum objectives as well as the design parameters for the desired system and Table 7 lists the major parameters and their corresponding values in optimization algorithm.

The three dimensions (3D) Pareto frontier solutions is plotted in Fig. 15. In order to distinguish the behavior of objectives, the 2D diagrams are plotted in Figs. 16–18.

The 2D Pareto frontier solutions of \( E_{AV, D} \) and \( C_{AV} \) for selecting the optimal point in order to maximize these objectives are represented in Fig. 16. As clearly revealed, the behavior of these two objectives in Fig. 16 are in conflict, so that any change of decision variables which causes the increment of the total avoidable exergy destruction rate leads to the decrement of the total avoidable exergy cost rate and vice versa.

Fig. 17 represents the 2D Pareto frontier solutions for the total avoidable exergy destruction rate and the total avoidable investment cost rate. Results show the same behavior for these objectives, so that the increment of avoidable exergy destruction rate causes the increment of the total avoidable investment cost rate.

The 2D Pareto frontier solutions of the total avoidable cost rates for desired system are plotted in Fig. 18. It is clearly observed that the objectives are in conflict and maximizing the total avoidable exergy destruction cost rate leads to minimizing the total avoidable investment cost rate and vice versa.

If the single objective optimization is carried out for each objective, the combination of these values indicates the ideal point which is not located on the 3D Pareto frontier. Therefore, the closest point of Pareto frontier to this ideal point can be considered as the optimum solution. This decision making process can be performed by LINMAP method [1]. The obtained optimum values of \( E_{AV, D} \), \( Z_{AV} \) and \( C_{AV} \) with corresponding decision variables are listed in Table 8. Moreover, this optimal point is illustrated in Figs. 15–18 applying red marker.

As observed from Table 8, optimization of the system leads to the improvement of the system performance from the exergy and exergoeconomic concepts so that the values of \( E_{AV, D} \), \( Z_{AV} \) and \( C_{AV} \) are improved within 76.78%, 38.66% and 103.38%, respectively relative to the base case.

The optimization results indicate that the freezer refrigerator inlet temperature does not change and the compressor mass flow rate is close to the base point value while the temperature of refrigerator evaporator increases and the temperature of freezer evaporator decreases significantly related to the base case. Since at the optimum conditions, the thermodynamic and economic performance of the system are improved, the irreversibilities of the system drop, so that the avoidable part of exergy destruction cost rate increases within 103.38% because under optimum and base conditions the value of \( Z_{AV} \) does not change significantly while \( Z_{AV} \) increases strongly owing to the high increment of heat exchanger areas. Moreover, the investment cost rate of compressor increases with the mass flow rate.

7. Conclusion

In this research, an IARC is analyzed, assessed and optimized based on advanced exergy and exergoeconomic concepts. Four major parameters, namely \( m_1, T_4, T_6 \) and \( T_{11} \) are selected as decision variables, while the total avoidable exergy destruction and cost rates are selected as three objective functions for multi-criteria optimization by applying the NSGA-II algorithm. The main conclusions that can be drawn from this work are listed below:

- The growth of \( T_3 \) has a significant positive effect on the total avoidable exergy destruction rate within 88.19% while the increments of other parameters have negative effects on it.
- Increments of \( m_1 \) and \( T_5 \) increase the total avoidable investment cost rate by about 126.92% and 3.68%, respectively.
- The total avoidable exergy destruction cost rate increases within 5.67% as \( T_3 \) rises while other parameters have negative impacts on it.
- Multi-objective optimization results indicate that the values of \( E_{AV, D} \), \( Z_{AV} \) and \( C_{AV} \) are improved within 76.78%, 38.66% and 103.38%, respectively relative to the base point.

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Optimum values of the decision variables and objective functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>Base case</td>
</tr>
<tr>
<td>Total avoidable exergy destruction rate, ( E_{AV, D} ) (kW)</td>
<td>2.308</td>
</tr>
<tr>
<td>Total avoidable investment cost rate, ( Z_{AV} ) ($/year)</td>
<td>653.213</td>
</tr>
<tr>
<td>Total avoidable exergy destruction cost rate, ( C_{AV} ) ($/year)</td>
<td>–2551.87</td>
</tr>
<tr>
<td>Compressor mass flow rate, ( m_1 ) (kg)</td>
<td>0.08</td>
</tr>
<tr>
<td>Condenser inlet temperature, ( T_1 ) (°C)</td>
<td>40</td>
</tr>
<tr>
<td>Refrigeration evaporator inlet temperature, ( T_6 ) (°C)</td>
<td>–5</td>
</tr>
<tr>
<td>Freezer evaporator inlet temperature, ( T_{11} ) (°C)</td>
<td>–58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Term</th>
<th>Base case</th>
<th>Multi optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total avoidable exergy destruction rate, ( E_{AV, D} ) (kW)</td>
<td>2.308</td>
<td>4.08</td>
</tr>
<tr>
<td>2</td>
<td>Total avoidable investment cost rate, ( Z_{AV} ) ($/year)</td>
<td>653.213</td>
<td>905.761</td>
</tr>
<tr>
<td>3</td>
<td>Total avoidable exergy destruction cost rate, ( C_{AV} ) ($/year)</td>
<td>–2551.87</td>
<td>86.341</td>
</tr>
<tr>
<td>4</td>
<td>Compressor mass flow rate, ( m_1 ) (kg)</td>
<td>0.08</td>
<td>0.08683</td>
</tr>
<tr>
<td>5</td>
<td>Condenser inlet temperature, ( T_1 ) (°C)</td>
<td>40</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>Refrigeration evaporator inlet temperature, ( T_6 ) (°C)</td>
<td>–5</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Freezer evaporator inlet temperature, ( T_{11} ) (°C)</td>
<td>–58</td>
<td>–58</td>
</tr>
</tbody>
</table>
• The optimum values of $m_1$, $T_4$, $T_6$ and $T_{11}$ leading to the increase of avoidable values of the system are obtained 0.08683 kg, 32 °C, 0 °C and −58 °C, respectively.

References

[34] Shah RK, Subbarao EC, Mashehkar RA. Heat transfer equipment design. CRC Press; 1988.