Derating of distribution transformers under non-linear loads using a combined analytical-finite elements approach

Milad Ghazizadeh1, Jawad Faiz2✉, Hashem Oraee1

1Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
2Centre of Excellence on Applied Electromagnetic Systems, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran, Iran
✉E-mail: jfaiz@ut.ac.ir

Abstract: Supplying non-linear loads causes increased losses in transformers which eventually leads to their reduced life spans. Therefore, transformers are derated in order to protect them against premature loss of life. To do this, load losses including ohmic loss and winding eddy current (WEC) loss need to be estimated. This study suggests an improved analytical approach using finite element method (FEM) which includes all material characteristics and geometrical structures in order to calculate WEC loss under non-sinusoidal load current in each winding individually. By adopting this procedure, harmonic loss factor as a dominant parameter in transformers derating is estimated. By emphasising the region in which the maximum WEC loss occurs, an additional factor (F\text{RECL}) based on the physical phenomena involved is introduced using FEM. On the basis of the aforementioned concepts, a new relationship is presented for transformer derating. Ultimately, the suggested method and the IEEE standard are applied to derate a distribution transformer and the results are discussed. In addition, the proposed method is validated using a thermal model and its advantages over the IEEE derating method are presented.

1 Introduction

Transformers are essential parts of any power systems and must operate efficiently and with high reliability [1]. Traditionally, transformers are designed for sinusoidal operation and their pre-defined life spans are achievable under these conditions. In recent years, non-linear loads in power systems, particularly at the distribution level, have been increased considerably which have led to non-sinusoidal current in the power system. This current increases the losses and leads to temperature rise within the transformers. Since the temperature rise directly decreases the equipment life spans, transformers operation under non-sinusoidal currents causes accelerated ageing of the transformers’ windings insulations. Hence, transformers must be protected against this condition. The most common approach in dealing with this problem is the derating of transformers [2–4].

There are three indices for estimation of harmonic content in a signal; namely crest factor, total harmonic distortion (THD) and K-factor [5] of which the last one is used in derating of transformers. Since harmonic frequencies play a vital role in the derating process and the issue has been included in the K-factor estimation, this index provides a more reliable estimate of distortion level. Computer-aided testing program has been followed by voltage and current signal processing in order to measure single- and three-phase losses of an equipment accurately [6]. At this end, core and ohmic losses of a single-phase transformer under linear and non-linear loads are measured and the results are compared with the short-circuit and open-circuit tests data. Two identical single-phase transformers under rated load and partial load are connected back-to-back and ohmic and core losses are measured on-line [7]. By this connection, the measurement error is reduced considerably. In [8], a transformer is derated by an improved method based on the measurement of equivalent resistance of the transformer for fundamental and all harmonic components. It has also been shown that the eddy current loss over various frequency ranges differs considerably. Hence, it is concluded that a single derating method may not suffice. In [9–12], losses are measured for transformer derating. An iterative-based method using analytical equations is proposed for core loss and ohmic loss estimation in order to calculate derating factor (DF) under non-sinusoidal supply voltage for different linear loads (R, RL and RC) [13]. Derating of a transformer under non-linear load and magnetic core asymmetry is studied in [14] based on a time-domain non-linear analytical model. Some researchers have paid attention to the derating of transformers based on thermal analysis using thermal-insulation concept [15, 16]. In [17], finite element method (FEM) is used to predict a three-phase transformer behaviour under non-linear load in time domain. In this method, internal electromagnetic equations and external equivalent circuit are simultaneously solved by a well-established computational package. In order to determine the impact of harmonic currents on the transformer thermal behaviour, thermal-electromagnetic coupling analysis is applied [18]. In [19], FEM is employed to estimate transformer losses under rated sinusoidal condition and non-linear load and the transformer is derated based on the assumptions and procedure proposed in [2]. To determine premature fatigue of transformers under non-linear loads, load losses of a single phase transformer is estimated using two-dimensional (2D) FEM and finally the transformer loss of life is presented based on calculated hot spot temperature [20]. In [21], a mixed derating concept is introduced to derate a 50 kVA distribution transformer under both non-linear load and unbalanced over-voltage which lead to the increase of load and no-load losses, respectively.

All the above-mentioned references can be categorised into four groups [22]: the IEEE standards [2–4], experimental [6–12], analytical [13–16] and finite elements (FE) analysis [17–21]. The major problem with the IEEE standards is that the proposed assumptions ultimately lead to conservative results. Considering the high cost of transformers, this drawback is not economically justifiable. On the whole in the experimental method, the total transformer loss is the basis for derating. In this procedure, to prevent early failure of transformers, the total loss under rated sinusoidal and harmonic current conditions must be equal. Therefore, in this method, considering the transformers as electromagnetic systems having lumped parameters, all transformer loss components are included.
2 Estimation of WEC loss

2.1 Governing principles

As shown Fig. 1, transformer losses consist of load losses and no-load loss [24, 25]. The load losses include ohmic loss and stray losses. Stray losses consist of eddy current losses in the windings called ‘WEC loss’ and in other parts of transformer such as tank, clamps and magnetic shield called ‘other stray loss’.

On the basis of the adopted procedure in the present paper, loss components in windings are included in the derating process. Hence, ohmic loss and WEC loss must be considered and other loss components including other stray loss and no-load loss are disregarded. In fact, ohmic loss is the dc ohmic loss ($R_{dc}I^2$) which is estimated considering winding conductor geometrical and material specifications and root-mean-square (rms) value of the harmonic current. On the other hand, passing alternative electrical current through transformer windings results in skin, proximity and geometric effects which lead to WEC loss [26].

By increasing the current frequency, more electrical current passes along the surface of the conductor; hence, the current density increases by moving from the conductor centre to the surface of the conductor (skin effect). The proximity effect is the impact of the magnetic fields of adjacent conductors on the current density of the conductor under consideration [26]. The impact of the geometric effect is due to the resultant magnetic field on the current density of the conductor. In fact, the resultant magnetic field is a function of winding geometry, its distribution and electromagnetic system structure which itself depends on the presence or otherwise of effects such as saturation, air gap and magnetic permeability. Hence, based on the interaction between these factors, magnetic field value and its distribution in the windings are determined. Fig. 2 shows the three factors causing WEC loss. As implied from Fig. 2, due to skin effect, AC current causes non-uniform current distribution along windings cross-sections which produces WEC loss. Furthermore, for the conductor under consideration, adjacent conductors (leading to proximity effect) as well as other magnetic circuit components (resulting in geometry effect) create time-variant stray flux in the conductor leading to WEC loss.

Knowing the WEC loss under sinusoidal condition, it can then be estimated under non-sinusoidal current as follows [2-4]

$$P_{WEC} = P_{WEC-R} \sum_{h=1}^{n_{max}} \left( I_{ho}^2 \right) \left( \frac{h}{k} \right)^2$$

where $P_{WEC}$ is the WEC loss under non-sinusoidal condition with $I_{max} = \sum_{h=1}^{N_{max}} \left( I_{ho} \right) = I_{ho}^2$, $P_{WEC-R}$ is the same loss under rated sinusoidal condition, $h$ is the harmonic order, $I_{ho}$ is the $h$-order harmonic current in per unit and $h_{max}$ is the maximum harmonic order existing in the non-sinusoidal current signal.

2.2 Improved analytical method using FE analysis

To include the three above-mentioned effects in calculating the WEC loss, the following analytical equation is recommended for estimation of load losses in a winding due to passing rms current $I$ with frequency $\omega$ [26]

$$P(\omega) = R_{dc}\rho_1 k_1 I^2 + \frac{4\pi N L}{\mu_0} (B_0^2) \rho_N k_N$$

$$= R_{dc} I^2 + R_0 \left[ \rho_1 k_1 - 1 \right] I^2 + \frac{4\pi N L}{\mu_0} \mu_0 (B_0^2) \rho_N k_N$$

where (*) is ohmic loss, (***) is the WEC loss, $R_{dc}$ is the winding dc resistance, $N$ is number of turns, $L$ is the mean length of one turn, $\sigma$ is the electrical conductivity (in $\Omega^{-1}m^{-1}$), $\mu_0$ is the magnetic permeability of air and $\rho_1$ (taking into account skin effect), $k_1$ (representing proximity effect), $\rho_N$ (function of the conductors'

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rated power, kVA</td>
<td>50</td>
</tr>
<tr>
<td>primary voltage, kV</td>
<td>20</td>
</tr>
<tr>
<td>secondary voltage, V</td>
<td>400</td>
</tr>
<tr>
<td>frequency, Hz</td>
<td>50</td>
</tr>
<tr>
<td>no. of HV winding turns</td>
<td>6580</td>
</tr>
<tr>
<td>no. of LV winding turns</td>
<td>76</td>
</tr>
<tr>
<td>HV winding resistance/phase, $\Omega$</td>
<td>247.44</td>
</tr>
<tr>
<td>LV winding resistance/phase, $\Omega$</td>
<td>0.0212</td>
</tr>
<tr>
<td>window width, mm</td>
<td>120</td>
</tr>
<tr>
<td>window height, mm</td>
<td>295</td>
</tr>
<tr>
<td>HV winding height, mm</td>
<td>231</td>
</tr>
<tr>
<td>LV winding height, mm</td>
<td>242</td>
</tr>
<tr>
<td>core material</td>
<td>M5</td>
</tr>
<tr>
<td>inner diameter of HV winding cylinder, mm</td>
<td>161</td>
</tr>
<tr>
<td>inner diameter of LV winding cylinder, mm</td>
<td>118</td>
</tr>
<tr>
<td>electrical conductivity of conductors, S/m</td>
<td>5.8 x 10^7</td>
</tr>
</tbody>
</table>

Table 1 Specifications and dimensions of the distribution transformer
radius and their skin depth) and \( k_B \) (including proximity effect) are the dimensionless coefficients which are determined based on the geometry and operating frequency as explained in [26]. \( \langle B_0^2 \rangle \) is estimated as follows

\[
\langle B_0^2 \rangle = \frac{1}{S_w} \int \int_{S_w} B_0^2(x, y) \, dx \, dy
\]

where \( B_0(x, y) \) is the effective value of the magnetic flux density and \( S_w \) is the total cross-section of the winding. The winding load losses are therefore calculated over a wide frequency range using (2).

There are two problems in transformer derating based on the mentioned method:

(i) Equation (2) can estimate load losses only for a single frequency while in the derating process, they must be estimated under non-sinusoidal conditions (combination of different frequencies).

(ii) In spite of applying simplifying assumptions in analytical estimation of \( \langle B_0^2 \rangle \) (such as \( \mu_{core} = \infty \)) in [26], it is still difficult to include non-uniform distribution of magnetic flux density.

To solve the first problem, IEEE standard assumption [2–4] is used in which superposition of WEC loss is applied in order to obtain the total WEC loss by summation of WEC losses at different harmonics. Therefore, WEC loss of each winding under different current harmonics is first estimated and then WEC loss in each winding under non-sinusoidal load current is calculated using superposition. To solve the second problem, a 2D-FEM is used to determine magnetic field distribution within transformer based on the geometry and magnetic properties. FEM is a powerful method for solving Maxwell equations in an electromagnetic system. In this method, the system domain is divided into a large number of mesh cells and simplified electromagnetic equations are mapped to the cells. Finally, all electromagnetic parameters are estimated in every cell with applying the boundary conditions of the system. While smaller mesh cells lead to more accurate results, the computation time is longer. Some of the recent FEM software packages become able to modify meshes to limit the computational error to a predefined satisfactory level. Having the magnetic field distribution, estimation of the magnetic flux density, forces exerted on the windings and induced voltages are possible [27, 28]. Fig. 3 shows the magnetic flux density distribution of the transformer under consideration using FEM (OPERA-2D/TR) [29]. Hence, by using 2D-FEM, magnetic flux densities in different parts of transformer are determined and \( \langle B_0^2 \rangle \) can be estimated.

Finally, using (** of (2), WEC loss in each winding due to harmonic current is estimated based on the physical specifications of low voltage (LV) and high voltage (HV) windings. By applying superposition, the WEC loss in each winding under non-sinusoidal current is determined.

As will be observed, ratio of \( \frac{P_{WEC}}{P_{WEC-R}} \) plays a major role in the derating process. Hence, the IEEE standards [2–4] introduce the following ‘harmonic loss factor’ in transformer derating procedure

\[
F_{HL} = \frac{P_{WEC}}{P_{WEC-R}}
\]

where \( P_{WEC} \) is caused by a non-sinusoidal load current with a value of one per unit.
Considering the fact that $P_{WEC}$ is not known in the IEEE derating procedure, the IEEE standards propose another equation for indirect estimation of $F_{HL}$, based only on harmonic characteristic of load current as follows

$$F_{HL} = \sum_{h=1}^{h_{max}} \left( \frac{h}{R_h} \right)^2 h^2$$

(5)

### 3 Transformer derating principles under non-linear load

#### 3.1 Derating strategy

In loading and derating of transformers, the term ‘life span reduction’ is prevalent. In fact, transformers life reduction directly depends on their windings insulation; hence, using the term ‘reduction of transformer winding insulation life’ is more appropriate [23]. The factor leading to windings insulation life reduction under non-linear load current is the increase of transformer load losses under non-sinusoidal current. In addition, harmonic currents flowing through system impedance also produces small distortion in the voltage waveform at the transformer terminals. These voltage harmonics also cause extra harmonic losses in a transformer’s core. However, operating experience has not indicated that core temperature rise will ever be the limiting parameter for determination of safe magnitudes of non-sinusoidal load currents [2-4]; hence, core loss is not included in the derating process.

In insulation life reduction of any equipment such as transformers, special attention is paid to the windings hot spot as a dominant parameter. Therefore, in choosing the derating method under non-sinusoidal current, the transformer winding losses must be considered. While cooling system highly affects the hot spot region; however, it is assumed that the hot spot emerges as a result of the point experiencing maximum load losses. On the basis of this assumption, which is consistent with the IEEE method, it is possible to introduce a derating procedure based only on electrical parameters. As shown in Fig. 1, ohmic loss and WEC loss are defined as effective load losses.

On the other hand, in transformer windings, loss distribution is non-uniform. Considering the fact that ohmic loss is distributed uniformly, the hot spot occurs within the winding region which is subjected to maximum WEC loss. Hence, the basis for derating is the equality of effective load losses in the winding under sinusoidal and non-sinusoidal load currents. Hence, the requirement for introducing a powerful method in transformer derating is the estimation of the relevant parameters within the region where maximum WEC loss occurs. Since the basis of the suggested method for transformer derating and the IEEE standards is similar, the IEEE standards are discussed briefly and then the suggested method is presented.

#### 3.2 IEEE standard method

IEEE has published three standards concerning derating of transformers under non-linear load in 1988 [2], 1998 [3] and 2008 [4]. In these standards, two methods for derating of transformers have been introduced. While the first method uses exact WEC loss distribution, the second one is based on the information extracted from typical no-load and short-circuit test results. The first method is therefore more accurate for transformer design engineers while the second one has less accuracy but is easily applicable. In fact, the difference between these two methods is related to the maximum WEC loss density estimation. Stray losses consisting of stray loss in winding and other parts of a transformer are evaluated by subtracting ohmic loss from the load losses [24, 25]. In the first version of the IEEE standards, all stray losses are taken to be equal to WEC loss. This assumption leads to conservative results.

This deficiency is somewhat improved in the second and third versions of the IEEE standards. Therefore, henceforth the two last versions of the IEEE standards are considered.

In the IEEE method, LV side ohmic loss under rated condition is defined as base power and the following assumptions are considered for an oil-immersed 20 kV/400 V transformer

(i) No-load and short-circuit tests data are available.

(ii) The WEC loss contribution from the stray losses at rated sinusoidal load is defined as follows:

$$P_{WEC-R} = 0.33 P_{SL-R}$$

(6)

where $P_{SL-R}$ is the transformer stray losses under rated sinusoidal load.

(iii) Ohmic loss is uniformly distributed in each winding.

(iv) Neglecting the winding turns ratio, for transformers with rated current lower than 1000 A, WEC loss is divided into 60 and 40% in LV and HV windings, respectively.

(v) Eddy current loss distribution in each winding is non-uniform.

(vi) Maximum loss occurs at the hot spot and its density is four times of the average.

According to the aforementioned assumptions, the maximum LV WEC loss of a three-phase transformer in per unit (pu) is as follows:

$$M_{WEC-R,LV}^{(pu)} = 4 \times 0.6 P_{WEC-R} \times \frac{R_{LV} I_{LV,R}^2}{3 R_{LV} I_{LV,R}^2} = \frac{0.8 P_{WEC-R}}{R_{LV} I_{LV,R}^2}$$

(7)

where $P_{WEC-R}/(3 R_{LV} I_{LV,R}^2)$ is the WEC loss in pu, $R_{LV}$ is the LV phase winding resistance and $I_{LV,R}$ is the LV rms rated current.

Finally, to prevent early destruction of transformer under non-linear load based on the mentioned principle, the transformer DF will be as follows

$$DF = F^{(pu)} = \frac{1 + M_{WEC-R,LV}^{(pu)}}{1 + F_{HL} \times M_{WEC-R,LV}^{(pu)}}$$

(8)

Although the IEEE standard offers a simple method and only requires common no-load and short-circuit test results, it has the following two drawbacks:

(i) Since stray losses largely depend on the materials property and geometrical structure, it is expected that second and fourth assumptions are conservatively included.

(ii) Test results and FE analysis show that in low-power transformers, the WEC losses are distributed more uniformly [3, 4]. Hence, it is expected that the sixth assumption is conservative and higher than the required value.

On the basis of the aforementioned drawbacks, it can be concluded that the two latter versions of the IEEE standards are still conservative. Considering the large number of distribution transformers used in power systems, this conservative method has considerable economic implications. Therefore, a more comprehensive and economic method for transformer derating, particularly distribution transformers, should be introduced.

#### 3.3 Suggested derating method using combined analytical-FE approach

On the basis of the strategy discussed in Section 3.1, the effective load losses must be considered under rated sinusoidal and non-sinusoidal conditions as follows

$$P_{LL,eff-R} = P_{Ohmic-R} + P_{WEC-R}$$

(9)

$$P_{LL,eff} = P_{Ohmic} + P_{WEC}$$

(10)
where $P_{LL, eff-R, LV}$ and $P_{LL, eff}$ are the effective load losses under rated sinusoidal and non-sinusoidal load current, respectively, and $P_{Ohmic-R}$ and $P_{Ohmic}$ are ohmic loss under the same conditions. Additionally, to include the thermal-insulation requirements, (9) and (10) must be rewritten for the winding in which maximum WEC loss occurs. The results obtained using (2) show that the WEC loss in LV winding is larger than that of the HV winding. Hence, (1), (9) and (10) for the LV winding are rewritten as follows (the subscript 'LV' represents the previously defined quantities in LV windings):

$$P_{LL, eff-R, LV} = 3R_{LV}I_{LV-R}^2 + P_{WEC-R, LV}$$  \hfill (11)

$$P_{LL, eff-LV} = 3R_{LV} \sum_{h=1}^{h_{max}} I_{h, LV}^2 + P_{WEC-R, LV} \sum_{h=1}^{h_{max}} \left( p_{h}^{(pu)} \right)^2 h^2$$  \hfill (12)

By choosing LV ohmic loss under rated condition as the base power, (11) and (12) have the following simple per unit forms

$$P_{LL, eff-R, LV}^{(pu)} = 1 + P_{WEC-R, LV}^{(pu)}$$  \hfill (13)

$$P_{LL, eff-LV}^{(pu)} = \sum_{h=1}^{h_{max}} \left( p_{h}^{(pu)} \right)^2 + P_{WEC-R, LV}^{(pu)} \sum_{h=1}^{h_{max}} \left( p_{h}^{(pu)} \right)^2 h^2$$  \hfill (14)

On the basis of (5), (14) is rewritten as follows

$$P_{LL, eff-LV}^{(pu)} = \sum_{h=1}^{h_{max}} \left( p_{h}^{(pu)} \right)^2 + P_{WEC-R, LV}^{(pu)} \sum_{h=1}^{h_{max}} \left( p_{h}^{(pu)} \right)^2 h^2$$  \hfill (15)

To include the region with maximum WEC loss, it is sufficient to use the maximum WEC loss in pu in the LV winding instead of the LV WEC loss in (13) and (15) as follows

$$P_{LL, eff-R, LV}^{M(\text{pu})} = 1 + P_{WEC-R, LV}^{M(\text{pu})}$$  \hfill (16)

$$P_{LL, eff-LV}^{M(\text{pu})} = \sum_{h=1}^{h_{max}} \left( p_{h}^{M(\text{pu})} \right)^2 + F_{HL} \sum_{h=1}^{h_{max}} \left( p_{h}^{M(\text{pu})} \right)^2 h^2$$  \hfill (17)

where $P_{LL, eff-R, LV}^{M(\text{pu})}$ and $P_{LL, eff-LV}^{M(\text{pu})}$ are the maximum effective load losses in pu at rated sinusoidal load and non-sinusoidal load current in LV winding, respectively.

To complete the second stage and establish a relationship between LV WEC loss at rated sinusoidal load ($P_{WEC-R, LV}^{(\text{pu})}$) and the maximum LV WEC loss ($P_{WEC-R, LV}^{(\text{M(\text{pu})})}$) in pu, a novel factor called ‘maximum WEC loss factor’ based on part (***) of (2) is proposed as follows

$$F_{MECL} = \frac{P_{WEC-R, LV}^{(\text{M(\text{pu})})}}{P_{WEC-R, LV}^{(\text{pu})}} = \frac{M_{WEC-R, LV}}{P_{WEC-R, LV}^{(\text{pu})}}$$

$$= \frac{R_{LV} I_{LV-R}^2 [p_k I_k - 1] + (4\pi N_{LV} I_{LV-R}/\sigma_{\mu_0}) B_{0, max}^2/p_{N}}{R_{LV} I_{LV-R}^2 [p_k I_k - 1] + (4\pi N_{LV} I_{LV-R}/\sigma_{\mu_0}) B_0^2/p_{N}}$$  \hfill (18)

where (*) models the losses due to the skin and proximity effects, (***) models the loss due to the geometric effect and $B_{0, max}$ is the maximum rms flux density in LV winding. Hence, $\langle B_{0, max}^2 \rangle$ is as follows

$$\langle B_{0, max}^2 \rangle = \frac{1}{S_w} \int_{S_w} B_{0, max}^2 \, dx \, dy = \frac{1}{S_w} \left( B_{0, max}^2 \times S_w \right) = B_{0, max}^2$$  \hfill (19)

The ‘maximum WEC loss’ can be interpreted according to mathematical and physical concepts. On the basis of the mathematical concept, the maximum LV WEC loss is the maximised function of WEC loss in this winding. Therefore, to obtain maximum LV WEC loss, parameters must be determined such that the loss in LV winding is maximised. Considering the parameters in the denominator of (18), it is clear that all parameters except ($B_0^2$) are fixed based on the transformer specifications; hence, ($B_0^2$) must be maximised. To maximise ($B_0^2$), the maximum $B_0(d, x, y)$ on the LV winding cross-section must be used which leads to the nominator of (18). From a physical point of view, in generating WEC loss, three phenomena including skin, proximity and geometric effects are involved. Skin and proximity effects depend on the physical and geometrical specifications of the winding conductors and the frequency; therefore, they are distributed uniformly in all winding conductors. However, the geometric effect depends directly on the conductor position and non-uniform distribution of the magnetic flux density which cause different loss in different conductors. To find the maximum WEC loss in LV winding, it is sufficient to consider the point with highest effective flux density (hot spot). Fig. 4 shows the magnetic flux density distribution in LV and HV windings of the transformer under consideration using FE analysis. $B_{0, max}$ of LV winding in (18) is therefore determined with high accuracy based on the suggested procedure using FEM. In fact, according to the sixth assumption of the IEEE standard, $F_{MECL}$ equals to 4 while

---

**Fig. 4** Magnetic flux density distribution in space of LV and HV windings of the transformer
based on (18), for the transformer under consideration this factor is 1.710. Considering (18), (16) and (17) are rewritten as follows

\[ p_{\text{MIL,eff-R.LV}}^{(pu)} = 1 + F_{\text{MECL}} \times p_{\text{WEC-R.LV}}^{(pu)} \]  

\[ p_{\text{MIL,eff-LV}}^{(pu)} = \sum_{k=1}^{h_{\text{max}}} \left( p_{k}^{(pu)} \right)^2 \left[ F_{\text{HL}} \times \left( F_{\text{MECL}} \times p_{\text{WEC-R.LV}}^{(pu)} \right) \right] \times \sum_{k=1}^{h_{\text{max}}} \left( p_{k}^{(pu)} \right)^2 \]  

\[ \text{DF} = \frac{1 + F_{\text{MECL}} \times p_{\text{WEC-R.LV}}^{(pu)}}{1 + F_{\text{HL}} \times F_{\text{MECL}} \times p_{\text{WEC-R.LV}}^{(pu)}} = \frac{\sum_{k=1}^{h_{\text{max}}} \left( p_{k}^{(pu)} \right)^2}{\sqrt{\sum_{k=1}^{h_{\text{max}}} \left( p_{k}^{(pu)} \right)^2}} \]  

Finally, based on the fundamental principle proposed for derating, we have (see (22))

In the suggested equation for transformer derating, the factor \( F_{\text{MECL}} \) has been included to improve the derating process. Therefore, derating of transformer is considered in 3D. Fig. 5 shows the DF for all transformers with equal LV WEC loss (in pu) under different \( F_{\text{MECL}} \) and \( F_{\text{HL}} \).

It is clear that (22) is the same as the IEEE standard (8), if \( F_{\text{MECL}} \) is assumed to be equal to 4. Hence, assuming \( F_{\text{MECL}} = 4 \), the only factor causing discrepancy between the suggested derating approach and the IEEE method is the estimated value of \( F_{\text{HL}} \). In spite of the similarity of the presented equations in the suggested method and the IEEE standard in the proposed approach, the first drawback of the IEEE standard is removed using an improved analytical method with the help of FE analysis in which WEC loss in each winding has been estimated individually. Also, the second drawback, which is significant in distribution transformers with low ratings, has been removed by introducing a novel factor \( F_{\text{MECL}} \).

### 4 Derating of the distribution transformer under non-linear loads

To study the performance of the distribution transformer under non-sinusoidal current, a six-pulsed power electronics converter is used as a non-linear load. This load generates harmonics \( 6k \pm 1 \) (\( k \) is an integer). Also, to study the distortion effect on the derating analysis, four non-linear loads are defined such that the harmonic component increases 3\% compared with its previous load. The harmonic content of the loads are given in Table 2 which shows that from the first non-linear load (N.L.1) to the fourth non-linear load (N.L.4), the THD increases from 18.22 to 37.33\%.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>N.L.1</th>
<th>N.L.2</th>
<th>N.L.3</th>
<th>N.L.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.72</td>
<td>0.16</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>7</td>
<td>0.09</td>
<td>0.12</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>11</td>
<td>0.04</td>
<td>0.07</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>0.06</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>17</td>
<td>0.01</td>
<td>0.04</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0.03</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>THD, %</td>
<td>18.22</td>
<td>24.04</td>
<td>30.53</td>
<td>37.33</td>
</tr>
</tbody>
</table>

The loss base description of \( F_{\text{HL}} \) states that when a non-sinusoidal current has the same rms value as that of the rated sinusoidal current, the harmonic loss factor is the ratio of the WEC loss produced by non-sinusoidal current and its corresponding loss produced by rated sinusoidal current. Hence, the values given in Table 2 should be rewritten such that all defined harmonic current values are 1 pu. The new harmonic current values are given in Table 3.

The WEC loss for rated sinusoidal load and any of the non-linear loads is estimated based on the improved analytical method. To include non-linear property of magnetic core in the FE study, the analysis is followed in time domain. In this regard, fixed time step of 1/6400 s is employed for analysis. Due to the time step, 128 samples are extracted in each period enabling an accurate analysing of the system. The employed software uses either the incomplete Cholesky conjugate gradient (ICCG) solver or the direct solver. In contrast with the ICCG, the direct solver is a slower solver, but always converges. To set initial conditions in the FE software, it is assumed that the magnetic core is demagnetised at \( t=0 \); hence, \( B_0 \) in the core is equal to 0. Therefore, a three-phase excitation is applied to the transformer at \( t=0 \) and the system is being analysed. Assuming the mentioned structure as a magnetically isolated system (meaning other magnetic sources are infinitely far away from the under consideration structure), a corresponding boundary condition is assigned to the outside edges of the background object attempting to simulate space extending to infinity. Except magnetic core, all other materials are assigned in the 2D model using the software’s library. For the magnetic core, MS is introduced in the software’s library by inserting B-H data. In addition, all materials are defined to be isotropic. In this study, adaptive analysis is selected in which the problem is solved in an iterative procedure refining the regions of the mesh cells having the highest error. By adopting this process, the mesh cells become denser in the area of the highest error leading to results with high accuracy. After any iteration, the solver calculates the total energy of the model and the energy caused by solution error. If both the error percent and the difference between errors percent in the two last iterations are below the permissible level, estimated electromagnetic quantities are valid; otherwise, the mesh cells are refined in order to achieve the desired accuracy. To achieve the required accuracy, the solver divides the under consideration system domain into about 19300 mesh cells. The problem was solved in about 320 min by a PC with CORE™ i7 CPU and 16GB RAM.

As expected, distortion rise in the non-linear load current causes WEC loss to increase as shown in Table 4 while ohmic loss remains equal to its rated value. The results indicate that by increasing the non-linear load current distortion, the WEC loss increases from 18.087 W under rated sinusoidal load to 164.412 W under the N.L.4 load which shows a considerable increase.
The IEEE standards [2–4] state that the assumption of proportionality of WEC loss with the square of frequency for transformers having small conductors and non-sinusoidal current with low distortion is accurate. Therefore, the IEEE method is accurate in estimation of $F_{\text{HL}}$ under such conditions. Also, as the load current distortion increases, the IEEE method leads to conservative results [2–4]. Taking into account the two above-mentioned points, $F_{\text{HL}}$ is estimated using (4) and (5) based on the improved analytical method and the IEEE method, respectively, and the results are summarised in Table 5. According to Table 5, for the transformer under consideration under non-linear load with low distortion, it is clear that the estimated results using the proposed method are in good agreement with the results of the IEEE method. In addition, due to the increase in load current distortion, the IEEE method tends to estimate $F_{\text{HL}}$ conservatively as shown in Fig. 6.

The suggested derating approach differs from the IEEE method in estimation of $F_{\text{HL}}$ and $F_{\text{MECL}}$. The suggested method is considered in the following two approaches in order to clarify the impacts of the two factors upon transformer derating:

(i) In the first approach (FA), according to the IEEE standard, $F_{\text{MECL}}$ is assumed to be 4. Hence, the discrepancy between the suggested method and the IEEE method is due to the different estimated $F_{\text{HL}}$.

(ii) In the second approach (SA), $F_{\text{HL}}$ and $F_{\text{MECL}}$ are used for transformer derating based on the introduced equations.

Therefore, FA and the IEEE methods estimate DF based on the calculated $F_{\text{HL}}$ using (4) (loss base definition) and (5) (harmonic current characteristic base definition), respectively. In addition, (22) is used to derate the transformer based on SA.

Finally, DF values based on the IEEE, FA and SA methods for the defined non-linear loads are calculated and summarised in Table 6. This table shows that for N.L.1, THD = 18.22%, the derated powers using the IEEE and the suggested FA are 46.705 and 46.820 kVA, respectively, which are in good agreement. By increasing the distortion, the difference between these two methods for N.L.4 rises to 6.48%. On the other hand, comparison of the results of the IEEE method and the suggested SA method indicates that the difference for N.L.1 is 3.25% while it is increased to 18.28% for N.L.4. Therefore, although the IEEE standard estimates both $F_{\text{HL}}$ and $F_{\text{MECL}}$ conservatively, such estimation of $F_{\text{MECL}}$ plays a major role in the unrealistic results of the IEEE standard. Therefore, the suggested method, based on the analytical equation, and FE analysis is more accurate and economical.

### Table 3: Harmonic characteristic of four non-linear loads (in pu) with rms value equal to rated current

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>N.L.1</th>
<th>N.L.2</th>
<th>N.L.3</th>
<th>N.L.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.984</td>
<td>0.975</td>
<td>0.956</td>
<td>0.937</td>
</tr>
<tr>
<td>5</td>
<td>0.148</td>
<td>0.175</td>
<td>0.201</td>
<td>0.225</td>
</tr>
<tr>
<td>7</td>
<td>0.089</td>
<td>0.117</td>
<td>0.143</td>
<td>0.169</td>
</tr>
<tr>
<td>11</td>
<td>0.039</td>
<td>0.068</td>
<td>0.096</td>
<td>0.122</td>
</tr>
<tr>
<td>13</td>
<td>0.030</td>
<td>0.058</td>
<td>0.086</td>
<td>0.112</td>
</tr>
<tr>
<td>17</td>
<td>0.010</td>
<td>0.039</td>
<td>0.067</td>
<td>0.094</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0.029</td>
<td>0.057</td>
<td>0.084</td>
</tr>
</tbody>
</table>

### Table 4: WEC losses for different loads

<table>
<thead>
<tr>
<th>Rated linear</th>
<th>N.L.1</th>
<th>N.L.2</th>
<th>N.L.3</th>
<th>N.L.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEC losses, W</td>
<td>18.087</td>
<td>39.969</td>
<td>68.043</td>
<td>111.498</td>
</tr>
</tbody>
</table>

### Table 5: $F_{\text{HL}}$ estimation using the IEEE standard and proposed methods for different loads

<table>
<thead>
<tr>
<th></th>
<th>N.L.1</th>
<th>N.L.2</th>
<th>N.L.3</th>
<th>N.L.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{HL(IEEE)}}$</td>
<td>2.259</td>
<td>4.258</td>
<td>7.775</td>
<td>12.568</td>
</tr>
<tr>
<td>$F_{\text{HL(ANAL. and FEM)}}$</td>
<td>2.210</td>
<td>3.762</td>
<td>5.909</td>
<td>9.090</td>
</tr>
</tbody>
</table>

### Table 6: Derating factors and derated powers of the distribution transformer based on IEEE, FA and SA of suggested methods under non-linear loads

<table>
<thead>
<tr>
<th>Load type</th>
<th>DF(IEEE)</th>
<th>DF(FA)</th>
<th>DF(SA)</th>
<th>$S_{\text{derated(IEEE), kVA}}$</th>
<th>$S_{\text{derated(FA), kVA}}$</th>
<th>$S_{\text{derated(SA), kVA}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.L.1</td>
<td>0.9341</td>
<td>0.9364</td>
<td>0.9693</td>
<td>46.705</td>
<td>46.820</td>
<td>46.820</td>
</tr>
<tr>
<td>N.L.2</td>
<td>0.8519</td>
<td>0.8703</td>
<td>0.9338</td>
<td>42.595</td>
<td>43.515</td>
<td>46.690</td>
</tr>
<tr>
<td>N.L.3</td>
<td>0.7483</td>
<td>0.7982</td>
<td>0.8906</td>
<td>37.415</td>
<td>39.910</td>
<td>44.530</td>
</tr>
<tr>
<td>N.L.4</td>
<td>0.6535</td>
<td>0.7183</td>
<td>0.8363</td>
<td>32.675</td>
<td>35.915</td>
<td>41.815</td>
</tr>
</tbody>
</table>

**Fig. 6** Calculated $F_{\text{HL}}$ based on the proposed and the IEEE methods.
oil natural air natural cooling system, the heat loss at the hot spot is

\[ Q_{\text{LOST,HS}} = u_{H,1} - u_{W,O,R} u_{H,R} - u_{W,O,R} \left[\frac{m_{\text{HS,R}}}{m_{\text{HS,1}}}\right]^{1/4} (P_{\text{HS,R}} + P_{\text{EHS,R}}) \Delta t \]  

(25)

where \( \mu \) is the viscosity of the fluid (temperature dependent), \( \theta_{W,O,R} \) is the temperature of oil in proximity of the winding hot spot, \( P_{\text{HS,R}} \) and \( P_{\text{EHS,R}} \) are the modified ohmic loss and WEC loss at hot spot temperature at rated load, respectively. Finally, the hot spot temperature at \( t_2 = t_1 + \Delta t \) is calculated as follows

\[ \theta_{H,2} = Q_{\text{GEN,HS}} - Q_{\text{LOST,HS}} + M_w C_{P_w} \theta_{H,1} M_w C_{P_w} \]  

(26)

The procedure for estimation of \( M_w C_{P_w} \) is given in [23].

According to the transformer data sheet, steady-state temperature of the hot spot at the ambient temperature of 30°C is 107.5°C.

Ohmic and WEC losses at the hot spot are estimated using the suggested method. In order to evaluate the accuracy of the method, the hot spot temperatures at reduced non-linear loads are estimated. Also, the hot spot temperature under each load is estimated based on the IEEE method. To calculate \( P_{\text{EHS}} \) using the suggested method and the IEEE method, the following equations are used, respectively,

\[ P_{\text{EHS}} = P_{\text{WEC,R}} \times F_{\text{MECL}} \times F_{\text{HL(MN and FEM)}} \]  

(27)

\[ P_{\text{EHS}} = P_{\text{WEC,R}} \times 4 \times F_{\text{HL(IEEE)}} \]  

(28)

Ohmic and WEC losses at the hot spot under rated load and reduced non-linear loads are summarised in Table 7. Analysis of the data presented in Table 7 indicates that the IEEE method has lower accuracy in loss estimation due to inaccurate estimation of \( F_{\text{HL}} \) and \( F_{\text{MECL}} \). SA and the IEEE method have errors in estimation of losses at the hot spot (compared with their rated conditions). However, the suggested method has high accuracy in determining losses at rated load. To illustrate this point, it is noted that the error for N.L.1 using the suggested and the IEEE methods is 2.31 and 4.66%, respectively, while the corresponding values for N.L.4 is increased to 11.48 and 20.95%, respectively.
Results of the estimated hot spot temperature at rated load and reduced non-linear loads using SA and the IEEE method are presented in Figs. 7a and b, respectively. In the thermal modelling, the hot spot temperature at rated load for both methods is taken to be 107.5°C.

Considering (23)–(26), it is clear that the predominant losses in determination of the hot spot temperature are ohmic and WEC losses. This rationale is in agreement with the concept and principles outlined in the suggested method. Another conclusion of the analysis is that SA can fully satisfy the thermal requirements; therefore, the presented principles and calculations are valid. Furthermore, as the thermal analysis clearly shows, although both methods are conservative, the margin is improved considerably in SA which indicates an economic justification compared with the IEEE method. Also as already noted, by increasing the distortion level of non-linear loads, the IEEE method derates the transformer more conservatively while this drawback is reduced in severity in SA.

6 Conclusions
To prevent premature failure of distribution transformers insulation under non-linear loads, the transformers must be derated. In this context, an improved analytical method using FE analysis is devised to estimate WEC loss in each winding under non-linear load current. It is shown that estimated harmonic loss factor \( F_{\text{HL}} \) based on the proposed method is always smaller than that of the IEEE standard and the difference becomes larger as the distortion of non-linear load current is increased. Considering more uniform distribution of WEC loss in low power transformers, a novel factor ‘maximum WEC loss factor’ \( F_{\text{MECL}} \) is introduced based on mathematical and physical concepts. It is emphasised that the estimated factor in distribution transformers using the IEEE standard is conservative. Finally, based on \( F_{\text{MECL}} \), a new equation for transformer derating is proposed. To study the impact of \( F_{\text{HL}} \) and \( F_{\text{MECL}} \) on the distribution transformers derating, the suggested method is followed by two approaches. Comparison of the results by applying these two approaches with the IEEE standard shows that the IEEE standard derates the transformer under non-linear loads conservatively and this becomes more significant as the distortion level is increased. Furthermore, although the IEEE method is conservative in estimation of both \( F_{\text{HL}} \) and \( F_{\text{MECL}} \), \( F_{\text{MECL}} \) plays a more prominent role in leading to conservative results. The suggested method is verified using a model and the thermal requirements are satisfied. In addition, the thermal model indicates that while the method is still conservative, the derating is enhanced compared with the IEEE method from an economic point of view. Therefore, in spite of higher computational cost of the suggested method, it leads to more accurate and economic results and should be adopted in transformer derating particularly for non-linear loads with high distortion levels.

7 References
29. Opera-2D Software, Vector Field Co., U.K.