Constitutive relationships for hot deformation of austenite

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Abstract

Constitutive equations were used to derive the flow stress of a 17–4 PH stainless steel during hot compression testing. Two general methods were used: (i) a conventional method of finding apparent materials constants; and (ii) a physically based approach which accounts for the dependence of the Young’s modulus and the self-diffusion coefficient of austenite on temperature. Both methods were critically discussed and some modifications and easy-to-apply methods were also introduced. The second approach was also performed for peak and critical stresses to find out the effect of dynamic recrystallization on the ideal theoretical values. The discussion of results proved that when the deformation mechanism is controlled by the glide and climb of dislocations, a constant creep exponent (n) of 5 can be used in the classical hyperbolic sine equation, and the self-diffusion activation energy can be used to describe the appropriate stress.

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1. Introduction

Hot deformation processing of steels, which is usually conducted in the stability range corresponding to the austenite phase, plays an important role in industry for the production of steels with a number of desirable mechanical properties while keeping production costs as low as possible. In order to achieve this goal, the parameters of the forming process must be carefully controlled. An understanding of the microstructural behavior of the steel under consideration is therefore required, together with the constitutive relation describing material flow.

The modeling of hot flow stress using constitutive equations is quite important in metal-forming processes from the mechanical and metallurgical standpoints, because any feasible mathematical simulation needs an accurate flow description. As a result, considerable research has been carried out to model the flow stress of metals and alloys, and steels in particular [1–15]. The simplest and most widely applied method in the literature is the modeling of flow stress using an expression which relates the Zener–Hollomon parameter (Z) [16] to the flow stress (σ):

\[ Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = f(\sigma) \]  

(1)

In Eq. (1), the Zener–Hollomon parameter is the temperature-compensated strain rate and Q is the activation energy of deformation. It was shown by Sellars and Tegart [8,9] and others [17–21], using the hyperbolic sine function suggested by Garofalo [22], that hot working can be considered as a thermally activated process and can be described by strain-rate equations similar to those employed in creep studies. Based on these works, the Z parameter can be related to the flow stress in different ways [23]:

\[ Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = f(\sigma) = A' \sigma^n \]  

(2)
where \( A', A'', A', n', n, \beta \) and \( \alpha(\approx \beta/n') \) are known as apparent material constants. The stress multiplier \( \alpha \) is an adjustable constant which brings \( \alpha \sigma \) into the correct range that gives linear and parallel lines in ln \( \dot{\varepsilon} \) vs. ln \( \sinh(\alpha \sigma) \) plots [18,23,24]. Some authors [25] consider \( \alpha \) as the inverse stress at which the power law breaks (Eq. (2)). This power-law description is preferred for relatively low stresses. Conversely, the exponential law (Eq. (3)) is suitable for high stresses. However, the hyperbolic sine law (Eq. (4)) can be used for a wide range of temperatures and strain rates. Using the definition of the Zener–Hollomon parameter, Eq. (4) can be rewritten as:

\[
\dot{\varepsilon} = A[\sinh(\alpha \sigma)]^n \exp(-Q/RT)
\]

Although the constants of this equation (\( A, \alpha, n, \) and \( Q \) depend on the material being considered, they are also usually referred to as apparent values, because no account is generally taken of the internal microstructural state and they are only derived from an Arrhenius plot with a linear range and the assumption that the microstructure remains constant. Therefore, this method is called the “apparent approach” throughout this paper. Cabrera et al. [25–28] showed that when the dependence of Young’s modulus (\( E \)) and the self-diffusion coefficient of austenite (\( D \)) on temperature are taken into account, a constant creep exponent \( n = 5 \) and self-diffusion activation energy (\( Q_{sd} \)) can be used to describe the appropriate stress. The latter is true as long as the deformation mechanism is controlled by the glide and climb of dislocations. Accordingly, the unified relation can be expressed as:

\[
\dot{\varepsilon}/D(T) = B[\sinh(\alpha/\sigma)(E(T))]^n
\]

where \( D(T) = D_0 \exp(-Q_{sd}/RT) \) (with \( D_0 \) a pre-exponential constant) and \( E(T) \) describes the dependence of the Young’s modulus on temperature. The values of \( D_0 \) and \( Q_{sd} \) can be taken from the tables given by Frost and Ashby [29,30]. In these tables, the dependence of the shear modulus (\( G \)) on temperature in the form of \( G(T) = G_0 \left(1 - \frac{T_M}{T_0} \right)^{\frac{T_0 - 300}{T_M}} \) is also available. Here, \( G_0 \) is the shear modulus at 300 K, \( T_M \) is the temperature dependence of modulus, and \( T_M \) is the melting temperature of the material. According to the relation of shear and Young modulus, the values of \( E(T) \) can be calculated. This method is referred to as the “physically based approach”.

In this paper, the apparent and the physically based approaches are used to derive the constitutive equations of a 17–4 PH stainless steel during hot compression testing. Both methods are critically discussed and some modifications and easy-to-apply methods are also introduced. 17–4 PH (AISI 630) is more common than any other type of precipitation-hardened stainless steel. Industrial hot deformation processing such as forging for this steel is conducted in the temperature range at which the austenite phase is stable. The ability of 17–4 PH alloy to develop very high strength without catastrophic loss of ductility and its superior corrosion resistance compared to other steels of similar strength, have made it very attractive to engineers.

2. Experimental materials and procedures

A 17–4 PH stainless steel with chemical composition of 0.03 wt.% C–15.14 wt.% Cr–4.53 wt.% Ni–3.4 wt.% Cu–0.25 wt.% Nb was used in this work. Single-hit hot compression tests at high strain rates were performed using a Baehr DIL-805 deformation dilatometer, while for tests at low strain rates, an Instron 4507 universal deformation machine was used. However, some tests were performed at intermediate strain rates using these two machines in order to check if both testing systems offered similar results. Therefore, in order to cover a wide range of temperatures (900–1150 °C) and strain rates (10^{-4}–10^{-1} s^{-1}), and to check the reproducibility of our results, these two types of deformation machines were used and both sets of resultant flow curves were used simultaneously for constitutive analyses.

2.1. Hot compression tests using the Baehr DIL-805 machine

Specimens 10 mm high and 5 mm in diameter were prepared. In order to minimize the occurrence of inhomogeneous compression due to the existence of friction between the anvils and the specimen surface, the Rastegaev design [24] was used, in which the entire end face of the specimen was machined away except for a small rim to form a reservoir. Subsequently, these reservoirs were filled with glass powder as a lubricant material and the specimen was placed between Al_{2}O_{3} anvils in the vacuum chamber. Inductive heating and Ar gas quenching were used for thermal treatments. The specimen was austenitized at 1180 °C for 10 min and cooled down at a rate of 1.5 °C s^{-1} to the deformation temperature and held there for 5 min before hot compression testing. Single-hit hot compression tests were carried out at temperatures of 950–1150 °C with strain rates of 10^{-3}–10^{-1} s^{-1}.

2.2. Hot compression tests using the Instron 4507 machine

Cylindrical specimens, 11.4 mm in height and 7.6 mm in diameter, were prepared for hot compression testing using the Instron 4507 universal deformation machine. Tantalum foils and boron nitride solution were used to reduce friction in this case. The specimen was austenitized at 1100 °C for 15 min and cooled down at a rate of 1.5 °C s^{-1} to the deformation temperature and held there for 5 min before hot compression testing. After deformation, the samples were immediately quenched in water. Single-hit hot compression tests were carried out at temperatures of 900–1100 °C with strain rates of 10^{-4}–0.1 s^{-1}.
2.3. Flow curve analysis

The analyses based on the work-hardening rate ($\theta$) were performed to reveal if dynamic recrystallization (DRX) occurred and also to determine the characteristic points of the flow curves. The $\theta$ values were obtained using the central difference approach [24]. The onset of DRX was detected from the inflections in plots of $\theta$ vs. $\sigma$ or from the minimum in plots of $-d\theta/d\varepsilon$ vs. $\sigma$ (before the peak stress for both type of curves) [31,32]. The latter curves were also used to detect the critical stress for initiation of DRX ($\sigma_C$). Moreover, the values of peak stress ($\sigma_P$) were detected from the first occurrence of $\theta = 0$ in the $\theta$–$\sigma$ curves. It should be mentioned that this technique requires differentiation of the stress–strain curve to determine the values of $\theta$, but short-range noise can render such differentiation calculus impossible. In order to solve this problem, a smoothing by fitting a high-order polynomial to the flow curve was used to eliminate the irregularities and fluctuations in the experimental curves. Therefore, after removal of the elastic portion of flow curves, each curve was fitted and smoothed with a 7th- to 9th-order polynomial.

3. Results

3.1. Stress–strain curves

Flow curves obtained at different temperatures and strain rates using the Baehr DIL-805 and Instron 4507 are shown in Fig. 1a and b. Most of the curves exhibit typical DRX behavior with a single peak stress followed by a gradual fall toward a steady-state stress [33]. However, the peak stress becomes less obvious when the strain rate is increased or the deformation temperature is decreased. There is no significant difference between the flow stress values obtained using the Baehr DIL-805 and Instron 4507 at the same deformation conditions. The slight difference observed can be ascribed to the difference in heating conditions (induction in the dilatometer, and radiation in the Instron), specimen sizes, and temperature control of the instruments.

The flow curves also show that the flow stress decreases as the deformation temperature increases or the strain rate decreases. The drop in flow stress with deformation temperature may be attributed to the increase in the rate of restoration processes and the decrease in the strain-hardening rate. Since the formation of DRX nuclei becomes easier at higher deformation temperatures, the critical strain for initiation of DRX decreases. Moreover, the mobility of grain boundaries increases with increasing deformation temperature and hence the rate of DRX increases. Therefore, both the peak and steady-state strains decrease with deformation temperature. The increase in the flow stress with strain rate can be ascribed to the decrease in the rate of restoration processes and the increase in the strain-hardening rate [34]. The rate of DRV also decreases with increasing strain rates. Since well-developed substructures due to DRV are observed in DRX microstructures and are the origin of DRX nuclei [35–38], the increase in the critical strain for initiation of DRX with increasing strain rate is reasonable. The mobility of grain boundaries decreases with increasing strain rate, which in turn increases the peak and steady-state strains.

A comparison between the hot deformation flow curves of 17–4 PH stainless steel and AISI 304 austenitic stainless steel has been made in Fig. 1c. These flow curves were taken from the literature [39–42]. However, in one case,
the hot compression test, based on the details of Section 2.2, was performed on an AISI 304 stainless steel sample with chemical composition 0.064 wt.% C–18.35 wt.% Cr–8.96 wt.% Ni. Since the 17–4 PH stainless steel is austenitic at hot working conditions, its flow behavior can be compared to a similar austenitic stainless steel such as AISI 304 stainless steel. Fig. 1c reveals that the level of stresses of 17–4 PH stainless steel is comparable to those of AISI 304 stainless steel. However, the observed difference is mainly due to the differences in the chemical compositions of the steels. In general, AISI 304 stainless steel shows higher flow stresses, which can be attributed to its higher level of alloying elements.

3.2. Apparent constitutive analysis

The description of flow stress by Eqs. (2)–(4) is incomplete, because no strain for determination of flow stress is specified. Therefore, characteristic stresses that represent the same deformation or softening mechanism for all flow curves, such as steady state, peak or critical stress for initiation of DRX, should be used for this purpose. It should be noted that the nature of material constants and equations are dependent on the characteristic stress used to derive them. In general, the peak stress is the most widely accepted parameter used to find the hot working constants. Using the definition of the Zener–Hollomon parameter in Eqs. (2)–(4), and taking the natural logarithm of both sides of the resulting equations, the following expressions can be derived for the peak stress:

\[
\ln \dot{\varepsilon} + \frac{Q}{R} \left( \frac{1}{T} \right) = \ln A' + n' \ln \sigma_p
\]  
(7)

\[
\ln \dot{\varepsilon} + \frac{Q}{R} \left( \frac{1}{T} \right) = \ln A'' + \beta \sigma_p
\]  
(8)

\[
\ln \dot{\varepsilon} + \frac{Q}{R} \left( \frac{1}{T} \right) = \ln A + n \ln \left\{ \sinh (a \sigma_p) \right\}
\]  
(9)

At constant deformation temperature, and assuming the activation energy is constant, partial differentiation of Eqs. (7)–(9) yields, respectively, \( n' = \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \sigma_p} \), \( \beta = \frac{\partial \ln \dot{\varepsilon}}{\partial \sigma_p} \), \( n = \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \left\{ \sinh (a \sigma_p) \right\}} \). It follows from these expressions that the slope of the plot of \( \ln \dot{\varepsilon} \) vs. \( \ln \sigma_p \) and the slope of the plot of \( \ln \dot{\varepsilon} \) vs. \( \sigma_p \) can be used to obtain the values of \( n' \) and \( \beta \), respectively. These plots are shown in Fig. 2. The linear regression of these data results in average values of 423, 499 and 457 kJ mol\(^{-1}\) for the activation energy according to Eqs. (10)–(12), respectively. Analysis of the correlation coefficient (\( R^2 \)) of these regression values (0.941, 0.922, and 0.935, respectively, for Eqs. (10)–(12)) reveals that Eq. (10) has a slightly better fit to experimental data. Therefore, the apparent activation energy of hot working can be taken as 423 kJ mol\(^{-1}\). However, since the values of these correlation coefficients are close to each other, the appropriateness of the fit given by these three equations is practically the same. Therefore, the average value of these three activation energies (460 kJ mol\(^{-1}\)) was used for further analyses. According to Eq. (4), the plot of \( \ln Z \) vs. \( \ln \left\{ \sinh (a \sigma_p) \right\} \) may be used to find the relationship between \( Z \) and \( \sigma_p \). The corresponding curve is shown in Fig. 4 and the resultant equation with new regression constants is as follows:

\[
\dot{\varepsilon} \exp \left( 460 \times 10^3 / RT \right) = 1.05 \times 10^{16} \times \left[ \sinh (0.0105 \sigma_p) \right]^{4.828}
\]  
(13)
Therefore, the final value of \( n \) that represents the aggregate of all the data points is about 4.8.

### 3.3. Physically based constitutive analysis

The apparent activation energy of hot working is 460 kJ mol\(^{-1}\). This value is very different from that found for self-diffusion in austenite, which is about 280 kJ mol\(^{-1}\) \([29,30]\). The apparent hot working activation energy values are often much larger than any imagined atomic mechanism. Therefore, a physically based approach based on Eq. (6) is also used in this work in order to directly deal with the atomic mechanisms.

In the Frost and Ashby tables, 304 stainless steel is the material most similar to the 17–4 PH stainless steel used here. Therefore, the Frost and Ashby data for 304 stainless steel were used in the present study. However, the average melting temperature of \( T_M = 1422^\circ C \) and also the Young’s modulus at 300 K of \( E_0 = 197 \) GPa \([43]\) were used in the above-mentioned formulas. Therefore, the following expressions were used for \( D(T) \) and \( E(T) \):

\[
D(T) = 3.7 \times 10^{-5} \times \exp(-280000/RT) \tag{14}
\]

\[
E(T) = 197000 \times \left(1 - 0.85 \frac{(T - 300)}{1695}\right) \tag{15}
\]

There are only two unknown parameters, \( B \) and \( \alpha' \), to be determined. In order to find the value of \( \alpha' \), a new approach is introduced in this paper, which only requires a linear regression and is easy to apply. A similar approach to Section 3.2 was used for this purpose, in which Eqs. (2) and (3) were modified as follows:

\[
\dot{\varepsilon}/D(T) = B' (\sigma_P/E(T))^n' \tag{16}
\]

\[
\dot{\varepsilon}/D(T) = B'' \exp(\beta'\sigma_P/E(T)) \tag{17}
\]

It follows from these expressions that the slope of the plot of \( \ln(\dot{\varepsilon}/D(T)) \) vs. \( \ln(\sigma_P/E(T)) \) and the slope of the plot of \( \ln(\dot{\varepsilon}/D(T)) \) vs. \( \sigma_P/E(T) \) can be used to obtain the values of \( n' \) and \( \beta' \), respectively. These plots are shown in Fig. 5. The linear regression of these data, results in the average values of 6.126 and 5885.4 for \( n' \) and \( \beta' \), respectively. This gives the value of \( \alpha' = \beta'/n' = 960.71 \). A visual comparison between Figs. 5 and 2 shows that in the physically based approach, unlike the apparent approach, there is no need to consider separate regression analyses for each deformation temperature in order to find the value of stress multiplier, \( \alpha \) or \( \alpha' \). This shows that consideration of the dependence of the Young’s modulus and the self-diffusion
coefficient of austenite on temperature and inserting these parameters into the constitutive equations simplifies further regression analysis.

According to Eq. (6), the slope of the plot of $f_e = D(T)g^{1/5}$ vs. $\sinh(a_0r_p/E(T))$ by fitting a straight line with an intercept of zero ($y = ax + 0$) was used to obtain the value of $B^{1/5} = 462.19$ (Fig. 6). Therefore, the resultant constitutive equation can be expressed as:

$$
\dot{\epsilon} \exp\left(\frac{280 \times 10^3}{RT}\right) = \frac{7.86 \times 10^4 \times \sinh(960.71)}{E(T)}
$$

(18)

4. Discussions

Both the apparent and physically-based methods described herein are capable of representing the flow stress of the material as a function of the deformation conditions. The base equations for these two types of analyses have the same origin, but inserting the $D(T)$ and $E(T)$ into the equations leads to the hot deformation constants in these two approaches having different natures, as can be seen in Eqs. (13) and (18). However, another source of this difference is the consideration of $n$ as a constant with the value of 5 and also consideration of the self-diffusion activation energy, which is true as long as the deformation mechanism is controlled by the glide and climb of dislocations [25–28].

The apparent value of $n$ is 4.828, which is near the integer value of 5. However, what happens if $n$ is considered as a variable in Eq. (6)? In order to investigate this, Eq. (6) was changed to its general form as follows:

$$
\dot{\epsilon}/D(T) = B[\sinh(\alpha_0\sigma_p/E(T))]^n
$$

(19)

Again, the value of $\alpha = 960.71$ was determined for the peak stress using the method described in Section 3.3. According to Eq. (19), the slope and the intercept of the plot of $\ln(\dot{\epsilon}/D(T))$ vs. $\ln[\sinh(\alpha_0\sigma_p/E(T))]$ were respectively used to obtain the values of $n$ and $\ln B$ (Fig. 7). These analyses gave values of $n = 4.796$ and $B^{1/5} = 491.273$. Therefore, the resultant constitutive equation can be expressed as:

$$
\dot{\epsilon} \exp\left(\frac{280 \times 10^3}{RT}\right) = \frac{10.59 \times 10^8 \times \sinh(960.71)}{E(T)}
$$

(20)

It appears that the best value of $n$, i.e. $\sim4.8$, is near to 5 but not exactly close to this value. From the metallurgical standpoint, by consideration of the self-diffusion activation energy, the value of 5 is true as long as the deformation mechanism...
mechanism is controlled by the glide and climb of dislocations. Due to this, the only softening mechanism considered is DRV. However, in the above-mentioned analyses, the peak stress points have been used to find the regression values after the initiation of DRX. As a result, the DRX softening is somehow affecting the stress values. This could lead to the decrease in value of $n$ from the theoretical value of 5 to 4.8. To prove this, the analyses were repeated for the critical stress for initiation of DRX. The same procedure was followed and resulted in Eqs. (21) and (22) using Eqs. (6) and (19), respectively. The corresponding figures are also shown in Figs. 8 and 9.

The calculated value of $n = 4.945$ for the critical stress by consideration of the self-diffusion activation energy is now close to 5, which confirms that the deformation mechanism close to the critical point for the onset of DRX is controlled by the glide and climb of dislocations. This is consistent with the theory of hot deformation and introduces an alternative method for representation of flow stress in hot working, which is not only effective and simple, but also has physical and metallurgical backgrounds. Therefore, the physically based approach based on Eq. (6) is highly recommended for calculation of material constants using the critical stress. As mentioned before, it is usual to use the peak stress for regression analyses in hot working. In many cases, the volume fraction of DRX is low at peak stress. As a result, Eq. (6), based on the method described in this paper, can be used safely for the peak stress.

5. Conclusions

(1) Most of the stress–strain curves of the 17–4 PH stainless steel, over the wide range of temperatures and strain rates used in this study, exhibited typical DRX behavior with a single peak stress followed by a gradual fall toward a steady-state stress.

(2) It was shown that for correct calculation of apparent deformation activation energy, the three expressions of $Z$, namely the power law, exponential law and hyperbolic sine law, should be analyzed. For the experimental alloy, these analyses yielded the following constitutive equation to express the hot working characteristics of the investigated alloy:

$$
\dot{\varepsilon} \exp\left(\frac{460 \times 10^3}{RT}\right) = 1.05 \times 10^{16} \times \left[\sinh\left(0.0105 \times \sigma_c / E(T)\right)\right]^{4.828}.
$$

(3) A new approach, termed “physically based”, is introduced in this paper for calculation of the materials constants; this approach only needs a linear regression and is easy to apply. For the 17–4 PH stainless steel, the following rate equation was determined:

$$
\dot{\varepsilon} \exp\left(\frac{280 \times 10^3}{RT}\right) = 7.86 \times 10^8 \times \left[\sinh\left(960.71 \times \sigma_c / E(T)\right)\right]^{5}.
$$

(4) By consideration of $n$ as a variable, the value of 4.8 was determined using the physically based approach, which is not exactly close to the ideal value of 5. This decrease was related to the occurrence of DRX in this material until the peak point of the stress–strain curves.

(5) The value of $n$ for the critical stress for initiation of DRX was determined to be 4.945, which is close to 5; this proved that when the deformation mechanism is controlled by the glide and climb of dislocations, a constant creep exponent ($n$) of 5 and self-diffusion activation energy can be used to describe the...
appropriate stress. This approach is a powerful technique for calculating material constants using the critical stress. For the 17–4 PH stainless steel, the following rate equation was determined:

\[ \dot{\varepsilon} \exp(280 \times 10^3/RT) = 7.42 \times 10^8 \times \left[ \sinh(1052.5 \times \sigma_c/E(T)) \right]^5. \]

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