Comment on: ‘Improving compact gravity inversion based on new weighting functions’, by Mohammad Hossein Ghalehnoee, Abdolhamid Ansari and Ahmad Ghorbani

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SUMMARY

The recent paper of Ghalehnoee et al., ‘Improving compact gravity inversion based on new weighting functions’, discusses weighting functions for the compact inversion of gravity data. We studied the paper with great interest but deduced that the paper presents minor changes to already published methods. In the manuscript, the model weighting function is the product of three diagonal matrices, that is, a depth weighting matrix, a compactness constraint and a scaling matrix. The authors claim that the scaling matrix is new and introduce the notation ‘kernel weighting’. Based on our knowledge and understanding of the ideas, not only all the matrix weighting matrices have been used before but also their combination has been used in many published research papers. Here we explain why we believe that the ideas in Ghalehnoee et al. are not new.

Key words: Inverse theory; Numerical approximations and analysis; Gravity anomalies and Earth structure; Asia.

1 INTRODUCTION

Compact inversion is a well-known method for the reconstruction of focused images of the subsurface in the area of potential field data inversion. The methodology was first presented as an approach for the minimization of the volume (area in 2-D) of the causative body by Last & Kubik (1983) and is equivalent to maximizing for the minimization of the volume (area in 2-D) of the causative body by Last & Kubik (1983) and is equivalent to maximizing

\[ \text{Compactness of the body. Guillen & Menichetti (1984)} \]

\[ \text{modifying the approach by introducing a functional which minimizes the moments of inertia with respect to the centre of gravity, or with respect to a given dip line passing through the centre of gravity. The minimum support constraint, as introduced by Portniaguine & Zhdanov (1999), includes a prior model in the constraint condition. In all these cases, the minimization of the functional generates a sharp and focused image of the subsurface. Portniaguine & Zhdanov (2002) then modified the compactness constraint through a scaling of the model vector by the integrated sensitivities of the kernel matrix; an idea which originated with the works of Li & Oldenburg (1996, 1998) in which a depth weighting function is applied to the potential field data so as to counteract the natural decay of the kernel. The scaling approach suggested by Portniaguine & Zhdanov (2002) is equivalent, but not equal, to a depth weighting function as presented at different depths were reconstructed well and the resolution was improved as compared with a smooth inversion methodology (Pilkington 2009).}

The supposedly novel technique suggested in Ghalehnoee et al. (2017) is, in fact, the multiplication of the depth weighting of Li & Oldenburg (1996, 1998) with the compactness constraint of Last & Kubik (1983) and the scaling of Portniaguine & Zhdanov (2002), but using the name kernel weighting matrix for the scaling matrix. In the following we explain why the results in Ghalehnoee et al. (2017) do not present anything new.

2 INVERSION METHODOLOGY

We start with the assumption, as given in Ghalehnoee et al. (2017), that the update of the linear inverse problem \( \mathbf{g}^{\text{obs}} = \mathbf{A}\mathbf{m} + \mathbf{e} \) at iteration \( k \) is given by

\[ \delta \mathbf{m}^{k+1} = W_m^{-1} A^T (A W_m^{-1} A^T + \mu W_e^{-1})^{-1} \delta \mathbf{g}^k, \quad \delta \mathbf{g}^k = (\mathbf{g}^{\text{obs}} - \mathbf{A}\mathbf{m}^k) \]  

\[ \mathbf{m}^{k+1} = \mathbf{m}^k + \delta \mathbf{m}^{k+1}. \]  

This is a well-known formula, used in many papers with similar but not always identical notation (Last & Kubik 1983; Guillen & Menichetti 1984; Barbosa & Silva 1994; Chasserau et al. 2014a,b, 2015). In all cases, the sources located at different depths were reconstructed well and the resolution was improved as compared with a smooth inversion methodology (Pilkington 2009).

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et al. (2017) used a diagonal depth weighting matrix \( W^{(c)}_{m} \) for the cells of depth \( j \) taking the form of Last & Kubik (1983) and Guillen & Menichetti (1984). For the model weighting matrix \( W^{(m)}_{a} \), which specifies the behaviour of the solution given by eq. (1), they used the product of three diagonal matrices, that is, \( W^{(m)}_{m} = W^{(c)}_{m} W_{a} W_{z} \). Here we analyse all three components of this matrix product, and show that they have all been used previously, either individually or as a product. We thus question the novelty of the approach presented in the noted publication.

In potential field inversion the reconstructed models tend to concentrate near the surface regardless of the true depth of the causative bodies. This happens because the solution is a linear combination of the kernel which decays rapidly with depth. The problem can be overcome by introducing a diagonal depth weighting matrix with diagonal entry \( (W^{(c)}_{m})_{ij} = (z_j + z_0)^{\beta/2} \), which balances the decay of the field resulting from sources at different depths (Li & Oldenburg 1996, 1998). Here \( z_j \) is the mean depth for the cells of depth \( j \) for the model discretization and \( z_0 \) depends both upon the cell size and the observation height of the data (Li & Oldenburg 1996). The parameter \( \beta \) is usually chosen based on the data type and dimensionality of the problem. Ghalehnoee et al. (2017) used \( (W^{(c)}_{m})_{ij} = (\sum_{i=1}^{N}(z_i + h_j))^{\beta} \) in which \( h_i \) is the measurement height for data point \( i \) from the surface. This immediately simplifies as \( (W^{(c)}_{m})_{i} = (N z_i + \sum_{j=1}^{N}(h_j))^{\beta} \). Now suppose that all the data are measured at the same distance from the surface \( h_i = h \), then \( (W^{(c)}_{m})_{j} = (N z_i + h)^{\beta} \) is a constant scaling by \( N^{\beta} \), at all levels \( j \), of that used by Li & Oldenburg, and thus has no impact on the relative impact of the depth weighting matrix.

The compactness constraint introduced by Last & Kubik (1983) and developed by Portniaguine & Zhidanov (1999) has the form \( W^{(m)}_{m} = \text{diag}((m - m_{aprior})^2 + \epsilon)^{-1} \) for \( p = 2 \) and \( m_{aprior} \), the prior model which can be estimated from previous investigations or might be taken to be the zero model (Li & Oldenburg 1996). Parameter \( \epsilon > 0 \) is small and introduced to avoid instability in the algorithm. Because \( W^{(m)}_{m} \) depends on the model parameters the inverse problem becomes nonlinear in the solution, requiring the use of an iterative scheme. Typically the model-space iteratively reweighted least-squares (IRLS) algorithm is used, (Ajo-Franklin et al. 2007). The sparsity of the solution is controlled by both \( \epsilon \) and \( p \). With respect to \( \epsilon \), the solution is sparse when \( \epsilon \) is small. On the other hand, for \( p = 2 \) stabilization is obtained using an \( L_2 \)-norm stabilizer and the solution is sparse, see Sun & Li (2014), but becomes smoother as \( p \) decreases. To understand the impact of the choice of \( p \), we refer to the paper by Sun & Li (2014) which discusses an adaptive \( L_p \) inversion for the simultaneous recovery of blocky and smooth features of a geophysical model. Matrix \( W_{a} \) used in Ghalehnoee et al. (2017) is exactly the compactness constraint with a flexible power \( p \), already discussed in Sun & Li (2014). Furthermore, the fact that sparse inversion increases resolution was discussed in Pilkington (2009).

Finally, matrix \( W_{z} \), denoted as a new kernel weighting matrix in Ghalehnoee et al. (2017), was first introduced by Portniaguine & Zhidanov (2002). Unlike Li & Oldenburg (1996), Portniaguine & Zhidanov (2002) do not define an explicit \textit{a priori} weighting based on the physics of the problem but scale the model by the integrated sensitivities of the kernel using (Portniaguine & Zhidanov 2002; Pilkington 2009)

\[
W_{a} = \text{diag}\left(\sqrt[4]{\sum_{i=1}^{N}A_{i}^{2}}\right).
\]  

The function provides equal sensitivity of the observed data to the cells located at different depths and at different horizontal positions (Portniaguine & Zhidanov 2002), and thus introduces appropriate corrections for the vertical and horizontal distribution of the anomalous density (susceptibility in the magnetic case). More details of both the depth weighting of Li & Oldenburg (1996) and the scaling of Portniaguine & Zhidanov (2002) are provided in the section ‘depth weighting’ in Pilkington (2009). The most important point from Pilkington (2009) is that the scaling is equivalent, but not equal, to a depth weighting. Therefore, both functions have nearly similar performance, and consequently there is no need to use both in an inversion methodology. The kernel weighting matrix used by Ghalehnoee et al. (2017) is \( W_{a} = \text{diag}(\sum_{i=1}^{N}A_{i}) \). It is close to the scaling introduced by Portniaguine & Zhidanov (2002), with a similar motivation in each case, that is, to use the entries of the kernel for weighting. Then, it is clear that, contrary to the claim of Ghalehnoee et al. (2017), the idea behind the use of the kernel weighting function lacks innovation.

It remains to note that the idea of using the product of these matrices is not new and has been adopted in many papers. Although, as mentioned, because it is known that depth weighting and scaling perform similarly, one usually scales or uses depth weighting, but does not use both components. For example, the product of a sparse constraint with a scaling matrix was used in Portniaguine & Zhidanov (2002) in magnetic data inversion, while the product of depth weighting with a sparse constraint was used in Boulanger & Chouteau (2001) and Vatankhah et al. (2014a,b, 2015) for gravity data inversion. We conclude that we cannot find the novelty in the presented approach of Ghalehnoee et al. (2017). Furthermore, there is another issue with the presented paper, which should be noted here. The authors used a fixed value for the damping / regularization factor \( \mu \). It has been demonstrated already, however, that the iterative nature of the solution of the nonlinear problem is not amenable to the use of a fixed parameter, see, for example, Farquharson & Oldenburg (2004) and Vatankhah et al. (2015).

3 CONCLUSIONS

We demonstrated that the work by Ghalehnoee et al. (2017) presents only minor changes to already published papers. We analysed all the presented weighting matrices and related them to their counterparts in the existing literature. In particular, the presented kernel weighting matrix is similar to the scaling matrix of Portniaguine & Zhidanov (2002) and offers nothing significant other than some minor changes to the scaling matrices used for compact gravity inversion, with results verified on some realistic test cases.

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REFERENCES


