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To cite this article: Abolfazl Yousef-Zamanian & Mohammad Neshat (2017) Investigation of polarization state of terahertz radiation from compact laser-induced plasma in air, Journal of Modern Optics, 64:3, 300-308, DOI: 10.1080/09500340.2016.1230235

To link to this article: http://dx.doi.org/10.1080/09500340.2016.1230235

Published online: 15 Sep 2016.

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Investigation of polarization state of terahertz radiation from compact laser-induced plasma in air

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ABSTRACT
A comprehensive theoretical study is presented on the polarization state of THz wave radiation from a two-colour laser-induced plasma, and its dependency on optical pulse polarizations and phase difference. We extend the so-called photocurrent model to calculate three-dimensional distribution of photocurrent from which the polarization state of the THz radiation is calculated. It is shown that the calculation results from the photocurrent model, is in agreement with the previously reported experiments. Moreover, we show the possibility of creating any desired polarization state (i.e. elliptical, circular or linear) for the THz radiation through such generation mechanism by adjusting proper parameters.

ARTICLE HISTORY
Received 28 February 2016
Accepted 19 August 2016

KEYWORDS
Gaussian beam; laser-induced plasma; photocurrent; THz generation; polarization

1. Introduction
In recent years, terahertz technology has risen from relative obscurity to the spotlight in science and engineering such as label-free DNA genetic analysis, cellular-level imaging, chemical/biological sensing, explosives detection, tomographic imaging and nondestructive testing to name a few (1–5).

There are various ways for terahertz generation and detection, such as using photoconductive antennas, built-in field in semiconductors, photo-Dember effect, optical rectification (OR), electro-optical sampling, tilted pulse front excitation and quasi-phase matching structures by the aim of femtosecond lasers as the main THz pulse driver (6). In many applications, sources with high power and broad bandwidth are desirable. Therefore, the strength of the THz field (or pulse energy) and bandwidth are two important factors that their enhancement has been a prominent topic in terahertz research. Two common techniques for THz pulse generation are the current surge in a large area photo-excited semiconductor (using either an external bias (7, 8) or the intrinsic surface properties (9, 10)), and OR in second-order nonlinear crystals (11). However, in both approaches, the emission saturation and/or device damage due to the high-power optical excitation have prevented them from high THz power emission. Moreover, the carrier dynamics in biased semiconductors and surface emitters, or phase-matching requirements in OR limit the emission bandwidth (12).

In contrast, THz generation based on compact laser-induced plasma in air does not suffer from the above mentioned deficiencies (12). The concern of damage due to the strong laser field is avoided by endlessly substitution of air in the plasma that is created under irradiation. Dry air has neither phonon bands nor boundary reflection surfaces, and free electrons as charge carriers can potentially provide continuous coverage over a broad bandwidth. These are superior advantages for generation of broadband high intensity THz pulses. Moreover, air plasma technique can be used for stand-off THz spectroscopy and sensing in relatively far distances (6) due to the low optical attenuation in the atmosphere as compared to high absorption of THz wave.

THz wave generation in compact-air plasma can be described via several different mechanisms. When a high-intensity laser pulse ionizes a gas, a THz transient is produced through a coherent plasma oscillation driven by either the ponderomotive force or by transition radiation from accelerated electron bunches driven by wakefield acceleration (13). Powerful coherent emission of broadband few-cycle terahertz radiation can be produced from a laser wakefield by linear mode conversion. This occurs when the laser pulse is incident obliquely to the density gradient of inhomogeneous plasmas. Other promising alternative for THz generation involves emission from a photo-induced plasma in a gaseous medium, as was first demonstrated by Hamster et al. (14, 15). In their experiments, THz emission mechanism was based on the radial
acceleration of the ionized electrons due to the ponderomotive force. Such force is generated by the radial intensity gradient of the optical beam, leading to a conical THz emission at an angle to the direction of laser propagation. Other compact plasma-based THz generation schemes have been demonstrated, which provide stronger THz emission than does the ponderomotive mechanism, and with the emission in the forward propagation direction. In analogy to biased solid state emitters, Löffler et al. (16, 17) applied an external DC bias to the plasma region to generate a transverse polarization (DC-bias method), that resulted in at least one order-of-magnitude increase in the THz field strength with the maximum attainable THz emission being limited by screening of the applied bias. Cook and Hochstrasser (18) demonstrated another method to introduce the required transverse bias, i.e. using a superposition of the fundamental and second-harmonic (SH) optical pulse fields to generate the plasma (ω − 2ω, or AC-bias method). Since the frequency in the optical AC-bias method is well above the plasma frequency, unlike the DC-bias method, it does not suffer from the strong screening effect. Hence, AC-bias method has emerged as the optimal choice for plasma-based THz generation, and can be employed even for pulse energies well into the mJ range (12). More recently, additional plasma-based THz techniques have been reported, for example, few-cycle pulses with < 10 fs pulse duration has been used without additional SH field. The amplitude of THz field in few-cycle laser method depends strongly on the carrier-envelope phase of the pulses (19).

Although there are numerous reports on the investigation of the mechanisms and calculation of the intensity in plasma-based THz generation, very few papers have investigated the polarization state of the generated THz wave from plasma. In applications such as THz polarimetry and ellipsometry (20), the polarization state of the THz wave plays a major role, and needs to be closely studied. In (21, 22), the polarization of the terahertz wave emitted during the ionization process has been calculated through three-dimensional quantum mechanical simulations, and measured through experiments. A continuous rotation of THz polarization has been experimentally observed in (23), by changing the dual-colour phase difference. In (24), the generation and control of elliptically polarized terahertz waves has been demonstrated from air plasma produced by circularly polarized few-cycle laser pulses. Through polarization sensitive THz emission spectroscopy, Wen and Lindenberg (25) have found that the THz polarization smoothly rotates through 2π radians as the relative phase between the two optical pulses is adjusted. In the case of in-line laser focusing, evolving of the THz polarization from linear to elliptical has been experimentally observed, with increasing plasma length (26).

In this paper, a comprehensive study is presented on the polarization state of the radiated THz wave from plasma, and its dependency on optical pulse polarizations and phase difference in ω − 2ω approach. To do so, we have extended the photocurrent model to calculate three-dimensional distribution of the photocurrent from which the polarization state of the THz radiation is calculated. It will be shown that the proposed method, without involvement of complex quantum mechanical calculations, can provide results that are in excellent agreement with the previously reported measured data. In Section 2, an overview is given on the modelling of plasma-based THz generation in the AC-bias method with the emphasis on the photocurrent model. The simulation results using such model for numerous cases are presented in Section 3, followed by conclusion remarks in Section 4.

2. Modelling

Three approaches have been mainly proposed to describe THz generation phenomenon based on compact laser-induced gas plasma. First approach is a phenomenological description of this phenomenon in terms of four-wave mixing (FWM) of ω, 2ω and THz waves in third-order non-linearity, χ(3), of a gas plasma (27). It soon became clear that FWM hypothesis may not give the full picture, e.g. third-order non-linearity of air plasma is much smaller than that required to justify radiated terahertz intensity. Moreover, terahertz generation is directly proportional to plasma formation, and it may involve non-linearity orders higher than three (28).

Second approach of describing AC-bias method, is through microscopic photocurrent model that accounts for electron motion using semi-classical equations in the optical field following the ionization (28). Such model gives more insight with quantitative parameters than does the first approach. The third approach for the most accurate analysis and modelling of this phenomenon is through quantum theory (29, 30).

In our study, we use the photocurrent model where its detailed formulation is given in Appendix 3. By focusing a femtosecond laser with sufficient pulse energy on a target gas, such as air, its molecules begin to ionize. As a result, the freed electrons start to accelerate under the laser irradiation and drift in addition to the oscillation at laser frequency. Accelerated motion of electrons emits electromagnetic wave. The dynamics of the accelerated motion (photocurrent) is such that the emitted radiation has spectrum in THz range.
Commonly, the pulse energy of femtosecond lasers used in air plasma method is such that, according to the Keldysh parameter, the tunnelling ionization (TI) process dominates (31). More details on Keldysh parameter is given in Appendix 1. For noble gases, the ionization rate is computed from Ammosov–Delone–Krainov (ADK) relation (12) (see Appendix 2). Nitrogen (N₂) constitutes 78% of atmospheric air. It has been shown that for structureless atomic-like molecules such as N₂, ADK is a good approximation even for multiple degrees of ionization (32). It should be noted that the ionization rate is related to the optical field in a highly non-linear fashion. Such non-linearity is the root cause of optical-to-terahertz frequency conversion. Free electrons absorb energy and accelerate in the optical field. However, the average electron collision time is about 1 ps that is much longer than the optical pulse duration (∼50 fs), therefore, the avalanche ionization does not occur under femtosecond laser excitation. Recombination of electron–ion is also negligible due to their relatively long lifetimes (about hundreds of picoseconds).

The dynamic of free electrons under the influence of the optical field can be modelled by the Lorentz relation. It is worth noting that the ions are assumed stationary because of their heavy mass. Moreover, it is assumed that the electron velocity is far from the relativistic regime, so the force exerted by the magnetic field is negligible. Given the electron density and velocity, the transverse photocurrent is calculated as done in (32, 33) (see Appendix 3). Finally, the radiated (terahertz) field from the transverse photocurrent is calculated according to the radiation theory (34) (see Appendix 4).

For 3-D modelling, we used the space–time profile of a Gaussian beam representation as (35)

\[
\sigma_{n t o}^2 = \sigma_{0, nt o}^2 \left[ 1 + \left( \frac{z}{z_{\alpha, nt o}} \right)^2 \right]
\]

where \( \sigma_0 \) is the beam waist, \( z_0 \) is the Rayleigh length, \( \tau_0 \) is 1/e pulsewidth, and \( n = 1 \) or 2 represents the fundamental or second harmonic. In order to model the optical polarization-dependent behaviour, the electric field is represented in a vectorial form as

\[
\vec{E}_k = \vec{E}_{k, \omega} + \vec{E}_{k, 2\omega}, \quad k = x, y
\]

\[
\vec{E}_k (r, z, t, \omega) = \beta_{k, nt o} e^{i\omega_{nt o} t} \vec{E}(r, z, t, \omega)
\]

(2)

where \( \eta \) is the second harmonic conversion efficiency, \( \phi \) is the phase difference (time delay) between fundamental and second harmonic. It should be noted that in (2) all possible polarization states for the fundamental and SH optical fields can be obtained by properly adjusting the parameters \( \beta_{k, nt o} \) and \( \alpha_{k, nt o} \).

3. Simulation results and discussion

We start with a case that the optical field is a combination of an x-polarized Gaussian pulse and its co-polarized second harmonic. Assuming the femtosecond lasers have the pulse duration of \( \tau_{\omega, 2\omega} = 50 \) fs, and 20% SH conversion efficiency, Figure 1 shows the normalized electron density and radiated field spectrum for different laser intensities. As illustrated in Figure 1(b), by increasing the intensity, the normalized THz peak decreases, whereas the bandwidth becomes broader. Kim et al. (32) have shown that the peak of THz intensity is maximized at \( \phi = \pi/2 \) and minimized at \( \phi = 0 \) with maximum to minimum ratio of around 100. Our calculations on the bandwidth spectrum show that the bandwidth is maximized at \( \phi = 0 \) and minimized at \( \phi = \pi/2 \) with maximum to minimum ratio of almost two. Therefore, \( \phi = \pi/2 \) is overall the optimum phase difference choice.

The effect of laser pulse duration on the field spectrum is illustrated in Figure 2. As can be seen in Figure 2(b), by decreasing the pulse duration, the bandwidth of the THz radiation and its normalized peak value increase.

Increasing the spectrum bandwidth due to higher intensity and lower pulse duration of the laser can be justified as follows. According to the transverse photocurrent relation given in (C3), the temporal waveform of the transverse photocurrent is determined by the generated differential plasma charge at time \( t' \), and its velocity at time \( t > t' \). In Figure 1(a) by increasing the laser intensity,
the slope of the normalized electron density becomes steeper with almost no change in the temporal width of the velocity curve as it is evident from (C2). This leads to a narrower temporal width for the transverse photocurrent, and since THz waveform is proportional to the time derivative of the transverse photocurrent, wider spectrum bandwidth is obtained. It is noteworthy that the wider bandwidth is as a result of the narrower temporal width of the waveform. In Figure 2(a) by decreasing the laser pulse duration, not only the slope of the electron density becomes steeper, but the temporal width of the velocity curve becomes narrower as it is evident from (C2). Both effects in this case lead to a narrower temporal width and wider spectrum bandwidth. Therefore, decreasing
**Figure 3.** Lasers profiles and THz polarization detection arrangement. Assuming laser pulse energy $E_j = 2 \mu J$ ($I_0 = 6 \times 10^{18} \text{ W/m}^2$), pulse duration $\tau_{\omega,2\omega} = 25 \text{ fs}$, 20% SH conversion efficiency, $\sigma_{\omega,2\omega} = 2 \mu \text{ m}$, and both optical beams have the same Rayleigh length.

**Figure 4.** (a), (b) Normalized THz intensity, passing through an imaginary polarizer, versus polarizer angle and phase difference $\phi$ between $\omega$- and $2\omega$-lasers. (c), (d) time evolution of THz electric field vector in principle polarization coordinate (for details refer to the text). Insets show optical polarization states.

**Figure 5.** Normalized THz intensity, passing through an imaginary polarizer, versus polarizer angle and phase difference $\phi$ between $\omega$- and $2\omega$-lasers. Laser polarization states are linearly (a) co-polarized, (b) cross-polarized as shown in the insets.
Figure 6. Time evolution of THz electric field vector in principle polarization coordinate. \( \Psi \) and \( \phi \) are the angle between polarization vectors and the phase difference, respectively. (a) \( \Psi \) changes in 15° steps in clockwise direction from 0° to 180°. The inset shows optical polarization states. (b) \( \phi \) changes in 45° steps from 0° to 90°. Legends show the ellipticity \( E_p \) and angle of THz polarization vector \( \Psi_{\text{THz}} \).

the laser pulse duration has twofold effect on increasing the spectrum bandwidth.

3.1. Polarization state

In this section, we present the calculated results of THz polarization state, and compare them with the experimental results reported in (21). Once the spatio-temporal photocurrent distribution is obtained, it is transformed into frequency domain by Fourier analysis. The radiation field at each frequency is obtained through far field calculations from the photocurrent in the frequency domain (34) (see Appendix 4).

In order to study the polarization state, we calculate the radiated THz electric field vector on the axis of the laser propagation and far from the plasma spot as depicted in Figure 3. The plasma length for the laser characteristics given in Figure 3 is around twice the Rayleigh length. Figure 4(a), (b), show THz intensity, passing through an imaginary polarizer, versus polarizer angle and phase difference \( \phi \) between \( \omega \) and \( 2\omega \)-lasers. In Figure 4(a), the \( \omega \)- and \( 2\omega \)-laser have circular and elliptical (11:1) polarization, respectively, and both right-handed. In Figure 4(b), the optical polarizations are the same as those in Figure 4(a) except that they are left-handed.

Figure 4(c) and (d) show time evolution of electric field vector in principle polarization coordinate. In Figure 4(c) and (d) \( \omega \)- and \( 2\omega \)-lasers have the same polarizations as described in Figure 4(a) and (b), respectively. As seen in Figure 4(c), THz polarization is very close to linear (ellipticity higher than 20), and its tilt angle rotates clockwise by changing the phase difference \( \phi \). In Figure 4(c) and (d), the red and green plots correspond to phase difference of \( \phi = 0° \) and \( \phi = 10° \), respectively, and the red arrow shows the direction of phase difference increase in 10° step. It is worth noting that the tilt angle can be fully controlled over 360°. Left-handed optical polarization in Figure 4(d) shows similar behavior as that in Figure 4(c) except that the tilt angle rotates counterclockwise. Our study shows that when the \( \omega \)- and \( 2\omega \)-laser have both circular polarizations, the THz peak pulse remains unchanged by varying the tilt angle (not shown). Such behaviour has been observed in the experiments and quantum theory simulations as well (21).

Figure 5 shows THz intensity, passing through an imaginary polarizer, versus polarizer angle and phase difference \( \phi \) between \( \omega \)- and \( 2\omega \)-lasers. In Figure 5(a), (b), lasers have parallel (co-) and perpendicular (cross-polarized) linear polarizations, respectively. The peak of THz intensity for the co-polarized lasers is around 20 times higher than that for cross-polarized lasers. Our simulations show that when the lasers are linearly co-polarized, the polarization state of THz field is also linear and in the same direction of laser polarization vectors no matter of the value of phase difference \( \phi \). However, when the lasers are linearly cross-polarized, the linear polarization state of THz field is along the direction of \( 2\omega \)-laser polarization vector. Such behaviour has been observed in the experiments and quantum theory simulations as well (21).

Figure 6 shows time evolution of THz electric field vector in the principle polarization coordinate. It is
assumed that the lasers have linear polarization with $\Psi$ as the angle between polarization vectors and $\phi$ as the phase difference. As shown in Figure 6(a), it is clear that the THz polarization state can be controlled by changing the $\Psi$ angle. In Figure 6(b), the ellipticity and the direction ($\Psi_{THz}$) of the major axis of the THz polarization ellipse can be controlled by changing the phase difference $\phi$.

Figure 7 shows the THz polarization state for the case where the $\omega$- and $2\omega$-lasers have circular and linear polarization states, respectively. As shown in Figure 7, the THz polarization is close to linear and its tilt angle can be rotated by changing the phase difference. It is worth noting that by rotating the THz polarization vector, the peak field slightly changes. Comparing Figure 7(a), (b), (e), it reveals that the direction where the maximum peak of the field happens can be controlled by the direction of the linear polarization vector of $2\omega$-laser (angle $\Psi$).

Figure 8 shows the case of orthogonal elliptical polarizations for $\omega$- and $2\omega$-lasers. As shown in Figure 8, one...
can control the ellipticity of the THz polarization down to semi-circular polarization state by changing the phase difference $\phi$.

4. Conclusion

In this paper, we studied the normalized THz peak and bandwidth of the radiation from a two-colour laser-induced plasma in terms of laser intensity and optical pulse duration based on photocurrent model. It was found that when the intensity rises, the bandwidth widens, but the peak intensity falls. Moreover, by decreasing the optical pulse duration, both the bandwidth and the peak intensity increase. Finally, the effect of optical polarizations, phase difference and tilt angle of linear polarization on THz polarization state were thoroughly investigated. It was found that any desired polarization state can be achieved by choosing proper values of the two-colour laser parameters.

Acknowledgements

The authors would like to acknowledge the financial support from Iran National Science Foundation (INSF) for this project.

Disclosure statement

No potential conflict of interest was reported by the authors.

References


Appendix 1. Keldysh parameter

Photo-ionization of gas molecules in intense optical fields stems from various processes. Dominate ionization process can be determined by Keldysh parameter ($\gamma$) defined as (31)

$$\gamma = \sqrt{\frac{U_{ion}}{2U_p}}$$

where $U_p$ is the ponderomotive (quiver) energy, i.e. the average kinetic energy of a free electron in a (monochromatic) electromagnetic field, and $U_{ion}$ is the ionization potential energy of the target.
Figure A1. Ionization processes according to the value of the Keldysh parameter, (a) tunnelling ionization process ($\gamma < 1$), (b) transition region ($\gamma \sim 1$), (c) multi-photon ionization process ($\gamma > 1$) (from (36)).

gas. Figure A1 illustrates various ionization processes. If the laser intensity can alter the coulomb potential curve of the atom (see Figure A1(a)) such that the barrier width becomes small enough for electron tunnelling through the binding potential, then TI process happens. For TI to happen, the condition $\gamma < 1$ should be satisfied. Also in such a case, if the electron acquires enough energy to surmount the barrier, OBI process occurs. For $\gamma > 1$, Coulomb potential curve does not change considerably, and therefore, an electron needs to absorb the energy of multiple photons in order to be liberated from the atom, and consequently MPI process occurs (see Figure A1(c)). Also in such a case, if the sum energy of multiple photons is more than the ionization threshold, ATI process applies. At the transition condition $\gamma \sim 1$, a combination of two processes may happen (see Figure A1(b)).

In avalanche ionization process, the liberated electron due to photo-ionization gains more energy by absorbing extra photons, and if irradiation continues, inverse Bremsstrahlung effect occurs. When such electron gains high enough energy, it strips off a new electron upon collision with a neutral atom, and ionizes it. Therefore, the initial free electron generates a new free electron, and it itself remains in the conduction band with reduced energy. Continuity of this process leads to an avalanche rise in the number of free electrons, and avalanche ionization (AI) process is formed (37).

Appendix 2. Ionization rate and charge density

The ADK relation for ionization rate is given by

$$ W_{ADK} (t) = \frac{\alpha_{ADK}}{|\xi (t)/k_a|^2} \exp \left\{ - \frac{\beta_{ADK}}{|\xi (t)/k_a|^2} \right\} \tag{B1} $$

where $\xi (t)$ is the instantaneous optical electric field. Other parameters in (B1) are introduced in Table B1 (12).

Charge or plasma density $\rho(t) = -eN_e(t)$ can be calculated by solving a rate equation as

$$ \frac{\partial N_e (t)}{\partial t} = W_{ADK} (t) (N_0 - N_e (t)) \tag{B1} $$

Table B1. ADK ionization rate parameters.

<table>
<thead>
<tr>
<th>$\alpha_{ADK}$</th>
<th>$\omega_{ion} (\gamma \Delta n)^2 (4\sqrt{2} \gamma_a)^{2n-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\Delta n</td>
</tr>
<tr>
<td>$U_0 = k^2 m_e^2 e^4 / h^8$</td>
<td></td>
</tr>
<tr>
<td>$\omega_{ion} = U_{ion} / h$</td>
<td></td>
</tr>
<tr>
<td>$\tau_\theta = U_{ion} / U_{ion} \hat{t}$</td>
<td></td>
</tr>
</tbody>
</table>

$\gamma_a = k^3 m_e^2 e^4 / h^6$

$\beta_{ADK} = \frac{4\sqrt{2}/3}{\gamma_a^2}$

$\Theta = \frac{2\pi}{n} (\gamma + 1) \Gamma (n + 1)^{1/2}$

$\omega = \frac{e^4}{m_e h^4}$

$\omega_{ion} = U_{ion} / h$

$\tau_\theta = U_{ion} / U_a$

where $N_0 = Z_e N_{AV} P / RT (\approx 2.5036 \times 10^{25} \text{m}^{-3})$ is the number density of air, $N_{AV} = 6.022 \times 10^{23}$ is Avogadro’s number, $Z_e$ is number of conduction electrons, $P, R, T$ are standard pressure, universal gas constant and temperature, respectively (39). The solution to the rate equation in (B1) is obtained analytically as

$$ N_e (t) = N_0 \left[ 1 - \exp \left( - \int_{t_0}^t dt' W_{ADK} (t') \right) \right]. \tag{B2} $$

Appendix 3. Photocurrent modelling

The free electron density in the laser-induced plasma is obtained from (B2). The dynamic of free electrons under the influence of the optical field can be modelled by the Lorentz relation as

$$ F = m_e \ddot{x} = -e (\vec{\xi} + \vec{v}_e \times \vec{B}) \tag{C1} $$

where $F$ is the exerted force on the electron with charge $e$ and mass $m_e$, $\vec{x}$ is the electron displacement, $\vec{v}_e$ is the charge carrier velocity, $\vec{\xi}$ and $\vec{B}$ are the electric and magnetic fields, respectively. It is worth noting that the ions are assumed stationary because of their heavy mass. Moreover, it is assumed that the electron velocity is far from the relativistic regime, so the force exerted by the magnetic field is negligible. Therefore, the velocity of an electron at time $t$ that has been released by photo-ionization at time $t' < t$ is calculated as

$$ \vec{v}_e (t, t') = - e \frac{\vec{\xi}}{m_e} \int_{t'}^t (\vec{\xi} (t')) d\omega \tag{C2} $$

Given the electron density and velocity, the transverse photocurrent is calculated as

$$ J_\perp (t) = \int \vec{v}_e (t, t') \, d\rho_e (t') \tag{C3} $$

where $d\rho_e (t') = - e dN_e (t')$ is the change of charge density between $t'$ and $t' + dt'$ and $\vec{v}_e (t, t')$ is the velocity of the charges (electrons) at $t$ that have been released at $t'$. According to the radiation theory, the radiated (terahertz) field from the transverse photocurrent in (C3) is proportional to its rate of change in time.

Appendix 4. Radiation field

The radiation field at each frequency is obtained through far field calculations from the photocurrent in frequency domain by calculating the magnetic vector potential as (34)

$$ A (r, \theta, \phi, \omega) = \frac{\mu_0 c^3 (\omega - kr)}{4\pi r} \int \hat{J} \left[ \hat{r} \left( \hat{\omega} \hat{r} \hat{\omega} \right) \right] e^{i k \hat{\vec{r}} \cdot \hat{\vec{r}}} d\vec{r} \tag{D1} $$

where $A$ and $J$ are magnetic vector potential and photocurrent vector distribution, respectively, $k = \omega / c$ is the propagation phase constant, $\mu_0$ is the air permeability, and

$$ \beta = \chi \sin \theta \cos \phi + \chi' \sin \theta \sin \phi + \chi'' \cos \theta. \tag{D2} $$

It should be noted that for the far field the radial component along the radiation direction is negligible, therefore, the radiation electric field is given by

$$ E(\omega) = -i \omega (A_{\theta} \hat{\theta} + A_{\phi} \hat{\phi}). \tag{D3} $$

By taking the inverse Fourier transform, the radiation field in the spectral domain is transformed into the time domain.