Incorporating price-responsive customers in day-ahead scheduling of smart distribution networks

Mohammadreza Mazidi, Hassan Monsef, Pierluigi Siano

School of Electrical and Computer Engineering, University of Tehran, Iran
Department of Industrial Engineering, University of Salerno, Fisciano, Italy

Abstract

Demand response and real-time pricing of electricity are key factors in a smart grid as they can increase economic efficiency and technical performances of power grids. This paper focuses on incorporating price-responsive customers in day-ahead scheduling of smart distribution networks under a dynamic pricing environment. A novel method is proposed and formulated as a tractable mixed integer linear programming optimization problem whose objective is to find hourly sale prices offered to customers, transactions (purchase/sale) with the wholesale market, commitment of distribution generation units, dispatch of battery energy storage systems and planning of interruptible loads in a way that the profit of the distribution network operator is maximized while customers' benefit is guaranteed. To hedge distribution network operator against financial risk arising from uncertainty of wholesale market prices, a risk management model based on a bi-level information-gap decision theory is proposed. The proposed bi-level problem is solved by recasting it into its equivalent single-level robust optimization problem using Karush-Kuhn-Tucker optimality conditions. Performance of the proposed model is verified by applying it to a modified version of the IEEE 33-bus distribution test network. Numerical results demonstrate the effectiveness and efficiency of the proposed method.

1. Introduction

Nowadays, power systems are developing toward smart grids specifically at the distribution level, aiming at providing financial and technical benefits for both system operators and customers [1]. With the development of smart grids at the distribution level, Distributed Energy Resources (DERs) such as renewable and non-renewable Distributed Generation (DG) units, Battery Energy Storage Systems (BEESs) and Demand Response (DR) programs are being integrated into the distribution network operation [2]. The presence of one or more of these equipments along with the uncertainties will create more complex and challenging tasks in day-ahead scheduling of smart distribution networks.

As the key feature of smart distribution networks, optimal scheduling have attracted great attention, which can be categorized into supplied side and demand side point of views. References [3–6] focus on the supplied side issues. A two-stage hierarchical framework for day-ahead scheduling of distribution network is proposed in [3]. The first stage of the proposed framework deals with electrical power purchasing from the wholesale market and commitment of DGs, whereas the decisions related to the dispatching of committed DGs, participating in real-time market and planning of curtable loads are made in the second stage. The authors of [4] propose an optimal power flow algorithm to develop a generalized formulation aiming at minimizing the total operation cost of smart distribution network considering network constraints. In [5], a multi-objective approach for day-ahead scheduling of distribution network is proposed based on stochastic optimization. The paper aims at minimizing the emission and operating costs. In [6], a unified operation model is proposed in which the network topology and hourly scheduling of DGs as well as curtable loads are determined in a way that total operation cost of distribution network is minimized. On the other hand, thanks to advancements in smart grid technologies, the demand side issues are also receiving growing attention. Authors of [7] propose a model for smart energy management of a residential customer in which electrical and thermal appliances are jointly scheduled to minimize electricity cost. In [8], customers’ opportunity to adjust their consumption patterns in response to real-time pricing, with the goal of decreasing their electricity bills, has been investigated. In [9], a day-ahead pricing model is proposed to maximize profit of energy providers where in consumers satisfaction is
considered. A decentralized optimization approach is developed in [10] to minimize total cost of utility company in a way that the most smooth demand profile is yielded.

In reality, day-ahead scheduling of smart distribution network is exposed to the uncertainties of electricity price (on the supply side) and load (on the demand side). Ignoring the risk of mentioned uncertainties may impose great financial losses to the Distribution Network Operator (DNO). In this regard, the stochastic optimization methods together with risk measures are often used to tackle financial risk arising from the uncertainties in which Probability Density Functions (PDFs) are utilized to represent uncertain parameters [11]. A hierarchical stochastic method is presented in [12] to minimize expected procurement cost of a distribution company subjected to a risk constrain. A coordinated model of curtailable loads and DGs is proposed in [13] based on a multi-stage stochastic optimization to minimize expected operation costs of microgrids and handle imposed risk using scheduled reserves. In [14], operation optimization of microgrids is established with the goal of profit maximization and risk management. The accuracy and optimality of the stochastic methods rely on the accuracy of the PDF and number of scenarios which are considered within the optimization framework. In [15], operation optimization of microgrids is established with the goal of profit maximization and risk management. The accuracy and optimality of the stochastic methods rely on the accuracy of the PDF and number of scenarios which are considered within the optimization framework. In [16], operation optimization of microgrids is established with the goal of profit maximization and risk management. The accuracy and optimality of the stochastic methods rely on the accuracy of the PDF and number of scenarios which are considered within the optimization framework. In [17], operation optimization of microgrids is established with the goal of profit maximization and risk management. The accuracy and optimality of the stochastic methods rely on the accuracy of the PDF and number of scenarios which are considered within the optimization framework. In [18], operation optimization of microgrids is established with the goal of profit maximization and risk management. The accuracy and optimality of the stochastic methods rely on the accuracy of the PDF and number of scenarios which are considered within the optimization framework.
of the optimization problem grows significantly. To address these issues, a robust optimization method using Information Gap Decision Theory (IGDT) is proposed in this paper. Compared with other methods, the robust IGDT-based optimization method has several advantages. At first, it only requires forecasted values as well as lower and upper bounds of uncertain parameters, which are easier to obtain from historical data. Besides, the robust IGDT-based optimization method certainly guarantees a pre-specified level of profit in opposite of stochastic optimization methods that provide probabilistic guarantee for constraint satisfaction.

The problem of day-ahead energy and reserve scheduling in the smart distribution networks and smart microgrids are studied in [5,13], respectively. The authors utilized scenario-based stochastic programing approach to optimize the proposed models in which the customers can participate in both energy and reserve scheduling using only load reduction DR program. However, suitable linearized models for incorporating price-responsive customers and also transactions with wholesale market have not been considered. Moreover, the exposure risk imposed by uncertain variables has not been modeled.

Effective studies are presented in the reviewed articles, but still a comprehensive investigation for day-ahead scheduling of smart distribution networks from both supply side and demand side perspectives is needed. This paper introduces a new approach for the DNO to incorporate price-responsive customers in day-ahead scheduling of smart distribution network. This contribution leads to maximize profit of the DNO (supply side) while customers' benefit (demand side) is guaranteed. Meanwhile, a risk management model based on the bi-level IGDT method is proposed to hedge the DNO against financial risk arising from uncertainty of wholesale market prices. The proposed bi-level problem is solved by recasting it into its equivalent single-level robust optimization problem using Karush-Kuhn-Tucker (KKT) optimality conditions. The proposed models are formulated as a tractable Mixed Integer Linear Programing (MILP) optimization problem in which the global optimal solution is guaranteed. Some novel contributions of this paper could be highlighted as follows:

- Proposing a model for incorporating price-responsive customers in day-ahead scheduling of smart distribution networks; this model provides a win-win situation in which both customers and the distribution network operator benefit, simultaneously.

- Introducing a risk management model based on a bi-level information-gap decision theory and recasting it into its equivalent single-level robust optimization problem using Karush-Kuhn-Tucker optimality conditions; in this model, profit maximization and risk management are performed simultaneously.

- Utilizing mixed-integer linear programing formulation that is efficiently solved by commercial optimization software.

The remainder of this paper is outlined as follows. In Section 2, modeling and mathematical formulation for day-ahead scheduling of smart distribution network are described. The model is then recast to a robust optimization problem using IGDT-based method in order to manage the imposed risk due to uncertain wholesale market prices, in Section 3. Numerical results are given in Section 4. Finally, the paper is concluded with some important finding in Section 5.

2. Model formulation

In this section, modeling and mathematical formulation for day-ahead scheduling of the smart distribution network are presented.

2.1. Distribution network model

Two buses of a radial distribution network are illustrated in Fig. 1, the complex power flow equations associated with bus \( n \) of the distribution network can be described as follows [15]:

\[
p^l_{(n)} = P^\text{flow}_{(m,n)} - r_{(m,n)} I_{(m,n)} - \sum_{f \in (n,f)} P^\text{flow}_{(f,n)} \quad \forall m, n
\]

\[
q^l_{(n)} = Q^\text{flow}_{(m,n)} - x_{(m,n)} I_{(m,n)} - \sum_{f \in (n,f)} Q^\text{flow}_{(f,n)} \quad \forall m, n
\]

\[
v_{(n)} = V_{(n)} - 2 (r_{(m,n)} P^\text{flow}_{(m,n)} + x_{(m,n)} Q^\text{flow}_{(m,n)}) + (r^2_{(m,n)} + x^2_{(m,n)}) I_{(m,n)} \quad \forall m, n
\]

\[
I_{(m,n)} = \frac{P^\text{flow}_{(m,n)} + Q^\text{flow}_{(m,n)}}{V_{(m)}} \quad \forall m, n
\]

where, \( I_{(m,n)} = |I_{(m,n)}|^2 \) and \( V_{(n)} = |V_{(n)}|^2 \).

2.2. Demand response model

In this work, both price-based and incentive-based DR programs are considered through the proposed model.

2.2.1. Price-based DR model

In price-based DR program, electricity price varies at different hours of the day and each customer individually responds to the time-differentiated prices by shifting its own demand from the high-price to the low-price hours. Recent researches reveal that exposing customers to hourly real-time prices is the most efficient program to encourage them to consume electricity more efficiently [16]. In this paper, Day Ahead-Real Time Pricing (DA-RTP) program which used by the Illinois Power Company, has been adopted [17]. To this end, an economic model for the price-responsive customers is taken from reference [18] and presented in (5). This equation illustrates how much consumers use electricity to achieve minimum bill during scheduling period while participating in DA-RTP program.

\[
p^l_{(i,c)} = P^\text{IL}_{(i,c)} \times \left\{ 1 + \frac{\epsilon_{V_{(i,c)}} \times \frac{P_{(i,c)} - 0_{(i,c)}}{\rho_{(c)}} + \sum_{c,t} \epsilon_{E_{(c,t)}} \times \frac{P_{(c,t)} - 0_{(c,t)}}{\rho_{(c,t)}}}{\rho_{(c)}} \right\} \quad \forall c, i, t
\]

2.2.2. Incentive-based DR model

Incentive-based DR programs give participating customers incentive payments. In this paper, the IL program is adopted in which candidate customers reduce their electricity consumption without shifting to any time period [16]. To this end, candidate customers submit a step-wise price-quantity offer package to the
DNO, as shown in Fig. 2 and formulated in (6)-(9). Each package includes the amount of reduction load and associated offered price. While the offers are accepted, ILs are called to reduce their load and receive the offered price [13].

\[
A_{dk} \leq \delta_{dk,k} \leq A_{dk,k-1} \quad \forall k, d, t
\]  
(6)

\[
0 \leq \delta_{dk,k} \leq (A_{dk,k} - A_{dk,k-1}) \quad \forall k, d, t
\]  
(7)

\[
P_{d(k)}^c = \sum_{k=1}^{N_k} \delta_{d(k),k} \quad \forall d, t
\]  
(8)

\[
A_{dk} \leq P_{d(k)}^c \leq A_{dk,k} \quad \forall d, t
\]  
(9)

### 2.3. Wind turbine model

The power generation of a wind turbine is calculated by using the wind turbine power curve as described by Eq. (10) [19]:

\[
P_W(\phi) = \begin{cases} 
    P^r & \text{if } \phi^l \leq \phi \leq \phi^c \\
    P^c & \text{if } \phi^c \leq \phi \leq \phi^u \\
    0 & \text{otherwise}
\end{cases}
\]  
(10)

where, \( P^r, \phi^l, \phi^c, \) and \( \phi^u \) are the rated power, cut-in speed, rated speed and cut-off speed of the wind turbine, respectively. In each time period the scheduled wind power should be obviously smaller than the available wind power generation.

\[
P_W^{(d,t)} \leq P_W^{(d,t)} \quad \forall w, t
\]  
(11)

### 2.4. Battery energy storage system model

The BESS can operate in three different modes including charging, discharging and idle. In charging modes, the BESS behaves as a load and absorbs energy. In discharging modes, it behaves as a DG unit and returns the absorbed energy. In idle modes, BESS neither absorbs energy nor returns the absorbed energy. The following constraints are considered to model the BESS [20]:

\[
bs^c + bs^d \leq 1; \quad bs^c, bs^d \in (0, 1) \quad \forall e, t
\]  
(12)

\[
SOC(\epsilon, t) = SOC(\epsilon, t-1) + \eta^r \times P_{\epsilon}^c - \eta^d \times P_{\epsilon}^d \forall e, t
\]  
(13)

\[
SOC(\epsilon, t) \leq SOC(\epsilon, t) \leq SOC(\epsilon) \forall e, t
\]  
(14)

\[
0 \leq P_{\epsilon}^c \leq \frac{P_{\epsilon}^c}{P_{\epsilon}^c} \times bs^c \forall e, t
\]  
(15)

\[
0 \leq P_{\epsilon}^d \leq \frac{P_{\epsilon}^d}{P_{\epsilon}^d} \times bs^d \forall e, t
\]  
(16)

### 2.5. Objective function

It is assumed that DNO is owner of distribution network. In such framework, DNO tries to maximize the total benefit which is defined as the difference between revenue and cost [21]. The revenue is earned by selling electricity to the customers and to the wholesale market. The costs include those due to the purchase of electrical energy from the wholesale market, operating costs of DG units and the payment costs for ILs. Therefore, the objective function can be written as follow:

\[
\text{Max} \sum_{t=1}^{T} \sum_{c=1}^{C} \sum_{n=1}^{N_c} \pi_{(t,c)}(x_c) P_{(t,c)}^d - \sum_{t=1}^{T} \sum_{c=1}^{C} \sum_{n=1}^{N_c} \pi_{(d,c)} x_c \neq \delta_{d(k),k} \]  
(17)

The first term of (17) represents the total income derived from selling energy to different types of customers. The second term is the total payment cost for ILs. The third term denotes the total costs or incomes due to the purchase/selling of electrical energy in the wholesale market. A positive value of \( P_{\epsilon}^c \) indicates power purchasing from the wholesale market and negative, indicates power selling to the wholesale market. The fourth term denotes the sum of start-up cost, fuel cost and shut-down cost of DG units.

In order to implement a linear programming approach, the non-linear terms of the objective function should be modified into their equivalent linear formats. As given in (5), electricity demand has a quadratic dependence with the DG units' power. These non-linear terms can be linearized using Special-Ordered-Sets-of-type-2 (SOS2) technique briefly explained in Appendix A.

### 2.6. Constraints

The smart distribution network should be scheduled and operated based on the following constraints:

#### 2.6.1. Power balance constraints

Reliable operation of distribution network necessitates a continuously balanced between the scheduled supply and demand. In this regard, the active and reactive power balance equations at bus \( n \) of the distribution network and at hour \( t \) are as follows:

\[
p_{(n,t)} = \sum_{j=1}^{J} \sum_{w=1}^{W} P_{(n,j,w)} - \sum_{e=1}^{E} (\eta^c \times P_{\epsilon}^c - \eta^d \times P_{\epsilon}^d)
\]  
(18)

\[
Q_{(n,t)}^l = \sum_{j=1}^{J} \sum_{w=1}^{W} Q_{(n,j,w)} - \sum_{e=1}^{E} Q_{(\epsilon)}^l
\]  
(19)

\[
v_{(n,t)} = v_{(n,t)} - 2r_{(n,t)} x_{(n,t)} + x_{(n,t)} Q_{(n,t)}^l + (r_{(n,t)}^2 + x_{(n,t)}^2)
\]  
(20)
\[ I_{(m,n,t)} = \frac{P_{(m,n,t)}^2 + Q_{(m,n,t)}^2}{V_{(m)}} \quad \forall m, n, t \] \tag{21}

Similar to the objective function, Eq. (21) can be linearized using SOS2 method, as detailed in [24].

2.6.2. Distribution network constraints

To ensure the safe operation of the distribution network, bus voltage and feeder current limits are considered as follows:

\[ V_{(n)}^2 \leq V_{(n,t)}^2 \leq V_{(n)}^2 \quad \forall n, t \] \tag{22}

\[ I_{(m,n,t)} \leq I_{\text{sub}} \quad \forall m, n, t \] \tag{23}

It has to be mentioned that the substation of the distribution network represents a controlled voltage bus which is remained at the constant value [25]. Moreover, limited capacity of substation transformers imposes an upper-bounded on the quantity of imported/ exported power from/to the upstream grid. Thus, the following additional constraints are considered when a substation is connected to bus 1 of the distribution network:

\[ V_{(n,1)} = \text{Constant} \quad n = \text{Substation bus} \quad \forall \] \tag{24}

\[ I_{(m,1,t)} \leq I_{\text{sub}} \quad m = 1, \quad n = \text{Substaion bus} \quad \forall t \] \tag{25}

2.6.3. DG unit constraints

The following constraints ensure that the operating point of each DG unit meets technical limits including maximum and minimum capacity limits, ramping up/down rates limits and minimum up/down duration limits [26]. Eq. (26) guarantees that the scheduled power of each DG unit satisfies its maximum and minimum capacity limits.

\[ p_{DG}^{\text{MAX}} \times u_{(t,j)} \leq p_{DG}^{(t,j)} \leq p_{DG}^{\text{MIN}} \times u_{(t,j)} \quad \forall j, t \] \tag{26}

Frequent start-up and shut-down of a DG unit result in excessive turbine shaft tension and rotor temperature and potentially higher operation and maintenance costs. Using minimum up-time constraint, the DG unit cannot be turned off for specific number of hours after turning on. Similarly using minimum down time constraint, the DG unit cannot be committed and turned on for specific number of hours after turning off [27]. The inclusion of these constraints leads to more uniform commitment of the DG unit during scheduling period and therefore, reduces the excessive turbine shaft tension and rotor temperature, and therefore lower operation and maintenance costs. These constraints are considered as follows [28]:

\[ p_{DG}^{\text{MIN}} \times u_{(t,j)} \leq p_{DG}^{(t,j)} \leq p_{DG}^{\text{MAX}} \times (1 - y_{(t,j)}) \quad \forall j, t \] \tag{27}

\[ p_{DG}^{(t-1,j)} - p_{DG}^{(t,j)} \leq DR_{(j)} \times (1 - z_{(t,j)}) + p_{DG}^{\text{MAX}} \times y_{(t,j)} \quad \forall j, t \] \tag{28}

The following constraints impose the up/down ramp rate limits, which determine the quantity of increase or decrease of a DG unit generation during one scheduling period, respectively.

\[ \sum_{h=t}^{t+\Delta T_{(t,j)}-1} u_{(h,j)} \geq UT_{(j)} \times y_{(t,j)} \quad \forall j, t \] \tag{29}

\[ \sum_{h=t}^{t+\Delta T_{(t,j)}-1} (1 - u_{(h,j)}) \geq DT_{(j)} \times z_{(t,j)} \quad \forall j, t \] \tag{30}

The following constraints are needed to specify the relationship between startup and shutdown indicators with commitment of DG unit.

\[ y_{(t,j)} - z_{(t,j)} = u_{(t,j)} - u_{(t-1,j)} \quad \forall j, t \] \tag{31}

\[ y_{(t,j)} + z_{(t,j)} \leq 1 \quad \forall j, t \] \tag{32}

The status of DG units is determined using the following constraints:

\[ u_{(t+1,j)} - u_{(t,j)} \leq u_{DG}^{\text{ON}} \quad \forall j, t \] \tag{33}

\[ u_{(t,j)} - u_{(t-1,j)} \leq u_{DG}^{\text{OFF}} \quad \forall j, t \] \tag{34}

\[ u_{(t+1,j)} - u_{(t,j)} = u_{DG}^{\text{ON}} - u_{DG}^{\text{OFF}} \quad \forall j, t \] \tag{35}

2.6.4. Reserve constraint

Due to lack of accurate forecasting methods, it is necessary to ensure that there is enough reserve to compensate hourly forecasted errors of wind power and load demand. To this end, a percentage of wind power (e.g. \( \sigma_{\text{OFF}}^{W} = 20\% \)) and load demand (e.g. \( \sigma_{\text{OFF}}^{L} = 10\% \)) is considered in each time period to determine the required reserve provided by DG units [29]:

\[ \sum_{j=1}^{N_{j}} (P_{DG}^{(j)} - P_{DG}^{\text{MIN}}) \geq \sigma_{\text{OFF}}^{W} \times \sum_{w=1}^{N_{w}} P_{W}^{(w)} + \sigma_{\text{OFF}}^{L} \times \sum_{i=1}^{N_{c}} P_{c}^{(i,c)} \quad \forall t \] \tag{36}

2.6.5. Welfare constraints

The electricity consumption of each customer should be in the specific interval, which is represented by constraint (37).

\[ p_{(t,c)}^{\text{MIN}} \leq p_{(t,c)}^{(t)} \leq p_{(t,c)}^{\text{MAX}} \quad \forall t, i, c \] \tag{37}

Eq. (38) represents that the daily electricity consumption of each customer should be greater than its minimum energy requirement.

\[ \sum_{t=1}^{N_{t}} p_{(t,c)}^{(t)} \geq E_{(c)} \quad \forall i, c \] \tag{38}

The following constraint imposes a price cap for the sale price offered to customer type \( c \) at hour \( t \).

\[ p_{(t,c)}^{\text{MAX}} \leq P_{(t,c)}^{\text{MAX}} \quad \forall t, c \] \tag{39}

In order to provide incentive for customers to participate in the DA-RTP program, the total income of selling electricity to each customer should be less or equal to the case that wholesale market prices are directly imposed to customers. To this end, payment coefficient in Eq. (40) should be less than or equal to one.

\[ \sum_{t=1}^{N_{t}} p_{(t,c)}^{(t)} \times P_{(t,c)}^{\text{MAX}} \leq \gamma \times \sum_{t=1}^{N_{t}} P_{W}^{(w)} \times P_{(t,c)}^{\text{MAX}} \quad \forall i, c \] \tag{40}

3. Risk-management model

In this section, a risk management model based on the IGDT method is described.

3.1. IGDT method

The IGDT method is a non-probabilistic risk-management method that looks for a robust solution in the optimization problem under uncertain variables. Comparing IGDT method with other risk-management methods results in the following advantages.
1. The proposed method has no assumption on the probabilistic estimation of uncertain variables, e.g., wholesale market prices in this paper. Therefore, this method is not sensitive to uncertain parameters and can be used to handle the variables with a high degree of uncertainties.

2. The risk management and optimal scheduling are performed simultaneously in the proposed method and there is no need to define any kind of risk index to handle the risk of uncertain variables, e.g., wholesale market price in this paper.

3. The proposed method certainly guarantees a pre-specified level of profit opposite the stochastic optimization methods that provide probabilistic guarantee for constraint satisfaction.

There are several kinds of models for uncertain parameters according to their properties in IGDT. These models include energy-bound models, envelope-bound models, Minkowski-norm model [30]. In this paper, envelope bound model is used to model uncertain wholesale market price. This is mathematically expressed in (41) and graphically described in Fig. 3.

\[
\text{RR}(\xi, \lambda_{\text{WS}}) = \left\{ I_1^{\lambda_{\text{WS}}} - \xi \leq \lambda_{\text{WS}} - I_0^{\lambda_{\text{WS}}} \leq \xi, \forall t \right\} \text{ (41)}
\]

As it can be seen in Fig. 3, the uncertain price varies around its forecasted value in the confidence interval with the length of \(\xi \times \lambda_{\text{WS}}\). By this definition, the larger the value of forecasted price, the greater range of confidence interval. Because the uncertainty of the forecasted price during high price periods is more than that of other periods, this type of uncertainty modeling is appropriate here.

The objective of IGDT method is to maximize the length of confidence interval while a pre-defined profit (i.e. expected profit) is guaranteed [30]. In this way, the uncertainty modeling and risk management are accomplished together. This can be mathematically expressed by (42).

\[
\text{Profit}^{\text{exp}} = \max_{\text{DV}} \left\{ \text{min} \text{ Profit} (\text{RR, DV}) \geq \text{Profit}^{\text{exp}} \times (1 - \alpha) \right\}
\] (42)

The IGDT-based optimization problem can be written as follows:

\[
\max_{\text{DV}} \xi \quad \text{Subject to: } \text{Profit} \geq \text{Profit}^{\text{exp}} \times (1 - \alpha)
\] (43)—(45) and (46)—(50)

\[
\text{Profit} = \min_{\text{WS}} \sum_{t=1}^{N_t} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \rho_{i(1)} \times P_{i(1)} - \sum_{t=1}^{N_t} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \pi_{d(k)} \times \delta_{(l, d, k)} - \sum_{t=1}^{N_t} I_{\text{WS}}^{\text{WS}} - \sum_{i=1}^{N_i} \text{SUC} (i) \times u_{i(h)}^{\text{ON}} - \{\zeta_j \times u_{i(h)}^{\text{OFF}} + b_j \times P_{i(1)} + a_j \times P_{i(1)}^2 \} - \text{SDC}(j) \times u_{i(h)}^{\text{OFF}}
\] (44)

Subject to:

\[
(1 - \xi) \times \lambda_{\text{WS}}^{\text{WS}} \leq \lambda_{\text{WS}} - I_{\text{WS}}^{\text{WS}} \leq (1 + \xi) \times \lambda_{\text{WS}}^{\text{WS}} \quad \xi \geq 0, \forall t
\] (46)

The above problem is a bi-level optimization in which the first level including (43) and (44) determines the decision variables of the day-ahead energy scheduling to maximize the confidence interval, while guaranteeing that the predefined profit is achieved. The second level, including (45) and (46), determines the worst case of wholesale market price constrained by the envelope-bound model described in (41).

Bi-level optimization problems cannot be solved directly via commercial software. In this paper, a solution procedure based on KKT condition is utilized to solve the proposed bi-level optimization problem, which is presented in the reminder of this section [31].

3.2. Solution procedure

Because the decision variables of the day-ahead scheduling are calculated in the first level, they are constant in the second level and this leads to a linear optimization problem. Accordingly, the minimum profit of the objective function arises in the one of the upper or lower bounds of the wholesale market price indicated in (46). This is explained as follows:

\[
\begin{align*}
\lambda_{\text{WS}}^{\text{WS}} &= \left\{ (1 + \xi) \times \lambda_{\text{WS}}^{\text{WS}} \text{ Profit}_{WS(i)}^{\text{WS}} \geq 0, \forall t \\
&\quad (1 - \xi) \times \lambda_{\text{WS}}^{\text{WS}} \text{ Profit}_{WS(i)}^{\text{WS}} < 0 \end{align*}
\] (47)

The KKT optimality conditions of the second level problem are as follows:

\[
\begin{align*}
\mu_{WS(i)}^1 + \mu_{WS(i)}^2 &= 0, \forall t \\
\mu_{WS(i)}^1 \times (\lambda_{\text{WS}}^{\text{WS}} - (1 - \xi) \times \lambda_{\text{WS}}^{\text{WS}}) &= 0, \forall t \\
\mu_{WS(i)}^1 \times (\lambda_{\text{WS}}^{\text{WS}} - (1 + \xi) \times \lambda_{\text{WS}}^{\text{WS}}) &= 0, \forall t \\
\mu_{WS(i)}^1 \mu_{WS(i)}^2 &= 1, \forall t
\end{align*}
\] (48)—(51)

If \(\text{Profit}_{WS(i)}^{\text{WS}} \geq 0\), according to (47), \(\lambda_{\text{WS}}^{\text{WS}}\) is equal to \((1 + \xi) \times \lambda_{\text{WS}}^{\text{WS}}\). Therefore, the second term in (49) is not zero and \(\mu_{WS(i)}^1\) takes a zero value. The value of \(\mu_{WS(i)}^2\) can be calculated using (48), as follows:

\[
\mu_{WS(i)}^2 = -\text{Profit}_{WS(i)}^{\text{WS}} \forall t
\] (52)

By substituting (47) in (50), the equivalent of the first term of (47) can be rewritten as follows:

\[
\begin{align*}
\text{Profit}_{WS(i)}^{\text{WS}} \times (\lambda_{\text{WS}}^{\text{WS}} - (1 + \xi) \times \lambda_{\text{WS}}^{\text{WS}}) &= 0, \forall t \\
\text{Profit}_{WS(i)}^{\text{WS}} \times (\lambda_{\text{WS}}^{\text{WS}} - (1 - \xi) \times \lambda_{\text{WS}}^{\text{WS}}) &= 0, \forall t
\end{align*}
\] (53)—(54)

Finally, using the equivalent of (47) obtained in (53) and (54), the proposed bi-level optimization problem can be recast as follows:

\[
\max_{\text{DV}} \xi \quad \text{Subject to: } \text{Profit} \geq \text{Profit}^{\text{exp}} \times (1 - \alpha)
\] (55)—(56)
\[
\text{Profit} = \sum_{t=1}^{N_t} \sum_{i=1}^{N_i} p_{W,ic}(t) \times p_{W,ic}(t) - \sum_{t=1}^{N_t} \sum_{i=1}^{N_i} \pi_{d,i}(t) \times \delta_{d,i}(t)
\]

\[\begin{align*}
- \sum_{j=1}^{N_{DG}} p_{DG,j}(t) & \times p_{DG,j}(t) - \sum_{j=1}^{N_{DG}} SUC_{DG,j} \times u_{DG,j}(t) \\
- \left[ c_{ij} + b_{ij} \times p_{DG,j}(t) + a_{ij} \times p_{DG,j}(t) \right] & - SDC_{ij} \times u_{OFF,j}(t)
\end{align*}\]

(5)\text{ }—(16) \text{ and } (18)—(40)

The risk level of day-ahead scheduling in the smart distribution network can be controlled thorough adjusting the profit deviation factor (i.e. \( \alpha \)) which is specified by the DNO based on its risk management policy. Equality constraints make hard conditions for optimization problems while in non-equality constraints the variables can easily change their values during solving process. So, the equality constraints of (53) and (54) have been converted into the non-equality constraints of (59) and (60). It should be mentioned that these conversions cannot change the results of optimization routine. Meanwhile, the non-linearity of (53) and (54) can be linearized using SOS2 technique explained in Appendix A.

4. Numerical results

4.1. System data

The proposed model is applied to the IEEE 33-bus distribution network [32]. The single line diagram of the modified test system is illustrated in Fig. 4. In this network, there are seven DG units including four diesel generators and three wind turbines of the same type whose parameters are obtained from [33,34] and are presented in Tables 1 and 2, respectively. Candidate buses for DG units’ installations are selected according to the results of the expansion planning study which is carried out in [35]. It is assumed that all DG units produce active power at a unity power factor.

The energy storage system consists of a battery with a capacity of 1 MW h. The minimum and maximum energy of the BESS is supposed to be 10% and 90% of its own capacity, respectively; and the charging and discharging ramp rate limits for each hour are both set to be 0.2 MW. The step-wise bid-quantity offer package of ILSs is presented in Table 3.

Based on load characteristics (i.e. price elasticity and load profile) three types of customers are considered in the distribution network including residential, commercial, and industrial ones. It is assumed that all customers at each bus of the distribution network are the same type and modeled as a single lumped load, as illustrated in Fig. 5. The price elasticity for each type of customer is not equal along the day and changes in different time periods. These time periods and associated price elasticity are wisely derived from [36] and shown in Table 4.

The appropriate scaled down forecasted loads and market prices at NYISOs PJM on Friday, July 19, 2013 [37] have been used to obtain all numerical results. Fig. 5 provides the hourly forecasted wholesale market prices and distribution network demand. The contribution of customers in the total distribution network demand is shown in Fig. 6 [38]. Moreover, the hourly forecasted wind speed is retrieved from [39] and shown in Fig. 7.

The simulations have been performed with CPLEX solver in generalized algebraic modeling systems (GAMS) environment [40] on a personal computer with 4 GB of RAM and Intel Core 2 due 2.50 GHz processor when the relative termination threshold for solving MILP problems is set to be 0.1%.

4.2. Simulation results

The proposed models are applied to different case studies to investigate their effectiveness.

Case A: This case provides a comparison benchmark to show benefits of incorporation price-responsive customers in day-ahead scheduling of the smart distribution network. To this end, a situation is simulated in which customers pay the same rate for the amount of electricity they consume (110 $/MW h) which is the average value of wholesale market prices on July 2013. Therefore, customers have no incentives to participate in a DA-RTP program and the scheduling of the smart distribution network is limited to DG units, BESS and transactions in wholesale market.

Fig. 8 represents the profit of the DNO versus the number of segments. It is illustrated that with increasing the number of segments the results become more precise, nevertheless the problem complexity is increased. However, the profit of the DNO is fixed with twelve segments (i.e. \( S = 12 \)) and therefore; the precision of the simulations is validated.

The optimal scheduling of DG units, WTs, BESS, ILS and transactions with the wholesale market are presented in Fig. 9. According to this figure, all DG units are scheduled particularly at peak hours to supply network demand and meet technical constraints. Although, DG2 has a higher marginal cost rather than DG4, its placement in an important part of network causes more power production. DG1 with lowest marginal cost is the most dispatchable unit during the scheduling period. Meanwhile, WTs as non-dispatchable units generate their maximum power based on the forecasted wind speed. The BESS charged during hours 4–6 and 21–24 when market prices are relatively low and discharged during the hours 1–3 and 15–18 when market prices are relatively high. The ILSs are mainly dispatched during scheduling period in the hours with higher network demand including 9–23. In low-price hours, namely between 1–11 and 21–24, the DNO purchases energy from the wholesale market to cover network demand. On the other hand, the DNO, in order to maximize its profit, sells energy to the wholesale market during relatively high-price hours including 12–20. The DNO profit in this case is 24,037 $.

Case B: This case is similar to case A, but participating customers in the DA-RTP program pay their electricity bill on the basis of hourly prices determined in the proposed model. It is supposed that 40% of customers participate in DA-RTP program. The maximum and minimum demand limits for elastic customers are equal to 115% and 85% of their forecasted load, respectively. Meanwhile, minimum energy requirement of each responsive customer is supposed to be 90% of its initial daily consumption. The payment coefficient is supposed to be 1. The optimal day-ahead prices offered by the DNO to
residential, commercial and industrial customers are illustrated in Fig. 10. As can be seen, the DNO offers low electricity prices during off-peak hours to motivate customers to shift portion of their consumption to these hours. Meanwhile, day-ahead prices for each type of customer are different in several hours. It is due to the fact that customers’ response to DA-RTP depends on their load characteristics (i.e. price elasticity and load profile) which are different for each type of customer.

Table 1
Parameter values of diesel generators.

<table>
<thead>
<tr>
<th>Unit</th>
<th>$c_i$ [$$/MW]</th>
<th>$b_i$ [$$/MW h]</th>
<th>$a_i$ [$$/MW h^2$]</th>
<th>Technical constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG1</td>
<td>27</td>
<td>79</td>
<td>0.0057</td>
<td>SUC = 15, 10, 1, 1</td>
</tr>
<tr>
<td>DG2</td>
<td>25</td>
<td>87</td>
<td>0.0068</td>
<td>SDC = 13, 9, 1.5, 1.5</td>
</tr>
<tr>
<td>DG3</td>
<td>28</td>
<td>92</td>
<td>0.0065</td>
<td>MDT = 2, 2, 1.5, 1.5</td>
</tr>
<tr>
<td>DG4</td>
<td>26</td>
<td>81</td>
<td>0.0062</td>
<td>DR = 2.5, 2.5, 2.5, 2.5</td>
</tr>
</tbody>
</table>

Table 2
Parameter values of wind turbines.

<table>
<thead>
<tr>
<th>Rated Power [MW]</th>
<th>Cut-in speed [m/s]</th>
<th>Rated speed [m/s]</th>
<th>Cut-out speed [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>15</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3
Step-wise bid-quantity offer package of ILs.

<table>
<thead>
<tr>
<th>IL</th>
<th>Interruption load [MW]</th>
<th>Offered price [$/MW h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL1</td>
<td>0–0.1</td>
<td>0.1–0.2, 0.2–0.4, 0.4–0.6</td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td>13.6, 17.9, 39.4</td>
</tr>
<tr>
<td>IL2</td>
<td>0–0.25</td>
<td>0.25–0.45, 0.45–0.75, 0.75–0.90</td>
</tr>
<tr>
<td></td>
<td>8.3</td>
<td>18.1, 23.9, 52.5</td>
</tr>
</tbody>
</table>

Fig. 4. Single line diagram of the modified IEEE 33-bus distribution network.

Fig. 5. Hourly forecasted of wholesale market prices and distribution network demand.
Fig. 11 compares the distribution network demand profile in case A with case B. It is evident that customers’ consumption during peak hours would be reduced and shifted to off-peak hours in case B, which leads to improve load profile. The load profile improvement described earlier, would provide technical benefits. Table 5 compares the technical performance in case A and case B. As can be seen, all the considered technical performances are enhanced after applying the proposed model. Therefore, incorporating price-responsive customers in day-ahead scheduling of smart distribution networks improves the reliability of energy delivery and the efficiency of network operation.

The optimal scheduling of DG units, WTs, BESS, ILs and transactions with the wholesale market are presented in Fig. 12. As can be seen, the BESS and ILs optimal scheduling are the same as case A. It’s due to the fact that wholesale market prices stay constant in this case. Meanwhile, lower network demand during peak-hours leads the DNO to sell more energy to the wholesale market and, therefore, the DNO profit in this case is 24,916 $ which is more than case A. It’s worth to mention that DG units which provide requirement reserve produce more power. In fact as network demand is reduced in this case, a lower reserve is required.

Financial aspect of the proposed model, including profit of the DNO and total customers’ bill versus payment factor are compared in Figs. 13 and 14, respectively. As it can be seen from Fig. 13, for $\gamma > 0.91$ the profit of the DNO is greater than the case in which customers pay the same rate for the amount of electricity they consume (case A). Therefore, the proposed model, besides the mentioned technical benefits, provides financial motivations for the DNO. In addition, Fig. 14 shows that the total customers’ bill decreases by lessening the payment factor which provides customers with the opportunity to pay less money for their electricity and actively participate in the DA-RTP program while the profit of the DNO is more than that of case A. Owning to these results, the proposed model is a win–win situation in which changing load profile leads to both customers and the DNO advantage, simultaneously.

Fig. 15 shows obtained load profiles for different participation level of customers in the DA-RTP program. As can be seen, a greater portion of network load is decreased during peak-hours and shifted to the off-peak hours as the participation levels of customers in the DA-RTP program increases. Moreover, the profit of the DNO considerably increases to 26,013$. This can be a significant incentive for the DNO to invest in the communication and automation infrastructures needed for demand response implementation.

### Table 4

<table>
<thead>
<tr>
<th>Load type</th>
<th>Time period</th>
<th>On-peak (7–11,17–19)</th>
<th>Mid-peak (11–17)</th>
<th>Off-peak (19–7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>On-peak</td>
<td>-0.18</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Mid-peak</td>
<td>0.07</td>
<td>-0.24</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>0.06</td>
<td>0.1</td>
<td>-0.15</td>
</tr>
<tr>
<td>Commercial</td>
<td>On-peak</td>
<td>-0.22</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Mid-peak</td>
<td>0.08</td>
<td>-0.26</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>0.08</td>
<td>0.06</td>
<td>-0.1</td>
</tr>
<tr>
<td>Industrial</td>
<td>On-peak</td>
<td>-0.24</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Mid-peak</td>
<td>0.12</td>
<td>-0.28</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>0.08</td>
<td>0.14</td>
<td>-0.12</td>
</tr>
</tbody>
</table>
Case C: The price uncertainty is taken into account in this case. The optimal robust profit of the DNO versus the maximized confidence interval (i.e., $\xi$) is shown in Fig. 16.

As it can be seen, these variables are inversely related, which means that a higher value of robust profit needs a lower level of uncertainty for the wholesale market prices. For instance, if $Profit = 22,000$, the corresponding $\xi$ is equal to 0.33, in other words the minimum profit will be equal to 22,000 if the forecasted error of the wholesale market price is less than 33%.

In order to study the effects of the price-responsive customers on the DNO’s profit, the risk management is also performed in the absence of them (i.e. case A) and the
curve of Profit vs. $\xi$ is presented in Fig. 16. As it can be seen, the robust profit in the case when considering price-responsive customers (i.e. case B) is higher than the case in which customers pay the same rate for the amount of electricity they consume (i.e. case A). This is due to the fact that a part of DNO’s risk imposed by forecasted error of the wholesale price is mitigated by the price-responsive customers. Moreover, a portion of the network demand is shifted from peak-hours to less risky off-peak hours, which results in a higher profit
and less exposure risk for the DNO.

Fig. 17 shows the worst values of the wholesale market prices for $\xi = 0.33$. As explained in (47), the worst value of price arises in one of the upper or lower bounds of the confidence interval, depending on the sign of exchanged energy between the distribution network and the wholesale market. When the DNO sells electricity to the wholesale market, the worst value of price occurs on the lower bound of the confidence interval. On the other hand, when the DNO buys electricity from wholesale market, the worst value of price occurs on the upper band of confidence interval. This fact is illustrated in Fig. 17 where also the confidence interval is shown. As it can be seen, during high price hours, namely between 12 and 21, the confidence interval is wider than others. Because, as expressed in (41), the length of the confidence interval (i.e. $\xi \times \lambda_{WS}^{\xi(\alpha)}$) is directly related to the forecasted price of the wholesale market.

The optimal scheduling of DG units, WTs, BESS, ILs and transactions with the wholesale market for $\xi = 0.08$ and $\xi = 0.33$ are presented in Figs. 18 and 19, respectively. Comparing Figs. 18 and 19 shows that increasing the amount of $\xi$ leads to fewer transactions with the
wholesale market, due to the risk-averse strategy of the DNO. In other words, the DNO relies on its own generations rather than on the wholesale market. 

To check the robustness of proposed model a Monte Carlo (MC) simulation has been conducted. It aims to verify the robustness of obtained solutions. To this end, the optimal day-ahead scheduling for $\xi = 0.33$, as indicated in Fig. 19, is used for robustness verification. The robust profit is $22,000$. 10,000 scenarios of the wholesale market prices (i.e. $w_{st}$, $t = 1, \ldots, 24$) are randomly generated in a way that Eq. (46) is satisfied. The profit of the DNO is calculated using (57). The MC simulation results are illustrated in Fig. 20. The minimum, average, maximum and standard deviation of simulated profits are 22,441, 24,394, 26,617, and 562,422 respectively. With regard to Fig. 20, it is observed that the DNO's profit is greater than the specified value (vertical line shown in Fig. 20 which is $22,000$). Therefore, the MC simulation confirmed that applying the day-ahead scheduling found by using the proposed IGDT-based model ensures the DNO that the profit will be greater than the specified value (i.e. $Profit^{exp} \times (1 - \xi)$), if the forecasted error of the wholesale market prices remains less than $\xi$.

### 5. Conclusion

In this paper, a novel modeling and mathematical formulation are proposed for the day-ahead scheduling of smart distribution networks in which the potential benefits of price-responsive customers is enabled through DA-RTP sale prices. This model provides a practical tool for the DNO to determine hourly sale prices offered to customers, transactions with the wholesale market, commitment of DG units, dispatch of BESS and planning of ILS. The model is formulated in a MILP format for which the global optimal solution is guaranteed. The results demonstrated that the proposed model not only provides a higher profit for the DNO but also leads to significant motivations for customers to modify their consumption and reduce their electricity bill. From the technical point of view, applying the proposed model resulted in flatter load profile, lower peak load and peak to valley distance, lower power losses, higher load factor and evenly voltage profile. The risk management model, based on the IGDT method, was established in a single level robust optimization problem to hedge the DNO against uncertainty of the wholesale market prices. Compared with other risk management methods, the proposed robust IGDT-based model guarantees a pre-specified level of profit and therefore, makes it suitable for risk-averse DNO. The obtained results indicate that applying the DA-RTP program simultaneously reduces the imposed risk due to forecasted error of the wholesale market prices and increases the profit of the DNO. Meanwhile, the robustness of IGDT-based model was confirmed using MC simulation and it was found that the profit obtained by DNO is always higher than the guaranteed profit. Finally, the utilization of linearized loads’ voltage dependency model into power flow equations of the proposed model could be the focus of the future research.

### Appendix A

To obtain an optimal solution under the MILP format, the non-linear equations of the proposed model including (17), (21), (59), and (60) must be linearized. Generally, the non-linear terms appear in mentioned equations can be expressed using following functions:

$$f(\theta) = \theta^2$$

$$g(\theta, \theta') = \theta \times \theta' = \frac{(\theta + \theta')^2 - \theta^2 - \theta'^2}{2} = \frac{f(\theta + \theta') - f(\theta) - f(\theta')}{2}$$

For the sake of simplicity, the process to linearize (62) is presented. The same rational can be utilized to linearize (63).

The SOS2 technique is used to represent Piecewise Linear Approximation (PWL) of functions of a variable. SOS2 is typically used to model non-linear functions of a variable in a linear model when embedded in a Branch and Bound algorithm enables truly global optima to be found and not just local optima as well as search procedure will generally be noticeably faster [22].

As shown in Fig. 21, the PWL of (62) can be achieved using a set of piecewise linear segments. This can be mathematically formulated as follows [23]:

$$\theta = \sum_{i=1}^{S+1} \alpha_i \times \theta_i$$

$$f(\theta) \equiv \sum_{i=1}^{S+1} \alpha_i \times f(\theta_i)$$
\[
\sum_{i=1}^{S} \alpha_i = 1 \tag{66}
\]

In the above formulations, \(S\) denotes linear segments and \((\theta_i, f(\theta_i))\) represents the endpoint of each linear segment. The continuous variable \(\alpha_i\) determines over which segments the value is within. To ensure that at most two \(\alpha_i\) variables be non-zero and adjacent, the following constraints are required:

\[
\theta_1 \leq \theta_i \leq \theta_{i-1} + 1 \quad \forall S = 2, \ldots, S \tag{67}
\]

\[
\theta_2 \leq \theta_{i-1} \tag{68}
\]

\[
\theta_S \leq \theta_{i-1} \tag{69}
\]

\[
\sum_{i=1}^{S} \beta_i = 1 \tag{70}
\]

where, \(\beta_i\) is a binary variable. If \(\beta_i\) lies within segment \(s\) the value of \(\beta_i\) is equal to 1, otherwise 0.

References


