Offering a model for determination of optimal warranty period and production price

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ABSTRACT: Nowadays warranty is an important element in marketing, especially for the products that enter the market for the first time. Usually customers do not have enough information about productions efficiency, in such conditions; warranty plays a significant role in persuading customers to buy the products. As customer's point of view, warranty is a sign of quality and reliability and when customers have different choices among the similar products, warranty will be an effective factor in decision making. Also warranty, as a means of marketing, has positive effects on manufacturer's sales and profit but in the other hand, it imposes some costs to the manufacturers. As failures occur randomly, calculation of warranty costs is complicated but estimation of these costs is essential cause of their effects on profit. This paper tries to offer a model for determination of optimal warranty period and production price in regards to effects of warranty on sales amount. For achieving this purpose a model that is included of three warranties polices, which was offered by Murthy, is used and it is optimized by Glickman and Berger equation and at the end an application of the model is explained.

Keywords: Warranty, Warranty Policy, Free Repair/Replacement warranty (RFRW), Pro Rata Warranty (PRW)

INTRODUCTION

Warranty is a contract between seller and purchaser, in which seller guarantees satisfactory function of product regarding what is defined as product function up to a specified date (Blischke and Morty 1992). Product warranty has a significant role in strategies of production and marketing in order to obtain market share and to control expenses depending on quality. Today, producers use warranty as a tool to gain competitive advantage as studies conducted show that purchasers consider mode of offering warranty as an indicator of quality and reliability of product and this is effective as to extent of purchase by purchasers and on extent of sale for producers. Although offering warranty has a positive effect on sale of producers, as it creates a liability for them, brings expenses to them as well. As such expenses are related to the extent of fault in product and such faults are accidental, therefore, these expenses are variable and accidental as well. Producers have various warranty policies to offer warranty. Among such policies is free replacement warranty, free repair warranty or pro rata warranty. What affects way of offering warranty on the part of producers is extent of failures of products and related expenses. Thus, producers are to determine life distribution function of product, its reliability and therefore, failure rate of product by collecting information related to failures. This way they can estimate expected expenses for each policy of offering warranty and therefore, determine length of offering warranty and sales price which is economically justified.

Literature review

Warranty Policies

Warranty policies are comprised of three compensatory measures, number of criteria regarded for determining length of warranty coverage and mode of counting length of warranty (Blischke etal 1975). Classification of such factors causing formation of various policies of warranty is as follows:

Compensatory Measures

When a product encounters failure during warranty, seller has a few compensatory options such as repair, replacement or paying compensation for the failure. According to these measures, warranty policies are classified as free repair/ replacement warranty (FRW), pro rata warranty (PRW) and combination warranty (combination of two previous policies).
Free Replacement / Repair Warranty
According to free replacement / repair warranty, seller agrees to replace or repair product during warranty as long as failure is not a result of misuse. Usually, the number of repairs and replacements is not limited. FRW is the most common warranty policy for durable products such as television, personal computers, automobile parts, etc.

Pro Rata Warranty
According to this warranty, seller and purchaser have equal share in expense of failures. Seller must replace product and pay for a part of sales price. That is, he/she must offer discounted product to purchaser. Expenses of replacement or discount for purchaser depend on product life time. For example, a linear function of paying expenses can be defined as follows:

\[ r(t) = \begin{cases} (1 - \frac{t}{W})P & t \leq W \\ 0 & \text{else} \end{cases} \]

Where, \( t \) is time of failure, \( W \) is warranty period and \( P \) is sales price. According to this policy, amount of discount considered based on linear function of time is reduced and after warranty period is finished will be equal to zero. PRW warranty is usually offered for not repairable products such as automobile tire and batteries which encounter failure due to oldness.

Combination Warranty
This warranty is a combination of FRW and PRW warranties. The most common usage of this warranty starts with a period of FRW and followed by a period of PRW. A combination warranty can be used to attract more consumers without raising expenses of warranty.

Criteria Considered for Determining Warranty Coverage Period

One-Dimensional Warranty Policies
In such policies, warranty coverage period is determined using one criterion. In practice, time or age is the most common variable among these criteria. Warranty coverage starts from purchase and after a specified period is finished since occurrence of most failures depends on age of products. In some cases, failures are just due to number of usages or extent of using product. Extent of usage, for example on Kilometer or Mile basis, number of usages or flying hours etc. can be named as criteria for determining length of warranty coverage.

Two-Dimensional Warranty Policies
In this case, coverage period is determined by two characteristics of product (e.g. age and extent of function). For majority of products, failures only occur due to oldness or high number of usage or a combination of these two factors. Thus, these two factors must be used together to determine length of warranty coverage (Morty and Eskandar 2003).

Warranty Coverage Periods
Non Renewable Warranty
Length of coverage for this sort of warranty is fixed and is determined at the time of purchase. Replacement or repair of product does not change length of initial warranty. Coverage of this warranty is finished at the end of initial warranty period. This sort of warranty is the most common warranty policy used in consumer products.

Renewable Warranty
In this warranty, when a failure occurs, product is replaced or repaired and along with that, length of warranty is recalculated. This warranty is usually offered for not repairable products along with PRW warranty. For example if automobile battery encounters a failure before warranty ends, it can be replaced with a new one according to joint expense liability of seller and purchaser.

Warranty Expense
When a product is guaranteed, high expenses are imposed on seller. If warranty plan is not fully assessed, expenses of warranty may lower profit of seller. Usually expenses of warranty are as follows:
Administrative expenses, 
Transportation expenses, 
Repair/replacement expenses, 
Product returns expenses, 
Expenses of warranty offering center, 
Inventory expenses for spare parts, 
Court expenses when it comes to lawsuits.

Moreover, when products encounter failure during warranty, intangible expenses such as losing satisfaction of customers are imposed on seller (Belshiki and Skiver 1975). Expense of warranty is hardly predicted since failure of product occurs accidentally and conditions of usage have been variable overtime and depend on customers.

**Product Failure**

No product is made to work for over and will finally encounter one or more failures during its life time. When product does not deliver function it was designed for, failures occur. Reason for failure can be internal, that is due to fault or oldness of product or external and due to excessive tension while using or misuse. If t is non-negative continuous random variable showing failure time, f(t) is failure density function and F(t) is related distribution function, reliability function will be as follows:

\[
R(t) = 1 - F(t)
\]

If h(t) is failure rate or hazard function, rate of hazard will be as conditional probability of failure occurrence in unit of time, Assuming that failure has not happened before.

\[
h(t) = \lim_{\tau \to 0} \frac{1}{\tau} \left\{ t < \tau \leq t + \tau | \tau > t \right\} = \frac{f(t)}{R(t)}
\]

Three phases are identified during life time of majority of products. In phase one or maturity stage or infancy period, failure rate is reduced over time. Failure during this phase is mainly due to faulty materials or poor production. Next phase is called useful or practical phase of natural life stage, in which, failure rate is usually fixed. Failures during this phase are due to accidental pressure on product. In third phase or oldness phase, failure is due to gradual weakness and accumulation of shocks. This cycle is called life cycle of product (Haj Shirmohammadi 2000).

**Product Repair**

Dependent on its effect on failure rate after repair, repair of products can be divided into three types

**Full repair**

After full repair of product, product is considered as new one having the same failure rate of any new product.

**Partial Repair**

Conditions of product after partial repair seem uncertain. That is, rate of failure after partial repair is a random variable. Repaired product may have higher or lower rate of failure than before failure.

**Minimum Repair**

After minimum repair, efficiency of product does not change. That is repaired product will have the same rate of failure as before failure happens (Morty and Blishki 1992).

**Burn in Test**

Burn in test is a proved method used when product has a reducing function of failure rate. During this test, initial failures, due to poor components or assembly errors are omitted in a way that product is delivered with healthy components. In conclusion, burn in test enhances reliability of product and reduces need to use warranty. Generally, burn in test imposes extra expenses on producer. Every time product encounters a failure during warranty, producer may encounter more expenses such as transportation and maintenance, expenses of losing customer or court expenses. Therefore, repairing a possible fault during burn in test comes with fewer expenses than during warranty (Nguyen, D.G and Morty, 1976).


**Warranty Elasticity**

Warranty elasticity is defined as percentage of change in sales relative to percentage of change in terms of warranty and has a direct effect on extent of sale. Increase in warranty elasticity is translated into enhancement of warranty significance for purchasers when purchasing a product (Glickman, T.S., and Berger, P.D 1976).

**Description of model offered by Morty et al**

Morty et al (2007), Cho et al (1992) and May (1997) all studied expense parameters in the same way in their own models and assumed that all parameters of expense have been estimated and are fixed. Burns in test expenses are classified according to reparable or non-reparable of product. For expenses of warranty, three warranty policies are explained: Free repair warranty, free replacement warranty and pro rata warranty. Practically, these warranties are the most common types of warranties offered by producers. According to cost-based classification conducted by Morty et al (Morty, Hong, Lieu, 2007) in their models, total expense for producer is classified as follows:

$c_m$: Expense of Producing Each Unit of Product

$c_1$: Fixed expense of burn in test of each unit of product which is independent of the number of failures occurring during burn in test and independent of burn in test time. This expense is defined as expenses of setting up and removing test equipment or expenses of unpacking and repacking etc.

$c_2$: Expense of burn in test dependent on time, for example expense of inspection and expenses of delay in completing production process

$c_3$: Expenses dependent on failure including expenses of repair for each failure

$c_4$: Extra expenses for each failure during warranty are expenses of transportation, maintenance and in case of lawsuit, expenses of court as well as intangible expenses such as losing satisfaction of customers.

**Burn in Test Expense**

**Burn in test expense for the when faulty product is repaired at the time of test**

In model described it is assumed that after completion of production process, products are tested for a specified duration of time and their faults are controlled. When failure occurs, failed product is repaired and retested. Time of burn in test continues from the point stopped due to failure. This process of test-repair-test-repair continues up to completion of burn in test. Also, the following hypotheses exist: 1- Time of repair is zero 2- Repair is minimum.

For majority of products, time of repair relative to interval between failures is very short. That is, while time of repair can be measured in hour, interval between failures is determined in month or year. In this model it is supposed that time of repair is negligible.

If B is the time of burn in test, expense of test plus expense of production is as follows:

$$c_m + c_1 + c_2 B + c_3 N(B)$$

Where, $N(B)$ is the number of expected failures during burn in test. In minimum repair conditions, $N(B)$ is this way (Blishki and Skiver 1981).

$$\int_0^B h(t)dt$$

Thereafter, expenses of production and expenses of burn in test are called expenses of burn in test. Therefore, expenses of burn in test predicted for each product are as follows:

$$C_B = c_m + c_1 + c_2 B + c_3 \int_0^B h(t)dt$$

**Expenses of burn in test in conditions where failed product is replaced during test.**

In this case, if during burn in test, product encounters a failure, it will be replaced with a new one. New product is tested and through adjusting burn in test hour to zero, test is started again. This process of test-replacement-test-replacement continues up to the point where burn in test is completed with no fault.

Expenses include expenses of production, fixed expense of burn in test and expense of product test which is determined dependent on the fact that if product has encountered failure before completion of burn in test or not and is calculated as follows:


Since failures happen randomly, expenses of burn in test for each product is a random variable and depends on the time spent for the test. Test time spent for each product is calculated as follows:

\[ \int_{0}^{B} R(t) \, dt \]

(Proof in attachment 1)

Where, \( R(t) \) is function of reliability of product

Thus, expected expense of production and burn in test for each product is as follows:

\[ c_m + c_1 + c_2 \int_{0}^{B} R(t) \, dt \]

Burn in test continues up to the point that part under test passes it successfully and therefore number of predicted tests until first product passes test is considered as a random variable. It is assumed that all products undergo a production process with the same failure distribution. Thus, number of tested products up to obtaining the first healthy product, is an random variable with geometrical distribution having probability of success \( p = R(B) \) and average of \( \mu = \frac{1}{R(B)} \), where \( R(B) \) is probability of product failure during burn in test. Thus, predicted expense of burn in test for each remained product of the test is total expenses of all tested units. This expense is calculated as follows:

\[ \frac{1}{R(B)} \int_{0}^{B} h(t) \, dt \]

**Warranty Expenses**

**Free Repair Warranty Expenses**

In this model, seller repairs failures during warranty. Also, it is assumed that repair during warranty is minimum. When a product encounters a failure during warranty, extra expenses of C4 plus repair expense C3 are imposed on seller.

If \( w \) is term of warranty, predicted expense of warranty for each product under minimum repair is calculated as follows:

\[ C_w = (c_3 + c_4) \int_{0}^{B + w} h(t) \, dt \]

If \( h(t) \) is a t decreasing function, warranty expense decreases along with passing burn in test time.

**Free Replacement Warranty Expense**

When failure occurs during warranty, failed product is freely replaced with another one.

In this case, failure distribution of products after burn in test is important. After completion of burn in test, product is on B age. Here, it is assumed that failure distribution is fixed. If \( F_B(t) \) and \( f_B(t) \) are conditional distribution function and function of probability density of failure time of \( t \) (supposing that product has passed burn in test time of B),

\[ F_B(t) = \frac{F(t + B) - F(B)}{R(B)} \quad t \geq 0 \]

\[ f_B(t) = \frac{dF_B(t)}{dt} = \frac{f(t + B)}{R(B)} \quad t \geq 0 \]

Expected number of replacements during warranty must be calculated to find predicted expenses of warranty. If \( \{t_n, n = 1, 2, 3, \ldots\} \) is a series of product failure times, each failed product will be replaced with a
new product of the same production process after passing burn in test. Therefore, it can be assumed that
failure times are independent and have equal distribution with a common distribution function \( F_B(t) \). If \( N(t) \) is
number of replacements in \([0,t]\):
\[
M_B(t) = E[N(t)]
\]
If \( M_B(t) \) is the number of expected replacements in the interval \([0, t]\), it can be shown that:
\( M_B(t) \) is called renewal function (Frazer 1986).
With the help of \( M_B(t) \), expected expense of warranty for each product is obtained as below:
\[
C_W = [C_B + c_i] M_B(W)
\]
Which \( C_B \) is burn test expense and \( M_B(W) \) is the number of expected replacements during warranty.

**Pro Rata Warranty Expense**

In this case, when failure occurs during warranty, rebate is paid to customer. Rebate depends on the
age of product at the time of failure and is determined based on sales price (P). The function usually used by
producer to pay for rebate is a linear rebate function.
\[
r(t) = \begin{cases} 
1 - \frac{t}{W} & \text{if } t \leq W \\
0 & \text{otherwise}
\end{cases}
\]
It should be noted that amount of rebate paid by producer decreases linearly through aging of product and
reaches zero in \( W \). thus, predicted warranty expenses for each product is equal to predicted rebates for
customer.
\[
C_W = \int_0^W r(t) f_B(t) dt
\]
\[
= \int_0^W \left( 1 - \frac{t}{W} \right) P f_B(t) dt = P \left[ F_B(W) - \frac{1}{W} \mu_B(W) \right]
\]
Where \( \mu_B(W) \) is calculated as follows:
\[
\mu_B(W) = \int_0^W t f_B(t) dt
\]
\[
= \frac{1}{R(B)} \left[ W.F(B + W) - \int_0^W F(t + B) dt \right]
\]
By inserting \( F_B(t) \) and \( M_B(t) \), the following result is obtained:
\[
C_W = P \left[ \frac{F(B + W) - F(B)}{R(B)} - \frac{1}{WR(B)} \left( W.F(B + W) - \int_0^W F(t + B) dt \right) \right]
\]
\[
= P \left[ 1 - \frac{1}{WR(B)} \int_0^W R(t + B) dt \right]
\]

**Optimizing Model Based on Price Elasticity and Warranty Elasticity**

If \( \Pi \) is total expected profit, it will be equal to number of expected sales in profit of each unit minus fixed
expense:
\[
\Pi = (P - C) Q - \text{Fixed Expense}
\]
Where, \( P \) is sales price of each unit
\( C \) is expected expense of each unit including warranty burn in test
\( Q \) is amount of expected sales in planning horizon
In total, \( Q \) can be a function of several different variables
For example: \( Q = f(P, W, A, D, R, F, P', W', A', D', R', F') \)

Where, \( W \) is term \( t \) warranty, \( A \) is advertisement, \( D \) is mode of distribution, \( R \) is quality, \( F \) is properties of product and \( P', W', A', D', R', F' \) are variables of competitors. Based on Glickman, T.S., and Berger, P.D equation, this study only considers term of warranty ad sales price as decision making variables. All other variables are considered as fixed ones (that is, decision maker has determined such values beforehand or has no control over them). Moreover, it is assumed that amount of sales has inverse relation with price and direct relation with term of warranty. That is:

\[
\frac{\partial Q}{\partial P} < 0, \frac{\partial Q}{\partial W} > 0.
\]

Based on Glickman and Berger equation, amount of sales is defined as a function of \( W \) and \( P \):

\[
Q = kP^{a}W^{a_2}
\]

Where, \( k>0 \) is a fixed coefficient \( a_1 < -1 \) is price elasticity \( 0 < a_2 < 1 \) is warranty elasticity and \( a1<(a2+1) \). The above relation is called Glickman and Berger equation which is a sort of multiplicative model. Amount of sales decreases exponentially along with price and increases along with term of warranty. It is assumed that \( a_2, a_1 \) and \( K \) are certain parameters and are estimated with regard to other marketing variables.

If \( P^*, W^*, B^* \) are optimum time of burn in test, optimum term of warranty and optimum sales price maximizing profit, conditions necessary for optimizing is that the first derivative must be zero. That is:

\[
\frac{\partial \Pi}{\partial P} = 0, \frac{\partial \Pi}{\partial W} = 0, \frac{\partial \Pi}{\partial B} = 0
\]

Result of expected profit function derivative calculation relative to sales price is as follows:

\[
\frac{\partial \Pi}{\partial P} = (P - C) \frac{\partial Q}{\partial P} + Q \left[ 1 - \frac{\partial C}{\partial P} \right] = 0
\]

\[
\frac{\partial Q}{\partial P} = k a_1 \: P^{a_1 - 1} W^{a_2} = Q \frac{a_1}{P}
\]

And after inserting the above value is obtained:

\[
(P - C)a_1 + P \left[ 1 - \frac{\partial C}{\partial P} \right] = 0
\]

Therefore:

\[
P^* = C \left( \frac{a_1}{a_1 + 1 - \frac{\partial C}{\partial P}} \right)
\]

Derivative calculation of profit function relative to \( W \) gives an answer:

\[
\frac{\partial \Pi}{\partial W} = (P - C) \frac{\partial Q}{\partial W} - Q \frac{\partial C}{\partial W} = 0
\]

\[
\frac{\partial Q}{\partial W} = k a_2 P^{a_1} W^{a_2 - 1} = \frac{Q a_2}{W}
\]

Through inputting the above value and optimum price in equation 5-3, optimum term of warranty will be as follows:

\[
W^* = \frac{C}{\frac{\partial C}{\partial W}} \left( \frac{a_1}{a_1 + 1 - \frac{\partial C}{\partial P}} \right) a_2
\]

After derivative calculation of profit function relative to \( B \) (burn in test term duration):

\[
\frac{\partial \Pi}{\partial B} = -Q \frac{\partial C}{\partial B} = 0
\]
Therefore, optimum duration of burn in test is obtained when:
\[
\frac{\partial C}{\partial B} = 0
\]

Now, through results obtained for optimum duration of burn in test \( \frac{\partial C}{\partial B} = 0 \), \( w^* \) and \( P^* \) according to expenses, three different warranty policies are studied as follows in order to obtain optimum duration of burn in test, optimum duration of offering warranty and optimum sales price for each of these three warranty policies.

**Optimum Duration of Warranty and Sales Price in Free Repair Policy**
Failures at the time of burn in test and warranty are repaired by the producer. Expense expected for each unit of product is obtained as follows:

\[
C = C_B + C_W = c_m + c_1 + c_2 B + c_3 \int_{0}^{B} h(t) dt + (c_3 + c_4) \int_{B}^{B+W} h(t) dt
\]

Optimum duration of burn in test-optimum:

\[
h(B^*) = \frac{c_2 + [c_3 + c_4] h(B^* + W^*)}{c_4}
\]

Term of warranty:

\[
W^* = C \left[ \frac{a_2}{c_3 + c_4 h(B + W)} \right] \left( -\frac{a_1}{a_1 - 1} \right)
\]

Optimum sales price

\[
P^* = C \left( \frac{a_1}{a_1 + 1} \right)
\]

**Optimum Term of Warranty and Optimum Sales Price in Free Replacement Policy**
At this state, failure during burn in test and warranty results in replacement of product. Expected expense for each unit of product based on equations 4-2 and 6-2 is calculated as follows:

\[
C = C_B + C_W
\]

\[
= C_B + \left[ C_B + C_4 \right] M_B(W)
\]

\[
= C_B \left[ 1 + M_B(W) \right] + c_4 M_B(W)
\]

\[
= \frac{1}{R(B)} \left[ c_m + c_1 + c_2 \int_{0}^{B} R(t) dt \right] \left[ 1 + M_B(W) \right] + c_4 M_B(W)
\]

And optimum burn in test:

\[
h(B^*) = -\frac{c_2 [1 + M_B(W)] + [C_B + c_4] \frac{\partial M_B(W)}{\partial B}}{C_B [1 + M_B(W)]}
\]

Optimum duration of offering warranty in this policy is as follows:

\[
W^* = \frac{C}{[C_B + c_4] h(W)} \left( -\frac{a_2}{a_1 - 1} \right)
\]

Optimum sales price of product in this policy is as follows:

\[
P^* = C \left( \frac{a_1}{a_1 + 1} \right)
\]
Optimum Duration of Warranty and Optimum Sales Price in Pro Rata Warranty Policy

In this case, seller replaces faulty product with a new one during test and pays for its compensation during warranty. Expected expense of each unit of product as follows:

\[
C = C_B + C_w = \frac{1}{R(B)} \left[ c_m + c_i + c_2 \int_0^B R(t) dt \right] + \frac{P}{R(B)} \left[ R(B) - \frac{1}{W} \int_0^w R(B + t) dt \right]
\]

Optimum duration of test is obtained as follows:

\[
h(B^*) = \frac{c_2 + \frac{P}{R(B^*)} \left[ F(B^* + W) - F(B^*) \right]}{\frac{P}{R(B^*)} \int_0^w R(B^* + t) dt - C_B}
\]

Optimum duration of warranty is as follows:

\[
W^* = \frac{C}{\frac{\partial C}{\partial W}} \left( \frac{a_1}{a_1 + 1 - \frac{\partial C}{\partial P}} - 1 \right) a_2
\]

Therefore, optimum term of warranty and optimum sales price are calculated from the following relations

\[
W^* = \frac{P}{W^{**2}} \left[ \int_0^{W^*} R(B^* + t) dt - W^* R(B^* + W^*) \right] \left( \frac{a_1}{a_1 + 1 + \frac{1}{W^*} \int_0^{W^*} R(B + t) dt - R(B^*)} \right) a_2
\]

\[
P^* = \frac{C}{\frac{a_1}{a_1 + 1 + \frac{1}{W^*} \int_0^{W^*} R(B + t) dt - R(B^*)} a_2}
\]

In this policy, gaining answers is realized through numerical methods (Tables1)

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**Table 1. Effects of Changing Price elasticity on Duration of Warranty and Sales Price**

<table>
<thead>
<tr>
<th>Pro rata warranty</th>
<th>Free replacement warranty</th>
<th>Free repair warranty</th>
<th>Price elasticity</th>
<th>Warranty elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum sales price (IRR.)</td>
<td>Optimum term of warranty (Month)</td>
<td>Optimum sales price (IRR.)</td>
<td>Optimum term of warranty (Month)</td>
<td>Optimum sales price (IRR.)</td>
</tr>
</tbody>
</table>

**Application of Model**

In this part, we use offered model in order to determine optimum term of warranty and optimum sales price for a product in automotive production industry with the following characteristics. To obtain optimum answers for three different policies (according to production and repair data), expense of producing each unit of product is IRR. 700,000, average expense of repair for such part amounts to IRR 200,000 and extra warranty expenses (transportation and maintenance) are about IRR. 50,000. After studying data relating failure of product and conducting goodness of fit test with 95% confidence interval, assumption that failure follows exponential increase, was rejected. Thus, rate of failure or speed of failure is the same amount of distribution parameter equal to 0.02838 = λ. Due to the fact that failure function follows exponential distribution and due to the fact that rate of failure is fixed, optimum equation for different policies after placing λ and simplifying equation will be as follows (Figure1):
Determining exact figures for price and warranty elasticity requires conducting economic studies. To this end, various figures of such elasticity which according to available studies, price elasticity, and warranty elasticity in part producing industry is located within that, are chosen and effect of such figures in term of warranty and price of product are studied. Results of such study are presented in the following table 2.

Table 2. Effects of Changing Price elasticity on Duration of Warranty and Sales Price

<table>
<thead>
<tr>
<th>Warranty Policy</th>
<th>Pro Rata Warranty</th>
<th>Free Replacement Warranty</th>
<th>Free Repair Warranty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale Optimal (RIALS) Price</td>
<td>Optimal Duration (Month)</td>
<td>Optimal Price (RIALS)</td>
<td>Optimal Duration (Month)</td>
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<td>2625000</td>
<td>2625000</td>
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CONCLUSION AND RECOMMENDATIONS

CONCLUSION

What presented in this article is concurrent study of optimum duration of burn in test, warranty duration and sales price for warranty-enjoyed products. It was assumed that sales price and term of warranty were among factors affecting amount of sales. Three policies offered by Morty et al were explained and mode of calculating expected number of repairs in free repair policy, free replacement policy expected were described in free replacement policy and pro rata warranty. Types of repairs were described and it was assumed that in all warranty policies, type of repair is minimum. For explaining correlation between sales price and term of warranty and amount of sales, Glickman and Berger relation was used. Moreover, optimum relations according to effect of Glickman and Berger relation on three said warranty policies were obtained. In all of these policies, price elasticity, warranty elasticity and expenses described were among factors affecting determining sales piece and term of warranty. Studies show that by increasing price elasticity, producer is obliged to decrease sales price in order to maintain his/her share of market and as decreasing sales price decreases profit of producer out of each unit of product, tendency of producers to offer warranty which has subsequent expenses, is lowered and thereby, term of warranty is reduced. Also, studies show that increase in warranty elasticity results in increase in the length of warranty duration and this increases possible expenses and finally results in increase in sales price of products.

As expenses of repair in the majority of cases are less than replacement expenses, term of warranty in free repair policy is more than two other policies. In two other policies, as replacement of product is followed, term of warranty is close to each other. Moreover, in pro rata policy as at the time of replacement, a part of expense is paid by purchaser, sales price is less than two other policies.
Recommendations

In this study, it was assumed that factors affecting extent of sales were only sales price and term of warranty. But in the future studies, other factors affecting extent of sales can be regarded for. Also, in this study, all expenses were assumed as fixed but variable expenses and inflation can be regarded in equations too. Moreover, present study only addresses one-dimen- sional warranties while two-dimensional warranties having many applications in automotive manufacturing industry can be studied. Repairs addressed by this study are minimum while other sorts of repairs and their effect on rate of failure and warranty can be investigated as well.

REFERENCES