Experimental and numerical study on heat transfer and flow resistance of oil flow in alternating elliptical axis tubes

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Abstract

As heat exchangers are used extensively in many engineering fields the increase of their heat transfer rate, while reducing their flow resistance has become one the most challenging aspects in heat transfer. In this study, heat transfer and flow resistance of alternating elliptical axis tubes is investigated both experimentally and numerically. The working fluid is heat transfer oil, and the flow’s Reynolds number ranges from 300 to 2000. The grid and numerical models are generated using Gambit 2.4.6 and Fluent 6.3, which are verified by the experimental results. The numerical results show that decreasing the aspect ratio and pitch length, increases heat transfer and flow resistance. However, in order to compare the heat transfer and flow resistance simultaneously, the non-dimensional heat transfer enhancement ratio is defined. The comparison of this ratio shows that alternating elliptical axis tubes perform better than the flattened or circular ones. It is observed that this ratio increases with the increase in Reynolds number.

1. Introduction

Heat exchangers are widely used in many engineering fields, so their design and development are considered as a major field of study in heat transfer knowledge. Several heat transfer enhancement techniques have been introduced to improve the overall thermo-hydraulic performance of heat exchangers resulting in reduction of the heat exchangers’ size and operating cost.

In general, there are two main approaches to enhance the heat transfer rate. The first one is called active methods and requires external power source such as surface vibration or electrostatic fields [1–4]. The second approach which is called passive methods performs the task without any direct input of external power. This type of enhancement is achieved by extended surfaces, adding nano-particles to the working fluid, use of tubes with special geometries, and so on [5–9]. The objective of using tubes with special geometries is to boost flow turbulence and to generate secondary flow. However, it should be noted that changing the geometry of tubes would result in more pressure drop. As a result, studying new geometries for tubes should consider both heat transfer enhancement and flow resistance.

Tan et al. [10] investigated convective heat transfer and fluid flow in twisted oval tubes experimentally and numerically. The experimental study showed that heat transfer and pressure drop increase simultaneously using these tubes compared to the smooth circular one. The effects of the geometrical parameters on the performance of the twisted oval tubes have been analyzed numerically. Their results revealed that the convective heat transfer coefficient and friction factor increase with the growth of aspect ratio. However, they decrease with the increase in twist pitch length.

Pethkool et al. [11] studied turbulent water flow inside a helically corrugated tube experimentally. Results showed that the heat transfer and thermal performance of a corrugated tube are better than those of a smooth circular tube. Rainieri et al. [12] investigated the forced convective heat transfer in straight and coiled tubes, having smooth and corrugated walls experimentally for Ethylene Glycol where the Reynolds number varied from 150 to 1500. Their main conclusion was that the wall curvature and corrugation enhance heat transfer. They obtained the largest increment in heat transfer using corrugated helical coils. They also suggested that the combined passive technique based on wall corrugation and curvature represents an interesting solution for Reynolds numbers ranging from 150 to 1500.

Meng et al. [13] investigated the alternating elliptical axis tubes with water as the working fluid experimentally. They showed that the heat transfer of the alternating elliptical axis tubes is more than
that of the twisted elliptical and the corrugated tubes for equal pumping power within a wide range of Reynolds numbers (500 to \(5 \times 10^5\)). Chen et al. [14] investigated flow in an alternating horizontal or vertical oval cross-section pipe with computational fluid dynamics. They showed that it is difficult to find an optimized geometry that can perform well for a wide range of Reynolds numbers. However, their results indicated that the pipe, if well designed, can perform better than a circular pipe for the flow conditions specified in their paper.

Despite the above mentioned studies, there is not sufficient work done about heat transfer and flow resistance of high Prandtl number fluids in noncircular tubes. The present work investigates both experimental and numerical results for heat transfer and flow resistance of heat transfer oil in alternating elliptical axis tubes. The effects of changing the geometry on these two main parameters are studied for the Reynolds numbers ranging from 300 to 2000.

2. Experimental setup

As the first step towards explaining the experimental setup, Alternating Elliptical Axis (AEA) tube is shown in Fig. 1. AEA tubes are developed based on boundary layer breakage principle. Regrading this principle, a circular tube is flattened alternately with 90° rotation of its cross-section in each segment. AEA tubes are put into practice with five different geometrical characteristics presented in Table 1. It should be noted that all the investigated tubes are made of copper, their outer diameter in the circular section is \(580\), and their thickness is 0.63 mm.

The experimental apparatus is shown schematically in Fig. 2. It mainly consists of a reservoir tank, a pump, a flow loop, a test section, a cooler, and a steam supplier tank. The transparent plastic reservoir tank with the capacity of 4 l is utilized to reserve the working fluid and monitor its height. In order to measure the tube’s wall temperature, six K-type thermocouples are mounted on it with equal distances from each other. The bulk temperature of the working fluid is measured by two other K-type thermocouples at the entrance and exit of the test section. Two adjusting valves control the flow rate, one at the end of the test section and the other at the by-pass line. A shell and tube heat exchanger is used as a cooler to reduce the temperature of the working fluid. The 50-l steam supplier tank is fitted with an 8 kW element heater to generate fully saturated vapor. In order to maintain the constant temperature boundary condition at tube’s wall, 120 cm of the test section is surrounded by the saturated vapor. The entire steam supplier is totally insulated by fiberglass cover with the purpose of minimizing the heat loss. The flow resistance along the test section is measured by an Endress Hauser differential pressure transducer with an uncertainty of ±1 Pa. A 1-l glass vessel with a drain valve and a stop watch with ±0.01 accuracy are utilized to calculate the flow rate.

### Nomenclature

\[
\begin{align*}
m & \quad \text{flow’s mass flow rate, kg/s} \\
W & \quad \text{pumping power, W} \\
A & \quad \text{outer minor axis of AEA tube, m} \\
a & \quad \text{inner minor axis of AEA tube, m} \\
A_e & \quad \text{tube’s cross sectional area, m}^2 \\
A_s & \quad \text{tube’s surface area, m}^2 \\
B & \quad \text{outer major axis of AEA tube, m} \\
b & \quad \text{inner major axis of AEA tube, m} \\
C_p & \quad \text{isobaric specific heat, J/kg K} \\
D & \quad \text{Hydraulic diameter, m} \\
f & \quad \text{friction factor} \\
G_z & \quad \text{Graetz number} \\
h & \quad \text{convection coefficient, W/m}^2\text{ K} \\
I & \quad \text{turbulence intensity} \\
k & \quad \text{kinetic energy, J} \\
L & \quad \text{tube’s length, m} \\
Nu & \quad \text{Nusselt number} \\
p & \quad \text{tube’s pitch length, m} \\
p & \quad \text{tube’s section perimeter, m} \\
Pr & \quad \text{Prandtl number} \\
T & \quad \text{temperature, K} \\
t & \quad \text{time, s} \\
U & \quad \text{flow’s mean velocity, m/s} \\
u, v, w & \quad \text{physical velocity components, m/s} \\
X & \quad \text{axial position within the tube, m} \\
x, y, z & \quad x, y, \text{ and axial direction coordinates} \\
y^+ & \quad \text{non-dimensional distance from the wall, } y/\left(\frac{\tau_{\text{wall}}}{\rho}/v\right) \\
\end{align*}
\]

### Greek symbols

\[
\begin{align*}
\Delta P & \quad \text{pressure drop, kPa} \\
\Delta T_b & \quad \text{bulk temperature difference, K} \\
\Delta T_m & \quad \text{logarithmic mean temperature difference, K} \\
\dot{V} & \quad \text{flow’s volume flow rate, m}^3/\text{s} \\
\varepsilon & \quad \text{dissipation rate, J} \\
\kappa & \quad \text{thermal conductivity, W/m K} \\
\mu & \quad \text{dynamic viscosity, Pa s} \\
\nu & \quad \text{kinematic viscosity, m}^2/\text{s} \\
\rho & \quad \text{density, kg/m}^3 \\
\psi & \quad \text{effectiveness} \\
\end{align*}
\]

### Subscripts

\[
\begin{align*}
b & \quad \text{bulk} \\
c & \quad \text{cross sectional} \\
D & \quad \text{hydraulic diameter} \\
\text{in} & \quad \text{inlet} \\
\max & \quad \text{maximum} \\
\text{out} & \quad \text{outlet} \\
s & \quad \text{surface} \\
t & \quad \text{turbulent} \\
\text{wall} & \quad \text{wall} \\
\end{align*}
\]

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**Fig. 1.** Alternating Elliptical Axis (AEA) tube.
In this study, the working fluid is considered to be heat transfer oil. Thermophysical properties of it are measured within the temperature range of 30–90 °C and presented in Table 2.

### 2.1. Data reduction

From Fourier Law and basic equations of heat transfer, the experimental mean convective heat transfer coefficient of flow is defined as,

$$\tilde{h} = \frac{mC_p(T_{out} - T_{in})}{A_l\Delta T_m}, \quad (1)$$

where \(T_{in}\) and \(T_{out}\) are the flow’s inlet and outlet mean temperatures respectively and \(A_l\) is the tube’s surface area that heat passes through. \(\Delta T_m\) is the logarithmic mean temperature difference calculated as,

$$\Delta T_m = \frac{\Delta T_{out} - \Delta T_{in}}{\ln \left( \frac{\Delta T_{out}}{\Delta T_{in}} \right)}, \quad (2)$$

In Eq. (2), \(\Delta T_{out}\) and \(\Delta T_{in}\) can be defined generally as,

$$\Delta T_{out} = T_{wall} - T_{b}, \quad \Delta T_{in} = T_{b} - T_{in}, \quad (3)$$

where \(T_{wall}\) and \(T_{b}\) are the wall temperature and the bulk temperature of the fluid respectively. The mean convective heat transfer coefficient is usually expressed in the form of mean Nusselt number \(\overline{Nu}\):

$$\overline{Nu} = \frac{\tilde{h}D}{\kappa}, \quad (4)$$

where \(D\) is the hydraulic diameter of the tube, and \(\kappa\) is the thermal conductivity of heat transfer oil. The hydraulic diameter of any tube is defined as,

$$D = \frac{4A_p}{p}, \quad (5)$$

where \(A_p\) and \(p\) are the sectional area and perimeter of the tube respectively.

As it is mentioned in Section 2, the pressure drop is measured using an Endress Hauser differential pressure transducer. Having the pressure drop of the tube, the friction factor can be calculated as,

$$f = \frac{2D \Delta P}{\rho U^2 L} \quad (6)$$

where \(\rho\), \(U\), and \(L\) are the heat transfer oil’s density, the flow’s velocity, and the tube’s length respectively.

### 2.2. Experimental validation

Before measuring the convective heat transfer coefficient and the pressure drop of the heat transfer oil inside AEA tubes, the reliability and accuracy of the experiment apparatus as well as data reduction is investigated using heat transfer oil as the working fluid inside a horizontal circular tube (Tube A).

Circular tube’s experimental mean Nusselt numbers for different flow rates are compared to the ones obtained from the well-known Kay’s correlation [15]:

$$\overline{Nu} = 3.66 + \frac{0.0668Gz_D}{1 + 0.04Gz_D}, \quad (7)$$

\(Gz_D\) is the Graetz number with respect to the hydraulic diameter of the tube and is defined as,

$$Gz_D = \frac{D}{X} Re Pr, \quad (8)$$

where \(X\) is an axial position within the tube.

Similarly, the experimental friction factors are compared to the ones attained by the analytical friction factor correlation for laminar flows defined as,

$$f = \frac{64}{Re}, \quad (9)$$

As can be seen in Fig. 3, the experimental results for oil flow in circular tube (Tube A) are in a good agreement with the prediction of the correlations (9% for Nusselt number and 13% for friction factor when flow is laminar).

### 3. Numerical setup

In order to investigate the heat transfer enhancement mechanism of the AEA tube, temperature distribution and flow resistance of tubes specified in Table 1 are comparatively analyzed with Fluent 6.3. This analysis is performed by utilizing the continuity, momentum, and energy equations in the Cartesian coordinates for internal fully developed fluid flow.

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x_i} &= 0, \quad (10) \\
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x_i} &= -\frac{\partial P}{\partial x_i} + \mu \left( \frac{\partial u}{\partial x_j} + \frac{\partial u}{\partial x_j} \right), \quad (11) \\
\rho \frac{\partial T}{\partial t} + \rho u \frac{\partial T}{\partial x_i} &= -\mu \frac{\partial u}{\partial x_i} + \kappa \left( \frac{\partial u}{\partial x_j} + \frac{\partial u}{\partial x_j} \right) \quad (12)
\end{align*}
\]
The fluid flow is simulated using the standard $k$–$\epsilon$ model [16]. The justification for using this model will be discussed in Section 4. The $k$ and $\epsilon$ equations can be written as,

$$\frac{\partial}{\partial t}(k) + \frac{\partial}{\partial x_j}(kt_j) = \frac{\partial}{\partial x_j} \left[ \left( v + \frac{v_i}{\sigma_t} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \epsilon,$$

$$\frac{\partial}{\partial t}(\epsilon) + \frac{\partial}{\partial x_j}(\epsilon t_j) = \frac{\partial}{\partial x_j} \left[ \left( v + \frac{v_i}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + C_{tk} \frac{\epsilon}{k} (P_k) - C_{2\epsilon} \frac{\epsilon^2}{k},$$

where $\mu_t$ is the turbulent viscosity. $\mu_t$ and $P_k$ are defined as,

$$\mu_t = \rho C_{\mu} \frac{k^2}{\epsilon},$$

$$P_k = -\rho \mu_t \frac{\partial u_i}{\partial x_j}.$$  

The $k$–$\epsilon$ model has some constants defined as, $C_{tk} = 1.44$, $C_{2\epsilon} = 1.92$, $C_{\mu} = 0.09$, $\sigma_t = 1$, $\sigma_\epsilon = 1.3$.

3.1. Boundary conditions

In this study, the whole simulation work is carried out with Fluent 6.3. SIMPLEC algorithm is employed to solve the continuity and momentum equations. The standard-deviation method [17] and second order upwind algorithm [18] are utilized to decertize the pressure and the velocity terms respectively. In addition, the enhanced wall treatment method simulates the boundary layer motion of the fluid [19] and the governing equations are solved with pressure-based coupled algorithm. First, a coupled system of equations comprising the momentum and the pressure based continuity equations is solved. Then, the energy and turbulence equations are added and solved.

Inlet, outlet, and wall are considered as the three different zones of the numerically investigated tube. The boundary conditions at the inlet and the outlet of the tube are mass flow inlet and outflow respectively. Hence, the flow properties in the inlet are defined as, $\dot{m}_{in} = \text{cte}$, $k_{in} = \text{cte}$, $c_{in} = \text{cte}$, $T_{in} = \text{cte}$, $I_{in} = 0.03$, and $D = 0.014615$ m. Since the temperature of the wall is constant and it is a stationary wall, the boundary condition at the wall is defined as, $T_{wall} = \text{cte}$, $u_{wall} = 0$, $v_{wall} = 0$, and $w_{wall} = 0$.

3.2. Computational grid

In order to solve the above mentioned equations the domain has to be discretized by a computational grid. Fig. 4 shows the generated grid in the circular and the flattened sections of the tube. Using this structured grid in the section, a fully structured grid is developed along the tube. The computational grid consists of 8,640,000 hexahedral cells and 8,744,871 nodes.

In order to have a reliable grid, the velocity and the thermal boundary layers have to be captured by it. Since the enhanced wall treatment method is employed in simulating the boundary layer motion, the value of non-dimensional $y^+$ parameter should be about or less than one [17]. The distance of the nearest node to the tube’s wall is 0.025 mm and $y^+$ varies from 0.1 to 0.6 for all of the investigated Reynolds numbers. Thus, it can be claimed that the grid captures the velocity boundary layer.

In addition to the velocity boundary layer, the grid should capture the thermal boundary layer. In order to investigate that, the local convection coefficient and the Nusselt number are calculated in the middle of each pitch length and compared to those of the smaller grids. Since the local convection coefficient and the Nusselt number remain constant with making the grid smaller, it can be concluded that the grid captures the thermal boundary layer too.

4. Results and discussion

At the first point, heat transfer and flow resistance of the experimentally investigated tubes should be discussed. Fig. 5 depicts the Nusselt number and the friction factor versus Reynolds number for tubes B (flattened tube) and C. It can be observed that both heat transfer and flow resistance increase with increase in Reynolds number; however, the increase of heat transfer is more than that of the flow resistance.

Before discussing the numerical results, they have to be verified by the experimental ones. Due to the low range of Reynolds numbers, it seems that the flow regime in tube C is laminar. However, the application of the laminar flow regime to the numerical model results in a high error with respect to the experimental results. As a result, the turbulence models are examined and the $k$–$\epsilon$ model shows the least error among them. The maximum error observed for the Nusselt number and the friction factor is 24% and 21% respectively, which is acceptable when comparing the experimental and numerical results. The turbulence generation in this tube
can be justified by the alternating expansions and contractions that the flow faces when passing the tube. It is noteworthy that this is in contrast to Chen et al.’s [14] laminar flow assumption for this range of Reynolds numbers. In opposition to tube C, the comparison of the experimental and numerical results shows that the flow regime is laminar in tube B (flattened tube). The laminar regime in the flattened tube, which does not have those expansions and contractions strengthens the above mentioned idea. Fig. 5 compares the experimental and numerical results for heat transfer and flow resistance in tubes B (flattened tube) and C.

According to the verification of the numerical model and with the purpose of boosting the heat transfer rate in less flow resistance, two other geometries for AEA tubes are investigated. These two new geometries are called tubes D and E in Table 1. The former has a pitch length ($P$) longer than tube C, while the latter has an aspect ratio ($\frac{a}{b}$) greater than tube C. The heat transfer and flow resistance of tubes B (flattened tube), C, D and E are going to be discussed thoroughly in following paragraphs.

Fig. 6 shows the Nusselt number and the convection coefficient of the numerically investigated tubes for different Reynolds numbers. The hydraulic diameter of tube E is different from the three other tubes. Thus, the comparison based on the Nusselt number cannot be accurate. Considering the convection coefficient, tubes C, D, E and B (flattened tube) has the most to the least rate of heat transfer. Investigating the results shows that any decrease in tube's aspect ratio ($\frac{a}{b}$) helps fluid's molecules with lower temperature to get closer to the tube's wall and cause an increase in the temperature gradient near the wall. In addition, the decrease of pitch length ($P$) results in more transition sections in which the cross section of the tube rotates 90°. The thermal boundary layer generated in the pitch length will dissipate in these transition sections and the small temperature gradient formed near the wall will change to a larger one. These two facts cause in an increase in the heat transfer rate.

Fig. 7(a) depicts the friction factor of numerically investigated tubes for different Reynolds numbers. It is obvious that the friction factor for tube B (flattened tube) is significantly less than that for the other tubes. However, due to different hydraulic diameters of the tubes, more accurate comparisons is performed in Fig. 7(b) using tubes’ pressure drops. It can be claimed that any decrease in tube’s aspect ratio ($\frac{a}{b}$) deviates tube’s shape from the circular one and increases the pressure drop. In addition, the decrease of tube’s pitch length ($P$) results in more transition sections that cause more pressure drop. Furthermore, it can be argued that any decrease in $P$ affects the flow resistance more than the decrease of $P$. As it is shown in Fig. 7(b) the pressure drop of tube E with greater aspect ratio is about to that of tube B (flattened tube). How-
ever, the pressure drop of tube D with longer pitch length is just a little bit less than that of tube C.

Finally, in order to compare the heat transfer and the flow resistance of tubes simultaneously, it is essential to introduce heat transfer enhancement ratio. First, the effectiveness of each tube is calculated as,

\[
e = \frac{q}{q_{\text{max}}} = \frac{mC_p(T_{\text{out}} - T_{\text{in}})}{mC_p(T_{\text{wall}} - T_{\text{in}})},
\]

(17)

where \(m\), \(C_p\), \(T_{\text{in}}\) and \(T_{\text{out}}\) are the fluid’s mass flow rate, specific thermal capacity, and inlet and outlet bulk temperatures respectively. Also, \(T_{\text{wall}}\) is the tube’s wall temperature. Second, the pumping power needed to overcome the tube’s pressure drop is calculated as,

\[
\dot{W} = \Delta P \times \dot{V},
\]

(18)

where \(\Delta P\) is the tube’s pressure drop and \(\dot{V}\) is the volume flow rate. Finally the tube’s heat transfer Enhancement Ratio (ER) is defined as,

\[
\text{ER} = \frac{e_{\text{TubeA}}/e_{\text{TubeB}}}{\dot{W}/\dot{W}_{\text{TubeA}}},
\]

(19)

In calculating ER, first the effectiveness and the pumping power of the investigated tube is compared to those of the circular tube independently, then ER is achieved by dividing the effectiveness and pumping power ratios to each other. It should be noted that in order to get reliable results, the effectiveness and pumping power of the circular tube (Tube A) should be calculated using identical mass flow rate, specific thermal capacity, inlet bulk temperature and wall temperature to that of the investigated tube. It is clear that tubes with ERs more than 1 are worth to be used as an alternation to the circular tubes in heat exchangers.

ER of four numerically investigated tubes is shown in Fig. 8. Tube B’s (flattened tube) ER is about 0.7 in all Reynolds numbers, which means in equal pumping power the circular tube’s heat transfer rate is more than it. Tube E shows the best ER among the other ones. However, if \(\frac{a}{b} = 1\), tube E would be a circular one and its ER would be 1. So, it seems that there is an optimum \(\frac{a}{b}\).
between the $\xi$ of tube C and that of the circular tube (Tube A). Tube C's ER is a little more than that of tube D. This can be justified with the fact that the number of transition sections in tube C is more than that of tube D. Fig. 8 also shows that in tubes C, D, and E, ER increases with the increase of Reynolds number.

5. Conclusion

In this paper a numerical and experimental study on heat transfer and flow resistance of alternating elliptical axis tubes is carried out. Although it seems that the flow regime for the Reynolds numbers in the range of 300–2000 is laminar, the comparison of the experimental and numerical results shows that the best fitting turbulence model in this range of Reynolds numbers is Standard $k-c$.

The most important parameter to be discussed is tubes' heat transfer. The experimental and numerical results show that the alternating elliptical axis tube performed better compared to the circular and flattened ones and increased the heat transfer rate. It can be concluded from the numerical results that any decrease in aspect ratio ($\xi$) or pitch length ($P$) increases heat transfer rate.

The other important parameter to be debated is flow resistance. Results show that alternating elliptical axis tube's pressure drop is more than that of the flattened one, and the flattened tube's pressure drop is more than that of the circular tube. Numerical results show that any decrease in the aspect ratio ($\xi$) or the pitch length ($P$) increases the pressure drop.

It can be inferred from the above paragraphs that the alternating elliptical axis tube shows a desirable heat transfer enhancement, while having a fair increase in flow resistance. In order to compare these conflicting parameters simultaneously, the non-dimensional heat transfer enhancement ratio is defined. Heat transfer enhancement ratio compares the heat transfer enhancement of the investigated tubes to that of the circular one in the same pumping power. Heat transfer enhancement ratio for flattened tube is less than 1 and this means that the flattened tube cannot be an economical substitution for the circular tube in heat exchangers. However, the alternating elliptical axis tubes perform better than the flattened ones. Their heat transfer enhancement ratios are more than 1 that means their heat transfer rate is more than that of the circular tube in the same pumping power. The impacts of tube's aspect ratio and pitch length on the enhancement ratio are investigated. The numerical results show that heat transfer enhancement ratio does not react monotonic to the decrease in tube's aspect ratio or pitch length. For the future studies, the optimization of alternating elliptical axis tube's aspect ratio and pitch length may result in higher heat transfer enhancement ratios.

Conflict of Interest

None declared.

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