Investigation of ductile rupture in U-notched Al 6061-T6 plates under mixed mode loading

A. R. TORABI and R. HABIBI
Fracture Research Laboratory, Faculty of New Sciences and Technologies, University of Tehran, Tehran, Iran
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ABSTRACT The aim of the present research is to evaluate ductile failure of U-notched components under mixed mode I/II loading conditions. For this purpose, first, several rectangular plates made of the aluminium alloy Al 6061-T6 and weakened by central bean-shaped slit with two U-shaped ends are tested under mixed mode I/II loading conditions, and the load-carrying capacity of the specimens are experimentally measured. Then, using the equivalent material concept, Al 6061-T6, which is a highly ductile material, is equated with a virtual brittle material, and the load-carrying capacity of the same U-notched specimens virtually made of the equivalent material is theoretically predicted by using two well-known stress-based brittle fracture criteria. Finally, the theoretical failure loads of the virtual specimens are compared with the experimental ones of the real Al 6061-T6 specimens. It is revealed that the experimental results could very well be predicted by means of both brittle fracture criteria without conducting time-consuming elastic–plastic analyses.

Keywords brittle fracture criteria; ductile rupture; equivalent material concept; mixed mode I/II loading; U-notch.

NOMENCLATURE

dc critical distance of the UMS criterion measured from the notch tip
dc, U critical distance of the UMS criterion measured from the coordinate origin
r0 distance between the coordinate origin and the notch tip
r critical distance of the UMTS criterion measured from the notch tip
rc, U critical distance of the UMTS criterion measured from the coordinate origin
E elastic modulus
K strain-hardening coefficient
KIC U-notch fracture toughness for a U-notch
KII U-notch crack growth resistance of material
KIC plane-strain fracture toughness of material
N strain-hardening exponent
UMS U-notch mean-stress
UMTS U-notch maximum tangential stress
β notch rotation angle
εf strain at crack initiation for the equivalent material
εp true plastic strain
εu engineering plastic strain at maximum load
εtrue plastic strain at maximum load
εp true plastic strain at yield point
εy elastic strain at yield point
ρ notch radius
σ true stress

Correspondence: A. R. Torabi. E-mail: a_torabi@ut.ac.ir

**INTRODUCTION**

Most of engineering components and structures are made of ductile materials and contain notches of various features. A notch decreases the load-carrying capacity of the notched member because of the stress concentration at the notch vicinity. Although the design of notched ductile structures under static and monotonic loading conditions is usually performed so that material in no part of the structure experiences yielding, it is possible in engineering structures that some undesired or unexpected loads are applied to the notched structural component and, as a result, a plastic region forms at the notch neighbourhood. It is possible also in some cases; mechanical designers may allow some controlled plastic deformations around the notch because of reducing the cost of structure. Depending on the magnitude of the applied load and hence on the size of plastic zone around the notch, crack(s) may initiate from the notch border. Therefore, it is essential to accurately determine the load-carrying capacity of notched components in the presence of considerable plastic deformations at the notch vicinity, which is normally obtained at the onset of crack initiation from the notch border.

Depending upon the notch plastic zone size, four various failure regimes are usually recognized for a notched component. They are known as the small-scale yielding (SSY), the moderate-scale yielding, the large-scale yielding (LSY) and the gross yielding.\(^1\) The failure analysis in the SSY regime is usually carried out by means of brittle fracture criteria that exist in the field of the linear elastic notch fracture mechanics (LENFM).\(^2\)–\(^16\) This can be performed by neglecting the plastic zone or modifying slightly the brittle fracture criteria according to the notch plastic zone size.\(^1\) For the other three regimes, the LENFM fracture criteria are no longer valid because the failure mechanism in these regimes is completely different from that in the SSY regime. In the SSY regime, the crack initiation from the notch (by small plastic zone) and the final fracture of the notched component are approximately simultaneous because the crack propagates rapidly, and fracture takes place abruptly. Conversely, stable crack propagation is normally seen in the moderate-scale yielding, LSY and gross yielding regimes as a result of significant plastic deformations around the notch. For these three regimes, ductile fracture criteria like elastic–plastic critical J-integral\(^17\) should be applied. Such criteria usually need elastic–plastic finite element (FE) analysis, and hence, they are time-consuming and relatively complicated. In order to avoid elastic–plastic FE analyses, a new concept, called the equivalent material concept (EMC), has recently been proposed by Torabi.\(^18\) According to EMC, a ductile material having valid fracture toughness \(K_c\) (or \(K_I\)) is equated with a virtual brittle material having the same elastic modulus and fracture toughness but various tensile strength. It can be found in the literature that the EMC together with two well-known brittle fracture criteria, namely, the point-stress and the mean-stress criteria, have been capable of successfully predicting the experimental results on mode I load-carrying capacity of V-notched, U-notched and O-notched ductile steel plates.\(^18\)–\(^20\) In an applied work, Torabi\(^21\) has estimated well the tensile load-carrying capacity of several bolts weakened by V-shaped threads and made of two various ductile steels. It is worth noting that the formulation of EMC is very similar to that of Glinka’s\(^22,23\) equivalent strain energy density method because in both of them, the value of strain energy density (SED) for an elastic–plastic material is equated with that for an equivalent elastic material. However, the main difference is that Glinka has utilized his concept for determining the stress value at the notch tip in an inelastic notched member, while Torabi\(^18\)–\(^21\) has equated the SED for elastic and plastic–plastic materials till the ultimate point in order to determine the tensile strength of the virtual brittle material. The equivalent strain energy density method has also been employed by other researchers to analyze stress and strains around notches of various features under localized and generalized plasticity.\(^24,25\) The applicability of the EMC to various ductile materials, notch features and loading conditions is currently under development. Because the EMC acts as a bridge between the ductile rupture and brittle fracture, it is expected that it can also be joined with various brittle fracture criteria, other than the stress-based point-stress and mean-stress criteria, e.g., the well-known SED criterion.\(^15,26–30\)

Definitely, aluminium alloys are one of the most widely used group of ductile materials in engineering components and structures. For instance, aluminium alloys of the 2xxx, 5xxx, 6xxx and 7xxx series are known as

\[
\begin{align*}
\sigma_c & \quad \text{critical stress} \\
\sigma_f^* & \quad \text{tensile strength of the equivalent material} \\
\sigma_{\text{in-plane shear}} & \quad \text{in-plane shear stress} \\
\sigma_{\text{plastic}} & \quad \text{tangential stress} \\
\bar{\sigma} & \quad \text{mean value of tangential stress} \\
\sigma_u & \quad \text{ultimate tensile strength} \\
\sigma_Y & \quad \text{yield strength}
\end{align*}
\]
aero-alloys because of their widespread applications in aero-structures. By applying particular heat treatment processes during fabrication, desired material properties such as high strength, appropriate ductility, good fracture toughness and fatigue strength can be achieved for aluminium alloys. The aero-structural components made of aluminium alloys must be designed in a way they can sustain different failure modes like yielding, buckling, and crack initiation and propagation. In the presence of notches in aero-structures made of aluminium alloys (which is widespread), to avoid crack initiation from the notch border, the notch fracture toughness (NFT) of such alloys should be carefully studied both experimentally and theoretically. Note that unlike material fracture toughness, the NFT depends not only on the material properties but also on the notch feature.

Dealing with static strength of aluminium alloys weakened by notches, two papers have recently been published in Refs 31,32. The mode I NFT of a commercial aluminium alloy has been well evaluated in Ref. 31 by performing fracture experiments on single-edge-notched-bend specimens weakened by U-shaped notches of different radii and using elastic stress distributions around a round-tip crack-like notch. Madrazo et al.32 have performed tensile fracture tests on the notched compact-tension specimens made of Al 7075-T651 and recorded the load-carrying capacity of the specimens. The experimental results have then been successfully predicted by means of the theory of critical distances, which really belongs to Taylor and his co-researchers (see, e.g. Refs 33–35).

In the present research, fracture tests are first carried out on U-notched rectangular Al 6061-T6 plates under mixed mode I/II loading conditions for various notch radii, and the load-carrying capacity of the plates are experimentally recorded. Then, the experimental results are predicted by using simultaneously the EMC and two stress-based brittle fracture criteria, namely, the U-notch maximum tangential stress (UMTS) and the U-notch mean-stress (UMS) criteria. It is demonstrated that the failure loads of the notched aluminium plates could well be predicted by means of both the UMTS-EMC and UMS-EMC criteria, and hence, one is needless of performing elastic–plastic analyses.

**Material**

The material utilized in the experiments is the aluminium alloy Al 6061-T6, which is a highly ductile material. To mechanically characterize the material, three different standard tests are carried out on plain specimens. They are the tensile tests in accordance with ASTM E8,36 the Poisson’s ratio tests according to ASTM E132-0437 and the fracture toughness tests for small thicknesses according to ASTM B646-12.38 Some of the tensile test specimens, which are known as dog-bone specimens, are shown in Fig. 1. The dimensions of the specimens can easily be realized by referring to ASTM E8,36 and the thickness of the specimens is equal to 4 mm. Figure 2 depicts samples of the engineering and true stress–strain curves for the tested Al 6061-T6. Moreover, Tables 1 and 2 present the chemical composition and the mechanical properties of Al 6061-T6, respectively.

**Specimen**

As shown in Fig. 3, the specimen is a thin rectangular plate weakened by a central bean-shaped slit with two U-shaped ends, which are subjected to mixed mode I/II loading by applying a remote tensile load to the plate. In order to produce combined tensile-shear deformations at the notch neighbourhood, the specimen is rotated counterclockwise by the angle \( \beta \). As \( \beta \) increases from zero (\( \beta = 0 \) results in pure mode I loading), the contribution of shear deformations enhances resulting in various mode mixity ratios.

The parameters \( \rho, 2a, L, W, \beta \) and \( P \) in Fig. 3 are the notch radius, twice the notch length (i.e. the total slit length), the specimen length, the specimen width, the notch rotation angle and the remote tensile load, respectively. The following values are considered in the fracture experiments: \( \rho = 0.5, 1 \) and \( 2 \) mm; \( 2a = 25 \) mm; \( L = 160 \) mm; \( W = 50 \) mm; \( \beta = 0, 30 \) and \( 60^\circ \). The thickness of the entire plates is equal to 4 mm. Three tests are conducted for each of the U-notch geometries and loading conditions with the aim to check the

![Fig. 1 Some of the tensile test specimens.](image)
repeatibility of the tests; all in all, 27 fracture tests are carried out in the present investigation.

To fabricate the test specimens, a large Al 6061-T6 plate of 4 mm thick is first provided. Then, the sketch of each specimen is drawn by commercial drawing software and given to a high-precision two-dimensional computer numerical control (2D CNC) water jet cutting machine. The specimens are finally cut from the large plate. Appropriate rasps are utilized for removing possible local stress raisers remained from the cutting process. The fracture tests are conducted under displacement-control conditions with the speed of $1 \text{ mm min}^{-1}$ providing monotonic loading conditions.

Experimental results

Some of the U-notched specimens are represented in Fig. 4 before, during and after the fracture tests.

In Fig. 4c, a large plastic region is seen with the naked eye at the ligament demonstrating LSY failure regime for the notched Al 6061-T6 plates. It was observed during the experiments that plastic region nucleates from the notch border and grows in large amount around the notch; a crack initiates from a point on the notch round border and propagates slowly till final fracture. Figure 5 shows a sample load-displacement curve resulted from the test machine for a U-notched Al 6061-T6 plate with $\rho = 0.5 \text{ mm}$ and $\beta = 30^\circ$.

Figure 5 indicates that a significant plastic zone forms around the notch before the peak load (i.e. the onset of crack initiation from the notch border) and the load decreases slowly to zero (i.e. the final rupture) meaning stable crack propagation. From the statements earlier, it is found that Fig. 5 properly confirms the experimental observations.

To measure experimentally the size of plastic region around the notch at crack initiation instance, some movies are provided from the experiments and analyzed by means of an advanced movie-analyzer system. Although it is not possible to exactly measure the plastic zone size, the pictures captured at crack initiation instance from different U-notched plates suggest that almost 80% of the ligament experiences plastic deformations. It is shown in forthcoming sections that this experimental finding is well confirmed by elastic-plastic FE analyses.

Table 3 presents the experimental critical loads of the U-notched Al 6061-T6 plates for different notch radii and various angles of rotation $\beta$. The parameter $P_i$ ($i = 1, 2, 3$) denotes the failure loads in the repeated tests, and $P_{av}$ is the average of the three failure loads.

The experimentally obtained load-carrying capacity of the tested specimens is predicted in forthcoming sections by means of the EMC together with two well-known mixed mode I/II brittle fracture criteria.

**A BRIEF DESCRIPTION OF THE EQUIVALENT MATERIAL CONCEPT**

The EMC has originally been proposed by Torabi\textsuperscript{18} with the aim to avoid elastic–plastic FE analyses in ductile failure prediction of notched components. According to EMC, a ductile material having valid K-based fracture toughness ($K_I$ or $K_C$) value is equated with a virtual brittle material having the same elastic modulus and fracture toughness but different tensile strength. The tensile strength of the equivalent material can be computed by assuming the same

<table>
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<tr>
<th>Element</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Zn</th>
<th>Ni</th>
<th>Cr</th>
<th>Pb</th>
<th>Sn</th>
<th>Ti</th>
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<tbody>
<tr>
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<td>0.02</td>
<td>0.003</td>
<td>0.2</td>
<td>0.001</td>
<td>0.001</td>
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<td>0.0002</td>
<td>0.003</td>
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Table 1 Chemical composition of Al 6061-T6

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$ (GPa)</td>
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</tr>
<tr>
<td>Poisson’s ratio</td>
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<tr>
<td>Tensile yield strength (MPa)</td>
<td>276</td>
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<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>292</td>
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<tr>
<td>Elongation at break (%)</td>
<td>11</td>
</tr>
<tr>
<td>Engineering strain at maximum load</td>
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<tr>
<td>True fracture stress (MPa)</td>
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<tr>
<td>Fracture toughness, $K_C$ (MPa$\sqrt{m}$)</td>
<td>38</td>
</tr>
<tr>
<td>Strain-hardening coefficient (MPa)</td>
<td>314</td>
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<tr>
<td>Strain-hardening exponent</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 2 Mechanical properties of Al 6061-T6
values of the tensile SED required for both real ductile and virtual brittle materials for the crack initiation to occur. Although the details of EMC have recently been presented in some published papers (see, for instance, Refs 18–21), a brief description of it is presented herein in order to provide more convenience for the readers.

Figure 6 shows an engineering stress–strain curve for a typical ductile material.

For a typical ductile material, the stress–strain relationship over the plastic zone can be written in a well-known power-law form as

\[
\sigma = K \varepsilon_p^n
\]  

(1)

In Eq. 1, \(\sigma\), \(\varepsilon_p\), \(K\) and \(n\) are the true stress, the true plastic strain, the strain-hardening coefficient and the strain-hardening exponent, respectively. The total SED is composed of elastic and plastic components, and it can be written as

\[
(\text{SED})_{\text{tot}} = (\text{SED})_e + (\text{SED})_p = \frac{1}{2} \sigma Y \varepsilon_Y + \int \sigma d\varepsilon_p \]  

(2)

In Eq. 2, \(\sigma Y\), \(\varepsilon_Y\) and \(\varepsilon_p^Y\) are the yield strength, the elastic strain at yield point and the true plastic strain at yield point, respectively. Substituting \(\varepsilon_Y = \frac{\sigma Y}{E}\) and Eq. 1 into Eq. 2 gives \((E\) is the elastic modulus of material) the following:

\[
(\text{SED})_{\text{tot}} = \frac{\sigma Y^2}{2E} + \int \frac{\varepsilon_p}{\varepsilon_p^Y} K \varepsilon_p^n d\varepsilon_p
\]  

(3)
Therefore,

$$
(\text{SED})_{\text{tot.}} = \frac{\sigma Y^2}{2E} + \frac{K}{n + 1} \left[ (\varepsilon_P)^{n+1} - \left( \varepsilon_P^f \right)^{n+1} \right]
$$

(4)

If \( \varepsilon_P^f \) is assumed to be equal to 0.002 (equal to 0.2% offset yield strength), Eq. 4 can be rewritten as

$$
(\text{SED})_{\text{tot.}} = \frac{\sigma Y^2}{2E} + \frac{K}{n + 1} \left[ \varepsilon_P^{n+1} - (0.002)^{n+1} \right]
$$

(5)

To compute the total SED associated with the crack initiation instance (i.e. the area under the \( \sigma-\varepsilon \) curve from beginning to the peak load; Fig. 6), \( \varepsilon_P \) should be substituted in Eq. 5 with \( \varepsilon_{a,\text{true}} \), that is, the true plastic strain at peak load, which could easily be calculated by the expression

$$
\varepsilon_{a,\text{true}} = \ln(1 + \varepsilon_u) \quad \text{where} \quad \varepsilon_u \quad \text{is the engineering plastic strain at the ultimate (peak) point. Thus,}
$$

$$
(\text{SED})_{\text{necking}} = \frac{\sigma Y^2}{2E} + \frac{K}{n + 1} \left[ \varepsilon_{a,\text{true}}^{n+1} - (0.002)^{n+1} \right]
$$

(6)

As mentioned at the beginning of this section, the SED for ductile material associated with the crack initiation point, that is, necking, should be set to be equal to that for the equivalent brittle material with the aim to compute the tensile strength of the equivalent material. Figure 7 depicts a typical uni-axial stress–strain curve for the equivalent material.

In Fig. 7, the parameters \( \varepsilon_P^f \) and \( \sigma_f^* \) are the strain at crack initiation and the tensile strength of material, respectively. The SED absorbed by the equivalent material at the onset of crack initiation is

$$
(\text{SED})_{\text{EM}} = \frac{\sigma_f^*}{2E}
$$

(7)

According to EMC, SED values for both real ductile and virtual brittle materials should be identical. Therefore, Eqs 6 and 7 are set to be equal, and hence,

$$
\frac{\sigma_f^*}{2E} = \frac{\sigma Y^2}{2E} + \frac{K}{n + 1} \left[ \varepsilon_{a,\text{true}}^{n+1} - (0.002)^{n+1} \right]
$$

(8)

The tensile strength of the equivalent material \( \sigma_f^* \) is finally resulted as

$$
\sigma_f^* = \sqrt{\frac{\sigma Y^2}{2E} + 2EK \varepsilon_{a,\text{true}}^{n+1} - (0.002)^{n+1}}
$$

(9)

Now, various brittle fracture criteria in the context of the LENFM that normally need the tensile strength and fracture toughness of material for their predictions can be employed to predict the load-carrying capacity of ductile U-notched members by using simultaneously \( \sigma_f^* \).

<table>
<thead>
<tr>
<th>( \beta ) (deg.)</th>
<th>( \rho ) (mm)</th>
<th>( P_1 ) (N)</th>
<th>( P_2 ) (N)</th>
<th>( P_3 ) (N)</th>
<th>( P_{av} ) (N)</th>
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Table 3 The experimental critical loads of the U-notched Al 6061-T6 plates
presented in Eq. 9 and the material fracture toughness $K_I$ (or $K_c$) without requiring any elastic–plastic analyses.

In the next section, two well-known mixed mode I/II brittle fracture criteria, namely, the UMTS and UMS criteria, are described, and the corresponding fracture curves are developed and plotted for the equivalent material to predict mixed mode I/II NFT for the U-notched Al 6061-T6 specimens.

**FRACTURE CURVES FOR U-NOTCHED COMPONENTS VIRTUALLY MADE OF THE EQUIVALENT MATERIAL**

**Fracture curves for the U-notch maximum tangential stress criterion**

Figure 8 represents schematically a U-shaped notch with its Cartesian and polar coordinate systems. The coordinates origin is located at the distance $r_0 = \rho/2$ behind the notch tip on the notch bisector line.

The linear elastic tangential stress distribution around a U-notch can be written as follows$^{39}$:

$$
\sigma_{\theta\theta} = \frac{1}{2\sqrt{2\pi r}} \left\{ K_{II}^U \left[ \left( \frac{3}{2} + \frac{\rho}{r} \right) \cos \frac{\theta}{2} + \frac{1}{2} \cos \frac{3\theta}{2} \right] \right. 
+ \left. K_{II}^U \left[ \left( \frac{3}{2} - \frac{\rho}{r} \right) \sin \frac{\theta}{2} + \frac{3}{2} \sin \frac{3\theta}{2} \right] \right\} 
$$

(10)

The parameters $K_{II}^U$ and $K_{II}^U$ are the mode I and mode II notch stress intensity factors (NSIFs), respectively. $\rho$, $r$ and $\theta$ denote the notch radius, the radial and the tangential components of the polar coordinate system located at the distance $r_0 = \rho/2$ behind the U-notch tip, respectively.

Brittle fracture takes place in accordance with the UMTS criterion when the tangential stress at a specified critical distance ahead of the U-notch border attains the
material critical stress. Trivially, fracture initiates from a point on the notch border for which the tangential stress is a maximum. The maximum conditions are mathematically satisfied when

\[
\frac{\partial \sigma_{\theta \theta}}{\partial \theta} = 0 \Rightarrow \theta = \theta_0 \\
\frac{\partial^2 \sigma_{\theta \theta}}{\partial \theta^2} < 0
\]

(11)

The angle \(\theta_0\) in Eq. 11 is known as the fracture initiation angle (FIA). Equation 11 provides a set of positive and negative roots. By checking the sign of the second derivative of \(\sigma_{\theta \theta}\) for the roots, it is obtained that only negative \(\theta_0\) values result in negative \(\frac{\partial^2 \sigma_{\theta \theta}}{\partial \theta^2}\). Therefore, negative \(\theta_0\) values are solely acceptable. Substituting Eq. 10 into Eq. 11 gives

\[
K_{I}^{U, \rho} \left[ - \left( \frac{3}{4} + \frac{\rho}{2r_{c,U}} \right) \sin \frac{\theta_0}{2} - \frac{3}{4} \sin 3\theta_0 \right] \\
+ K_{II}^{U, \rho} \left[ \frac{3}{4} - \frac{\rho}{2r_{c,U}} \cos \frac{\theta_0}{2} + \frac{9}{4} \cos 3\theta_0 \right] \\
= 0
\]

(12)

Note that the parameter \(r\) in Eq. 10 is replaced with \(r_{c,U}\) (U-notch critical distance) according to the UMTS failure concept. Figure 9 depicts schematically the U-notch critical distances \(r_{c,U}\) and \(r_c\), which are measured from the coordinate origin and the notch tip, respectively. Note that only \(r_c\) is physically meaningful because it lies on material.

Because of the symmetry in geometry and loading conditions, crack initiates under pure mode I loading from the U-notch tip and propagates along the notch bisector line, and hence, the FIA becomes zero. Under pure mode II loading conditions, \(K_{II}^{U, \rho}\) is zero. Therefore, Eq. 12 can be simplified to

\[
\left( \frac{3}{4} - \frac{\rho}{2r_{c,U}} \right) \cos \frac{\theta_0}{2} + \frac{9}{4} \cos 3\theta_0 = 0 \Rightarrow \theta_0 = \theta_{0II}
\]

(13)

Equation 13 suggests that the mode II FIA \(\theta_{0II}\) depends on both the critical distance \(r_{c,U}\) and the notch radius \(\rho\). As mentioned, the main hypothesis of the UMTS criterion proposes that brittle fracture takes place for a U-notched component when the tangential stress at a specified critical distance ahead of the notch border reaches necessarily to the critical stress of material (\(\sigma_c\)). Therefore, one can write Eq. 10 at brittle fracture instance as

\[
\sigma_c = \frac{1}{2\sqrt{2\pi r_{c,U}}} \left\{ K_{I}^{U, \rho} \left[ \frac{3}{2} + \frac{\rho}{r_{c,U}} \cos \frac{\theta_0}{2} + \frac{3}{2} \cos 3\theta_0 \right] \\
+ K_{II}^{U, \rho} \left[ \frac{3}{2} - \frac{\rho}{r_{c,U}} \sin \frac{\theta_0}{2} + \frac{3}{2} \sin 3\theta_0 \right] \right\}
\]

(14)

It has widely been reported in the literature that the critical stress for brittle materials can successfully be assumed to be equal to tensile strength of material.\(^{40-46}\)

To eliminate \(\sigma_c\), from Eq. 14, a simple relationship reported in Refs.\(^5,40\) between \(\sigma_c\) and the mode I NFT \(K_{Ic}^{U, \rho}\) is utilized. It is as follows:

\[
\sigma_c = \frac{\left( 2 + \frac{\rho}{r_{c,U}} \right) K_{Ic}^{U, \rho}}{2\sqrt{2\pi r_{c,U}}}
\]

(15)

Substituting Eq. 15 into Eq. 14 gives the following:

\[
K_{I}^{U, \rho} \left[ \frac{3}{2} + \frac{\rho}{r_{c,U}} \cos \frac{\theta_0}{2} + \frac{3}{2} \cos 3\theta_0 \right] \\
+ K_{II}^{U, \rho} \left[ \frac{3}{2} - \frac{\rho}{r_{c,U}} \sin \frac{\theta_0}{2} + \frac{3}{2} \sin 3\theta_0 \right] \\
= \left( 2 + \frac{\rho}{r_{c,U}} \right) K_{Ic}^{U, \rho}
\]

(16)
It is worth noting that the parameter $K_{Ic}^{U}$ (i.e. the mode I NFT), which can be determined either experimentally by performing fracture tests or theoretically by using mode I brittle fracture criteria, depends not only on the material properties but also on the U-notch radius. For known values of the critical distance $r_c$ and the mode I NFT $K_{Ic}^{U}$, one can divide both sides of Eqs 12 and 16 by $K_{Ic}^{U}$ and solve simultaneously these two equations for any value of $\theta$ between 0 and $\pi/2$ and draw the variations of $K_{Ic}^{U}/K_{Ic}$ (vertical axis) versus $K_{Ic}^{U}/K_{Ic}$ (horizontal axis) over the entire mixed mode I/II domain from pure mode I to pure mode II in order to obtain mixed mode fracture curves for U-notches of various radii. Determination of the critical distances for the UMTS-EMC and UMS-EMC criteria, an expression is presented to compute the values of the critical distance $r_c$ for U-notches that weaken the equivalent material. Also, the method of computing the experimental value of the mode I NFT $K_{Ic}^{U}$ is elaborated in Section on Results and Discussion.

Fracture curves for the U-notch mean-stress criterion

According to the UMS criterion, brittle fracture occurs when the average value of the tangential stress over a specified critical distance ahead of the notch border reaches to the material critical stress. The procedure of formulating the UMS criterion is very similar to that of the UMTS criterion, except that in the UMS criterion, the mean value of the tangential stress over the critical distance should attain the critical stress rather than the tangential stress at a specified point ahead of the notch border. To formulate the UMS criterion, the mean value of the tangential stress over a specified critical distance should first be computed. Figure 10 represents the critical distances of the UMS criterion, $d_{c, U}$ and $d_{c}$, which are measured from the coordinate origin and the notch tip, respectively. From Fig. 10, it is clear that $d_{c, U} = d_{c} + r_0$ where $r_0$ is equal to $\rho/2$ for U-notches.

The mean stress over the critical distance $d_c$ can be written as follows:

$$\sigma_{\theta \theta} = \frac{1}{d_c} \int_{r=r_0}^{r=d_{c, U}} \sigma_{\theta \theta} dr$$  \hspace{1cm} (17)

Substituting Eq. 10 into Eq. 17 and integrating, we have

$$\sigma_{\theta \theta} = \frac{K_{Ic}^{U}}{2d_c \sqrt{2\pi}} \left\{ \left(3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right) \left(\sqrt{d_{c, U} - \sqrt{r_0}} \right) - 2\rho \cos \theta \left( \sqrt{d_{c, U} - \sqrt{r_0}} \right) + K_{IIc}^{U} \right\} \left\{ \left(3 \sin \frac{\theta}{2} + 3 \sin \frac{3\theta}{2} \right) \left(\sqrt{d_{c, U} - \sqrt{r_0}} \right) + 2\rho \sin \frac{\theta}{2} \left( \sqrt{d_{c, U} - \sqrt{r_0}} \right) \right\}$$  \hspace{1cm} (18)

In order to present Eq. 18 in a more concise form, the following auxiliary parameters are defined:

$$X = \sqrt{d_{c, U} - \sqrt{r_0}}, \hspace{0.5cm} Y = \frac{1}{\sqrt{d_{c, U}}} - \frac{1}{\sqrt{r_0}}$$  \hspace{1cm} (19)

Considering Eq. 19, Eq. 18 can be simplified to

$$\sigma_{\theta \theta} = \frac{K_{Ic}^{U}}{2d_c \sqrt{2\pi}} \left\{ \left(3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right) X - 2\rho \cos \frac{\theta}{2} Y \right\} + \frac{K_{IIc}^{U}}{2d_c \sqrt{2\pi}} \left\{ \left(3 \sin \frac{\theta}{2} + 3 \sin \frac{3\theta}{2} \right) X + 2\rho \sin \frac{\theta}{2} Y \right\}$$  \hspace{1cm} (20)

To reach the mean stress to a maximum, Eq. 21 should be satisfied.

$$\frac{\partial \sigma_{\theta \theta}}{\partial \theta} = 0 \Rightarrow \theta = \bar{\theta}_0$$

$$\frac{\partial^2 \sigma_{\theta \theta}}{\partial \theta^2} < 0$$  \hspace{1cm} (21)

The angle $\bar{\theta}_0$ is known as the FIA for the UMS criterion. Substituting Eq. 20 into Eq. 21 gives

![Fig. 10 A U-notch with the critical distances of the U-notch mean-stress criterion.](image-url)
The FIA is trivially equal to zero under mode I loading because of symmetry in geometry and loading conditions. Under pure mode II loading, $K_{II}^{U}$ becomes zero, and thus, Eq. 22 is reduced to

$$
\frac{K_{II}^{U,\rho}}{2d\sqrt{2\pi}} \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X + \rho Y \cos \frac{\theta_0}{2} = 0 = \overline{\theta}_{0II}
$$

(23)

The root of Eq. 23 is the mode II FIA predicted by the UMS criterion ($\overline{\theta}_{0II}$).

For brittle fracture to take place in accordance with the UMS criterion, the mean stress $\overline{\sigma}$ should attain the material critical stress. Thus,

$$
\sigma_c = \frac{K_{Ic}^{U,\rho}}{2d\sqrt{2\pi}} \left\{ \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X - 2\rho Y \cos \frac{\theta_0}{2} \right\} + \frac{K_{II}^{U,\rho}}{2d\sqrt{2\pi}} \left\{ \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X + 2\rho Y \sin \frac{\theta_0}{2} \right\}
$$

(24)

Similar to Eq. 15, a closed-form expression is required for $\sigma_c$ in terms of the critical distance and the mode I NFT (i.e. $K_{Ic}^{U,\rho}$) for eliminating $\sigma_c$ and presenting the equations in terms of the dimensionless NSIFs $K_{Ic}^{U,\rho}/K_{Ic}^{U,\rho}$ and $K_{II}^{U,\rho}/K_{Ic}^{U,\rho}$. For this purpose, one can simply apply the mode I fracture requirements to Eq. 24 as follows:

$$
\overline{\theta}_0 = 0
$$

$$
K_{Ic}^{U,\rho} = K_{Ic}^{U,\rho} \Rightarrow \sigma_c = \frac{K_{Ic}^{U,\rho}}{2\sqrt{2\pi}d_c}(4X - 2\rho Y) = \sqrt{\frac{2}{\pi d_c}} K_{Ic}^{U,\rho}
$$

$$
K_{II}^{U,\rho} = 0
$$

(25)

Substituting Eq. 25 into Eq. 24 gives

$$
\frac{K_{Ic}^{U,\rho}}{2d_c \sqrt{2\pi}} \left\{ \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X - 2\rho Y \cos \frac{\theta_0}{2} \right\} + \frac{K_{II}^{U,\rho}}{2d_c \sqrt{2\pi}} \left\{ \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X + 2\rho Y \sin \frac{\theta_0}{2} \right\} = 1
$$

(26)

By dividing both sides of Eq. 22 by $K_{Ic}^{U,\rho}$, one obtains

$$
\frac{K_{I}^{U,\rho}}{K_{Ic}^{U,\rho}} \left( \frac{3}{2} \frac{\theta_0}{2} + \frac{3\theta_0}{2} \right) X + \rho Y \sin \frac{\theta_0}{2} = \overline{\theta}_0
$$

$$
\frac{K_{II}^{U,\rho}}{K_{Ic}^{U,\rho}} \left( \frac{3}{2} \cos \frac{\theta_0}{2} + \frac{9}{2} \cos \frac{3\theta_0}{2} \right) X + \rho Y \cos \frac{\theta_0}{2} = 0
$$

(27)

Equations 26 and 27 form a linear system of equations in which the ratios $K_{Ic}^{U,\rho}/K_{Ic}^{U,\rho}$ and $K_{II}^{U,\rho}/K_{Ic}^{U,\rho}$ are unknown. Once the critical distances and the mode I NFT are known, the system can be solved for various values of $\overline{\theta}_0$ between zero and $\overline{\theta}_{II}$, and the fracture curves of the UMS criterion can ultimately be resulted in terms of $K_{Ic}^{U,\rho}/K_{Ic}^{U,\rho}$ and $K_{II}^{U,\rho}/K_{Ic}^{U,\rho}$.

### Determination of the critical distances for the UMTS-EMC and UMS-EMC criteria

A brief review of the governing equations of the UMTS and UMS criteria presented earlier indicates that the sole unknown parameters are the critical distances. It has several times been demonstrated in the literature that the critical distances of a sharp crack are valid also for notches of various features, for example, V-notches, U-notches, V-notches with end holes (VO-notches) and key-hole notches.8-12,44,45 Thus, the expressions of the critical distances required for drawing the fracture curves are as follows:

$$
r_{c,U} = r_0 + r_c = \frac{\rho}{2} + 2 \frac{K_{Ic}^{U,\rho}}{\sigma_c} ; \quad d_{c,U} = r_0 + d_c
$$

(28)

With the aim to predict the onset of crack initiation from the U-notch border in the ductile Al 6061-T6 plates tested under mixed mode I/II loading, the EMC should be combined with the UMTS and UMS brittle fracture criteria. For this purpose, it is just required to substitute $\sigma_c$ in Eq. 28 with $\sigma_c^f$ from Eq. 9. Moreover, the plane-strain fracture toughness $K_f$ may be replaced with the fracture toughness $K_c$ for thin Al 6061-T6 notched components. Considering the data presented in Table 2, the values of the parameters $\sigma_c^f$, $r$, and $d_c$ are calculated by using Eqs 9 and 28 to be equal to about 1066 MPa, 0.2 mm and 0.8 mm, respectively.

In the next section, the experimentally obtained critical loads of the U-notched Al 6061-T6 plates are predicted by means of the two combined fracture criteria, namely, the UMTS-EMC and UMS-EMC criteria.
RESULTS AND DISCUSSION

Because the NFT values depend normally on the material properties and the notch geometry, and they are usually independent of the overall geometry of the notched component, researchers prefer to present their brittle fracture criteria in terms of the critical values of the NSIFs, so-called the NFTs. The two combined fracture criteria presented in this study (i.e. the UMTS-EMC and UMS-EMC criteria) are also formulated in terms of the critical NSIFs. Therefore, it is essential to convert the experimental critical loads of the U-notched Al 6061-T6 plates to the corresponding values of the critical NSIFs. Thus, it is necessary to divide the critical NSIFs obtained from Eqs 29 and 30 by the mode I NFT $K_{Ic}^{U\rho}$. With the aim to compute the experimental value of $K_{Ic}^{U\rho}$ for each U-notched plate, the average of the three mode I critical loads (see the last column in Table 3 for $\beta = 0$) should be applied to the FE model of the plate, and the corresponding tangential stress at the notch tip is computed. Then, the computed value is substituted into Eq. 29 instead of $\sigma_{\theta\theta}(\xi, 0)$, and the critical $K_{Ic}^{U\rho}$ is calculated. The mean experimental values of $K_{Ic}^{U\rho}$ for the U-notched Al 6061-T6 plates are presented in Table 4.

The fracture curves of the UMTS-EMC and UMS-EMC criteria are represented in Fig. 11 together with the experimental results of the U-notched Al 6061-T6 plates for various notch radii.

Figure 11 illustrates that although the fracture curves of the UMTS-EMC criterion are generally closer to the experimental results than those of the UMS-EMC criterion, both criteria predict well the experimental results of the mixed mode I/II NFT for various notch radii and different mode mixity ratios. Moreover, the fracture curve of the UMS-EMC criterion locates always under that of the UMTS-EMC criterion meaning that the UMS-EMC criterion is more conservative than the UMTS-EMC criterion.

In order to compare the theoretical and experimental results quantitatively under mixed mode loading conditions, a dimensionless parameter, called the effective notch fracture toughness ratio (ENFTR) $K_{\text{eff}}^{U\rho}$, is defined as follows:

$$K_{\text{eff}}^{U\rho} = \sqrt{\left(k_{I}^{U\rho} / k_{Ic}^{U\rho}\right)^{2} + \left(k_{II}^{U\rho} / k_{Ic}^{U\rho}\right)^{2}}$$

(31)

It is worth mentioning that $K_{\text{eff}}^{U\rho}$ is, in fact, the chord drawn from the coordinate origin to any point on the fracture curve (theoretical value) or to any experimental data (experimental value).

Table 5 presents the theoretical values of the ENFTR together with the average values of the experimental ENFTR for U-notches of different radii. Also, included in Table 5 are the discrepancies between the theoretical and experimental results. As seen in Table 5, with the total average discrepancies of about 5% and 8% between the theoretical results of the UMTS-EMC and UMS-EMC criteria and the mean experimental results of the U-notched Al 6061-T6 plates, respectively, it is demon-
stratified that both criteria could successfully predict the mixed mode NFT of ductile U-notched plates.

As presented in Section on Fracture Experiments on Aluminium Plates, about 80% of the ligament (0.8 * 12.5 = 10 mm) encounters plastic deformations at the onset of crack initiation from the U-notch border.

With the aim to simulate the experiments and confirm the experimental observations, several elastic–plastic FE analyses are performed herein for computing the plastic zone size around the notch border at the onset of crack initiation. Note that the true stress–strain curve of Al 6061-T6 depicted in Fig. 2 is utilized in the FE analyses. Moreover, the load applied to each FE model is the mean experimental failure load. Figure 12a and b represents sample plastic regions around the U-notch border associated with the specimens of $\rho = 2 mm$ and $\beta = 0^\circ$ (i.e. horizontal U-notch), and $\rho = 1 mm$ and $\beta = 30^\circ$ (inclined U-notch), respectively. As can be seen in Fig. 12a and b, about 9.43 and 10.9 mm of the ligament experience plastic deformations in horizontal and inclined U-notches, respectively, meaning about 74.4% (9.3 * 100/12.5) and 87.2% (10.9 * 100/12.5) of the ligament, respectively. Comparing these results with that obtained from the experiments (i.e. about 80%) indicates that FE results properly confirm the experimental results, which both prove the LSY failure regime for the U-notched Al 6061-T6 specimens loaded under mixed mode I/II.

Regardless of the failure criterion utilized for predicting ductile fracture in notched members, for example, the critical J-integral, elastic–plastic FE analysis is essential to determine stress, strain and displacement distributions around the notch. As is well known, this kind of analysis is more complex and time-consuming than the linear elastic analysis. Therefore, engineers are highly interested in using rapid and accurate failure criteria in design of notched structural components based on the LENFM. Accordingly, it is attempted in the present investigation to check to see if we could predict mixed mode I/II NFT of ductile U-notched components by utilizing mixed mode brittle fracture criteria in the context of the LENFM needing solely the linear elastic stress analysis. The main result obtained from this investigation is that mixed mode I/II NFT of U-notched plates could successfully be predicted under LSY conditions by using simultaneously the EMC and the UMTS and UMS brittle fracture criteria (i.e. the UMTS-EMC and UMS-EMC criteria) without performing elastic–plastic FE analyses. It should be also highlighted that the two brittle fracture criteria are only capable of predicting the load-carrying capacity of ductile U-notched components and they are not able to predict the growth path of the crack emanating from the notch border or to determine the fracture plane and so on, because fracture behaviors of ductile and brittle materials are fundamentally different.

Regarding the quality of the tensile experiments, a likely question may be raised paying attention to Fig. 4. As seen in Fig. 4, the clamps do not cover the entire heads but centrally cover a significant portion of the heads. This might influence the notch neighbourhood...
stress distributions and hence the experimental results regarding the load-carrying capacity of the specimens. To check to see whether this influence is significant or not, we applied in the FE analyses the remote tensile load ‘P’ in a distributed manner first to the entire head and then to the clamped area, and it was found by comparing the results that the difference in the stress values around the notch is quite low (less than 3%). The results demonstrated that the notch vicinity is not considerably influenced by the experimental devices. However, the main point in performing the experiments is that the upper and lower clamps must be aligned in a way the specimen is not affected by additional bending stresses resulted from the misalignment. As can be seen in Fig. 4c, good alignment is present for the tested Al 6061-T6 plates.

As is well known, the thickness is a key geometric parameter that influences the value of the fracture toughness of material. Because of the wide applications of aluminium thin sheets and plates in engineering structures, particularly aero-structures, the international standard code ASTM B646-1238 has been proposed for measuring experimentally the fracture toughness \( K_c \) of aluminium alloys of finite thicknesses. The resulted \( K_c \) value can easily be replaced in the UMTS-EMC and UMS-EMC criteria to compute the value of the critical distances. Therefore, the two criteria proposed in the present work can fundamentally be applied also to the plates with finite thicknesses. Although the significant three-dimensional (3D) effects have been well demonstrated on brittle and quasi-brittle fracture of notched plates with finite thicknesses,54–57 no paper or technical report has been published in the literature dealing with such effects on ductile rupture of plates and sheets. However, because linear elastic stress analysis is performed in the present study for the U-notched plates virtually made of the equivalent material, it may need to evaluate likely effects of 3D FE modelling on the stress distributions around the U-notches. The 3D FE modelling revealed that a minor difference is obtained between the results of the stress distributions for 2D and 3D analyses.

**CONCLUSIONS**

The load-carrying capacity of ductile U-notched plates is investigated both experimentally and theoretically under mixed mode I/II loading conditions. Experiments

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### Table 5

The theoretical values of the ENFTR together with the average values of the experimental ENFTR including the discrepancies

<table>
<thead>
<tr>
<th>( \beta = 30^\circ )</th>
<th>( \rho ) = 0.5 (mm)</th>
<th>( \rho ) = 1</th>
<th>( \rho ) = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean experimental results</td>
<td>1.5</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>UMTS-EMC criterion</td>
<td>1.4</td>
<td>1.8</td>
<td>2.1</td>
</tr>
<tr>
<td>UMS-EMC criterion</td>
<td>1.3</td>
<td>1.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Mean discrepancy of UMTS-EMC criterion (%)</td>
<td>5</td>
<td>3.2</td>
<td>5</td>
</tr>
<tr>
<td>Total: 4.7%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean discrepancy of UMS-EMC criterion (%)</td>
<td>11</td>
<td>11.5</td>
<td>3</td>
</tr>
<tr>
<td>Total: 8.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \beta = 60^\circ )</th>
<th>( \rho ) = 0.5 (mm)</th>
<th>( \rho ) = 1</th>
<th>( \rho ) = 2</th>
</tr>
</thead>
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<tr>
<td>Mean experimental results</td>
<td>1.14</td>
<td>1.21</td>
<td>1.26</td>
</tr>
<tr>
<td>UMTS-EMC criterion</td>
<td>1.07</td>
<td>1.15</td>
<td>1.21</td>
</tr>
<tr>
<td>UMS-EMC criterion</td>
<td>1.04</td>
<td>1.11</td>
<td>1.18</td>
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<tr>
<td>Mean discrepancy of UMTS-EMC criterion (%)</td>
<td>6.1</td>
<td>5.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Total: 8.1%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mean discrepancy of UMS-EMC criterion (%)</td>
<td>8.8</td>
<td>8.3</td>
<td>6.3</td>
</tr>
<tr>
<td>Total: 8.1%</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

ENFTR, effective notch fracture toughness ratio; UMTS, U-notch maximum tangential stress; UMS, U-notch mean-stress; EMC, equivalent material concept.

**Fig. 12** Elastic–plastic finite element results regarding the distribution of Von-Mises stress in MPa around the U-notches associated with the specimens of \( \rho = 2 \) mm and \( \beta = 0^\circ \) (a) and \( \rho = 1 \) mm and \( \beta = 30^\circ \) (b).
are carried out on Al 6061-T6, which exhibits elastic-plastic behaviour with a significant strain to failure in tension, and the critical loads of the U-notched plates are recorded. The EMC is utilized to equate the ductile material with a virtual brittle material with the aim to allow us to analyze ductile failure of U-notched Al 6061-T6 plates by means of brittle fracture criteria. Two well-known brittle fracture criteria, namely, the UMTS and UMS criteria, are utilized to predict the load-carrying capacity of the U-notched plates virtually made of the equivalent material. It is found that both the UMTS-EMC and UMS-EMC criteria could well predict the experimental results. Both experimental observations and the results of the FE analyses indicated that the U-notched plates fail by LSY regime and hence, it could be concluded that the EMC can successfully be used in conjunction with stress-based brittle fracture criteria to predict mixed mode I/II crack initiation from the U-notch borders in ductile materials (e.g. Al 6061-T6) encountering LSY without needing elastic–plastic analysis. Because fracture behaviors of ductile and brittle materials are basically different, the UMTS-EMC and UMS-EMC criteria are not able to predict the growth path of the crack emanating from the notch border in aluminium plates or to determine the fracture plane and so on, and they are solely capable of predicting the load-carrying capacity of ductile U-notched components.

REFERENCES