Analytical solutions to nonlinear oscillations of micro/nano beams using higher order beam theory

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Abstract. In this study, the nonlinear oscillations of micro/nano beams, modeled by Timoshenko beam theory, actuated by suddenly applied electrostatic forces are investigated. The effects of electrostatic actuation, residual stress, midplane stretching and fringing field are considered in modeling. In order to develop the governing equations and the boundary conditions, the Hamilton's principle is employed. After combining governing equations, the Galerkin's decomposition method is used to convert the governing nonlinear partial equation to a nonlinear ordinary differential equation. The Homotopy Analysis Method (HAM) is used to present semi-analytical solutions to the strongly nonlinear behavior of system. To verify the present model, in special limiting cases, the results are compared with numerical results, and in low values of beam thickness, the results are compared with those obtained with assumption of Euler-Bernoulli beam theory, which are available in literature. Some numerical results are presented to investigate the effects of high thickness and different values of residual stress on the nonlinear frequency and the midpoint deflection of the beam.

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1. Introduction

One of the most significant phenomena associated with micro/nano electromechanical systems, actuated by electrostatic forces, is known as pull-in instability phenomenon which has been investigated by many researchers. Nathanson et al. [1] and Taylor [2] reported pull-in instability in 1960s. Microbeams actuated with electrostatic forces are widely used for different applications such as signal filtering and mass sensing [3].

When the applied voltage exceeds a critical value, called pull-in voltage (V_{pi}), the flexible microbeam deflects toward the rigid plate. In microbeams, pull-in instability continues to become increasingly important for the design of electrostatic MEMS and NEMS devices. Pull-in analysis can be categorized into two groups according to its state. When the rate of voltage variation is low, inertia has no effect on the microsystem behavior, the critical value of voltage is known as static pull-in voltage (V_{ps}). In contrast, when the rate of voltage variation is considerable, the effect of inertia must take into account and the critical voltage value is called dynamic pull-in voltage (V_{pd}). The pull-in instability associated with this situation is called dynamic pull-in instability [4,5]. References [6-14] give more details about this phenomenon. In most nonlinear studies, structures are modeled by Euler-Bernoulli theory. Moghimi Zand et al. obtained pull-in voltage of a microbeam, considering the Euler-Bernoulli beam theory [15]. In another study, Moghimi