Mixed-mode ductile failure analysis of V-notched Al 7075-T6 thin sheets

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**Abstract**  
First, several rectangular thin sheets made of Al 7075-T6 and weakened by blunt V-notches were tested for mixed mode I/II fracture, and the load-carrying capacity and the fracture initiation angles were experimentally recorded. Then, the Equivalent Material Concept (EMC), proposed originally by the first author, was utilized in conjunction with the maximum tangential stress (MTS) and the mean-stress (MS) criteria to predict the experimental results. By approximately 9% and 6.5% discrepancies for the VMTS-EMC and VMS-EMC criteria, respectively, it was found that both criteria could predict successfully the mixed-mode ductile fracture of Al 7075-T6 thin sheets without performing elastic–plastic analyses.

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**1. Introduction**

Aluminum alloys of the 7xxx series are extensively used in engineering structures, particularly in aerospace structures. By using special heat treatment processes like the age-hardening, high strengths, appropriate ductility, good fracture toughness and fatigue strength can be achieved for these alloys. The most well-known alloy in the grade 7xxx is certainly Al 7075 (with different heat treatments such as T6 and T651) which is used in various parts of aerial structures, e.g. spar, stringer, etc. Due to the importance of the health of aero-structures, they must be designed so that they can sustain material and structural failure modes like yielding, buckling, crack initiation and propagation. The most important mechanical properties of aerial aluminum alloys are probably the fatigue strength, the crack growth rate and the fracture toughness which are widely associated with the maintenance and repair of aerial vehicles. Therefore, these properties have been extensively studied in the past by many researchers and engineers both experimentally and theoretically.

A large bulk of the researches in the context of structural integrity of aerial vehicles has been performed on fatigue life prediction and fatigue crack growth rate of aluminum alloys (for example, one can refer to Refs. [1–10]). Dealing with evaluation of fracture toughness of aerial aluminum alloys under various loading conditions e.g. mode I, mode II, mode III and mixed mode loading, several papers have also been published [11,12].

Unlike cracks, defects, and scratches which are all harmful for aerial vehicles, notches can be simultaneously useful and harmful. Their usefulness is mainly due to the fact that they are introduced for joining two or more components (recall a large number of riveted joints and many V- and U-threaded screws in aero-structures). Their harmfulness, however, is because they concentrate stresses at their vicinity which may lead to crack nucleation from the notch edge. The nucleated crack may propagate rapidly or slowly depending on the brittleness and ductility of material which can lead to final fracture.
To avoid crack initiation from the notch border in aerial aluminum alloys, the notch fracture toughness (NFT) of such alloys should be studied both experimentally by performing fracture tests on test specimens and theoretically by means of appropriate fracture criteria.

Two of the most recent papers on evaluation of the NFT in aluminum alloys are those published in Refs. [13,14]. Vratnica et al. [13] investigated successfully the mode I NFT of a commercial aluminum alloy by conducting fracture tests on single-edge-notched-bend (SENB) specimens containing U-shaped notches of various notch radii and by using the linear-elastic stress distributions around a blunt crack-like notch. A combined experimental-theoretical study has also been performed by Madrazo et al. [14] on mode I fracture of Al 7075-T651 weakened by notches in which tensile fracture tests have been conducted on the compact-tension (CT) specimens and the experimental NFT values have been theoretically predicted by means of the Theory of Critical Distances (TCD). It has been shown in Ref. [14] that the TCD could predict the test results successfully.

The key parameter in fracture analysis of notched plates and sheets made of aluminum alloys is the thickness. For relatively large thicknesses (like those investigated in Refs. [13,14]), the component experiences nearly or exactly plain-strain fracture conditions which is recognized by small-scale yielding around the notch. In this state, $K_{Ic}$-based theories of the linear-elastic notch fracture mechanics (LENFM) are valid. However, for thin sheets, moderate or large-scale yielding may usually be realized at the notch neighborhood as a result of plain-stress conditions. In this case, although the fracture
toughness $K_c$ may be valid, the LENFM principles are not permitted to be used in the NFT predictions because of the relatively large size of the plastic zone around the notch.

The fracture toughness of cracked and notched elements encountering large plastic deformations is normally analyzed by means of some well-known methods such as the J-integral, the crack tip opening displacement (CTOD), the crack tip opening angle (CTOA), and the resistance curve (R-curve) [15] which all are complicated, relatively expensive and time-consuming methods. Torabi [16] has suggested recently a new concept, called the Equivalent Material Concept (EMC), by which a ductile material having valid fracture toughness $K_{IC}$ (or $K_c$) could be equated with a virtual brittle material having the same elastic modulus and the same fracture toughness, but different tensile strength. The EMC together with two well-known brittle fracture criteria, namely the point-stress (PS) and the mean-stress (MS) criteria, could be successfully utilized to predict the experimental results on mode I ductile failure of V-, U-, and O-notched steel plates [16–18]. More recently, in an applied work, the tensile load-carrying capacity of ductile steel bolts containing V-shaped threads has also been predicted well by Torabi [19] using the EMC-MS criterion.

In this investigation, the goal was twofold. First, to macroscopically study the fracture behavior of Al 7075-T6 thin sheets weakened by blunt V-notches and subjected to mixed mode I/II loading, and to gather some experimental results regarding load-carrying capacity and fracture initiation angle of such sheets. Second, to check to see if it is possible to predict the experimental results by means of the EMC combined with two well-known mixed mode I/II brittle fracture criteria, namely the maximum tangential stress (MTS) and the mean-stress (MS) criteria. It was found that both the EMC-MTS and EMC-MS criteria could predict successfully the mixed mode notch fracture toughness as well as the fracture initiation angle of Al 7075-T6 thin sheets without performing elastic–plastic analyses.

2. Experiments

2.1. Material

First, an aluminum alloy Al 7075-T6 sheet of 2 mm thick was provided. Then, three various standard tests were performed to determine the material properties. They were the tensile tests according to ASTM E8 [20], the Poisson’s ratio tests according to ASTM E132-04 [21], and the fracture toughness tests (for small thicknesses under plane-stress conditions) in accordance with ASTM B646-12 [22]. The engineering and true stress–strain curves for the tested Al 7075-T6 are shown in Fig. 1. The chemical composition and the mechanical properties of Al 7075-T6 are also presented in Tables 1 and 2, respectively.

2.2. Test specimen

The test specimen was a rectangular plate containing a central rhombic hole with four V-shaped corners; two of which (the extreme right and left corners) are subjected to mixed mode I/II loading (i.e. combined tensile-shear loading). To produce combined tensile–shear deformations at the notch tip vicinity, the specimen is rotated counterclockwise by the angle $\beta$. As $\beta$ varies from zero ($\beta = 0$ results in pure mode I loading), the contribution of shear deformations increases resulting in different mode mixity ratios. The specimen is schematically depicted in Fig. 2 including its geometric parameters.

In Fig. 2, the parameters $2\alpha$, $\rho$, $2\alpha$, $L$, $W$, $\beta$, and $P$ are the notch angle, the notch radius, twice the notch length (i.e. the total slit length), the specimen length, the specimen width, the notch rotation angle, and the remotely applied tensile load, respectively. The following values were considered in the fracture tests: $2\alpha = 30$, 60, and 90 (deg.); $\rho = 1$, 2, and 4 mm; $2\alpha = 25$ mm; $L = 160$ mm; $W = 50$ mm. The values of $\beta$ for the notch angles of 30, 60 and 90 (deg.) were also equal to (0, 30 and 60 (deg.)).

![Fig. 1. The engineering and true stress–strain curves for the tested Al 7075-T6.](image-url)
and (0 and 30 (deg.)), respectively. The thickness was equal to 2 mm for the entire specimens. In order to check the repeatability of the tests, three tests were carried out for each of the notch geometries and loading conditions; all in all, 63 fracture tests were conducted in this study.

For fabricating the notched test samples, an Al 7075-T6 sheet of 2 mm thick was first provided. Then, the sketch of the specimens was prepared by commercial drawing software and given to a high-precision 2D CNC water jet cutting machine. Ultimately, the specimens were cut from the sheet. To remove possible local stress raisers remained from the cutting process, the cut surfaces were precisely polished by using some appropriate rasps. The fracture tests were carried out under displacement-control conditions with the speed of 1 mm/min providing monotonic loading conditions.

2.3. Experimental results

2.3.1. Macroscopic observations

Fig. 3 shows some of the specimens before, during and after the mixed mode I/II fracture experiments.

As seen in Fig. 3c, large plastic deformations can be recognized with naked eye at the ligament suggesting large-scale yielding (LSY) failure regime. It was observed during the tests that plastic zone nucleates from the notch border and grows in large amount around the notch and finally, fracture takes place suddenly so that the instance of crack initiation from the notch border could not be captured by naked eye. The observations indicated that great plastic deformations do not necessarily mean stable crack propagation. Macroscopically, the crack initiation and propagation behaviors of a ductile material can be simply interpreted by means of typical engineering stress–strain curve. Despite overall strain to failure which is typically large for ductile materials, the strain at peak (i.e. at ultimate strength) is a key parameter in determining the material cracking behavior. Small strain interval between the ultimate and the final rupture points (like for the present aluminum alloy tested; see Fig. 1) means that crack initiates in the material by large plastic deformations and the material fractures rapidly or suddenly by unstable crack propagation. Conversely, if the strain at ultimate covers a small portion of the total strain to rupture, crack initiates rapidly with small plastic deformations and grows slowly by large plastic deformations till final rupture. As mentioned at the beginning of this subsection, large plastic zone was observed at the notch tip vicinity at the onset of crack initiation from the notch border and sudden fracture was recorded for the notched specimens under mixed mode loading, confirming correctly the statement above about the cracking behavior of ductile materials.

Fig. 4 depicts a sample load–displacement curve resulted from the test machine.

As seen in Fig. 4, an obvious but relatively small non-linear portion is seen in the curve between the end of the proportional limit and the peak. Also seen in Fig. 4 is sudden drop of the load from the maximum to zero. The sudden fracture of the notched specimens was justified above; however, why do we see small non-linear portion in the curve for the notched specimens encounter large plastic deformations at the crack initiation instance? Note that unlike un-notched specimens in which the entire cross-sectional area experiences uniformly the plastic deformations (and hence, great non-linear portion is normally seen in load–displacement curves), in notched components, plastic deformations are localized at the notch neighborhood resulting in small or moderate non-linear portion of the load–displacement curve. The other important reason could be the fact that the strain to failure for this specific material (about 4% presented in Table 1) is relatively small with respect to typical Al 7075-T6 alloys. It is also necessary to note that this amount of non-linear deformations could not be resulted from a small-scale yielding (SSY) regime, since in the SSY, no clear non-linear portion is usually realized in the load–displacement curve (the curve of SSY is normally very similar to that of the ideally brittle fracture).

<table>
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<tr>
<th>Element</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Zn</th>
<th>Ni</th>
<th>Cr</th>
<th>Pb</th>
<th>Sn</th>
<th>Ti</th>
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<td>0.004</td>
<td>0.2</td>
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<tbody>
<tr>
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<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.007</td>
<td>0.017</td>
<td>0.003</td>
<td>0.001</td>
<td>90.5</td>
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Table 1
Chemical composition of Al 7075-T6.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$ (GPa)</td>
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</tr>
<tr>
<td>Poisson’s ratio</td>
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<tr>
<td>Tensile yield strength (MPa)</td>
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<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>583</td>
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<tr>
<td>Elongation at break (%)</td>
<td>5.8</td>
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<tr>
<td>Engineering strain at maximum load</td>
<td>0.047</td>
</tr>
<tr>
<td>True fracture stress (MPa)</td>
<td>610</td>
</tr>
<tr>
<td>Fracture toughness, $K_f$ (MPa $\sqrt{m}$)</td>
<td>50</td>
</tr>
<tr>
<td>Strain-hardening coefficient, (MPa)</td>
<td>698</td>
</tr>
<tr>
<td>Strain-hardening exponent</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Table 2
Mechanical properties of Al 7075-T6.
The experimentally-obtained load-carrying capacity and fracture initiation angle (FIA) of the V-notched Al 7075-T6 specimens are presented in Tables 3 and 4, respectively for different notch angles and radii, and various angles of rotation $\beta$. $P_i$ and $\theta_i$ ($i = 1, 2, 3$) denote each failure load and FIA in the repeated tests, respectively and $P_{av}$ is the average of the three failure loads.

In the forthcoming sections, it is attempted to predict the experimental results by means of the Equivalent Material Concept (EMC) together with two well-known mixed mode I/II brittle fracture criteria.

3. The Equivalent Material Concept

The Equivalent Material Concept (EMC) has been elaborated in Refs. [16–18]. However, a brief description of it is presented herein to make the present manuscript convenient for the readers.

By using the EMC, a ductile material having valid fracture toughness ($K_{IIc}$ or $K_c$) value is equated with a virtual brittle material having the same elastic modulus and the same fracture toughness, but different tensile strength. Note that the phrase valid fracture toughness value is referred to a fracture toughness $K_c$ value which is valid according to the international standards like ASTM. The tensile strength of the equivalent material can be determined by assuming the same values of the
tensile strain energy density (SED) required for both real ductile and virtual brittle materials for the crack initiation to take place. The mathematical interpretation of the above statements is presented below.

A typical stress–strain curve for a ductile material is depicted in Fig. 5.

As is well-known, Eq. (1) is valid for a ductile material obeying power-law strain-hardening relationship in the plastic zone:
In Eq. (1), \( \sigma, \dot{\varepsilon}_p, K, \) and \( n \) are the true stress, the true plastic strain, the strain-hardening coefficient, and the strain-hardening exponent, respectively. The total SED is composed of elastic and plastic components and it can be written as

\[
\text{(SED)}_{\text{tot}} = \text{(SED)}_e + \text{(SED)}_p = \frac{1}{2} \sigma \dot{\varepsilon}_p + \int_{\dot{\varepsilon}_p}^{\varepsilon_p} \sigma \, d\dot{\varepsilon}_p
\]  

(2)

In Eq. (2), \( \sigma, \dot{\varepsilon}_p \) and \( \varepsilon_p \) are the yield strength, the elastic stress at yield point (i.e., equal to the proportional limit) and the true plastic strain at yield point, respectively. Substituting \( \dot{\varepsilon}_p = \frac{\sigma}{E} \) and Eq. (1) into Eq. (2) results in (\( E \) is the elastic modulus of material):

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma^2}{2E} + K \int_{\dot{\varepsilon}_p}^{\varepsilon_p} \dot{\varepsilon}_p d\dot{\varepsilon}_p
\]  

(3)

Thus

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma^2}{2E} + \frac{K}{n+1} \left[ (\dot{\varepsilon}_p)^{n+1} - (\dot{\varepsilon}_p)^{n+1} \right]
\]  

(4)
If $\epsilon_p$ is assumed to be equal to $0.002$ (obtained from 0.2% offset yield strength), then

$$ (\text{SED})_{\text{tot}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1} \left[ \epsilon_p^{n+1} - (0.002)^{n+1} \right] $$

(5)

For calculating the total SED corresponding to the onset of crack initiation (i.e. the area under the $\sigma$–$\epsilon$ curve from beginning of loading to the maximum load, so-called necking; see Fig. 5), one should replace $\epsilon_p$ in Eq. (5) with $\epsilon_u, \text{true}$, i.e. the true plastic strain at maximum load; which could simply be computed by the expression $\epsilon_u, \text{true} = \ln(1 + \epsilon_u)$ where $\epsilon_u$ is the engineering plastic strain at the ultimate point. Hence

$$ (\text{SED})_{\text{necking}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1} \left[ \epsilon_u, \text{true}^{n+1} - (0.002)^{n+1} \right] $$

(6)

The reason for calculating the SED value till the necking and not the final rupture can briefly be explained as follows. The necking in an unnotched tensile specimen can be simply considered as a crack initiation from the straight edges of the specimen that reduce the area and decrease the load-carrying capacity of the specimen. As is well-known, at necking instance (equal to a crack initiation), the ultimate load is achieved. Since it is intended in this investigation to predict the
load-carrying capacity of the notched components, which is obtained at crack initiation instance from the notch border, the SED must be computed till the ultimate point and not the final rupture.

As previously mentioned, the equivalent material considered by the EMC is a virtual brittle material with the same values of the elastic modulus $E$ and the fracture toughness $K_{IC}$ (or $K_c$), but unknown value of the tensile strength. Fig. 6 represents schematically a typical uni-axial stress–strain curve for the equivalent material.

In Fig. 6, the parameters $\varepsilon_I^*$ and $\varepsilon_{II}^*$ are the strain at crack initiation (i.e., the final fracture due to the brittleness of material) and the tensile strength of material, respectively. The SED absorbed by this brittle material at the onset of crack initiation is therefore

$$(\text{SED})_{EM} = \frac{\varepsilon_I^*}{2E}$$

(7)

Based on the EMC, SED values for both real ductile and virtual brittle materials should be equal. Thus, Eqs. (6) and (7) are set to be equal and hence

$$\frac{\varepsilon_I^*}{2E} = \frac{\varepsilon_{II}^*}{2E} + \frac{K}{n + 1} \left[ \frac{\varepsilon_{II}^{\text{true}}}{\varepsilon_{II}^{\text{true}}} - (0.002)^{n+1} \right]$$

(8)

Ultimately, $\sigma_I^*$ is resulted as follows

$$\sigma_I^* = \sqrt{\frac{2EK}{n + 1} \left[ \frac{\varepsilon_{II}^{\text{true}}}{\varepsilon_{II}^{\text{true}}} - (0.002)^{n+1} \right]}$$

(9)

The closed-form expression of $\sigma_I^*$ presented in Eq. (9) can be used together with the material fracture toughness $K_{IC}$ (or $K_c$) in different mixed mode I/II brittle fracture criteria for predicting theoretically the load-carrying capacity and the fracture initiation angle (FIA) of ductile V-notched components.

In the next section, two well-known mixed mode brittle fracture criteria, namely the maximum tangential stress (MTS) and the mean-stress (MS) criteria, are described and some fracture curves and also the curves of fracture initiation angle are developed by combining such criteria with the EMC to predict mixed mode NFTs and FIAS for ductile V-notched components.

4. Conjunction of the EMC with mixed mode brittle fracture criteria

4.1. Linear elastic stress distributions around a blunt V-notch

Filippi et al. [23] have developed a closed-form expression for mixed mode I/II linear elastic stress distributions around a blunt V-notch shown in Fig. 7.

The mixed mode I/II stresses can be written in polar coordinate system as

$$
\begin{bmatrix}
\sigma_{\theta\theta} \\
\sigma_{\theta\tau} \\
\sigma_{\tau\tau}
\end{bmatrix}
= K_i^{\theta\theta} \sqrt{2\pi r^{1-J_1}} \begin{bmatrix}
f_{\theta\theta}(\theta) \\
f_{\theta\tau}(\theta) \\
f_{\tau\tau}(\theta)
\end{bmatrix}^{(I)}
+ \left( \frac{r}{r_0} \right)^{\mu_1-J_1} \begin{bmatrix}
g_{\theta\theta}(\theta) \\
g_{\theta\tau}(\theta) \\
g_{\tau\tau}(\theta)
\end{bmatrix}^{(I)}
+ K_{II}^{\theta\theta} \sqrt{2\pi r^{1-J_2}} \begin{bmatrix}
f_{\theta\theta}(\theta) \\
f_{\theta\tau}(\theta) \\
f_{\tau\tau}(\theta)
\end{bmatrix}^{(II)}
+ \left( \frac{r}{r_0} \right)^{\mu_2-J_2} \begin{bmatrix}
g_{\theta\theta}(\theta) \\
g_{\theta\tau}(\theta) \\
g_{\tau\tau}(\theta)
\end{bmatrix}^{(II)}
$$

(10)

where $K_i^{\theta\theta}$ and $K_{II}^{\theta\theta}$ are the mode I and mode II notch stress intensity factors (NSIFs), respectively. The parameter $r_0$ is the distance between the origin of the polar coordinate system and the notch tip. The functions $f_{ij}(\theta)$ and $g_{ij}(\theta)$ have been reported in the Appendix of Ref. [24] and the eigenvalues $\lambda_i$ and $\mu_j$ which depend on the notch angle have been reported in Ref. [23]. According to a relation between the Cartesian and the curvilinear coordinate systems, the distance $r_0$ can be written as [23]:

$$r_0 = \frac{\pi - 2\alpha}{2(\pi - \alpha)}$$

(11)

where “$2\alpha$” and $\rho$ are the notch angle and the notch radius, respectively. The expressions for the NSIFs are [25]:

$$K_i^{\theta\theta} = \sqrt{2\pi} \sigma_{\theta\theta}(r_0, \theta) r_0^{1-J_1} \left( 1 + \omega_1 \right)$$

(12)

$$K_{II}^{\theta\theta} = \lim_{r \to r_0} \sqrt{2\pi} \sigma_{\theta\theta}(r, \theta) r^{1-J_2} \left( \frac{1}{r_0} \right)^{\mu_2-J_2}$$

(13)

where $\sigma_{\theta\theta}$ and $\sigma_{\tau\tau}$ are the tangential and the in-plane shear stresses, respectively. The parameter $\omega_1$ has been presented in the Appendix of Ref. [24]. The NSIFs can be computed for any V-notched member by utilizing the finite element (FE) method as elaborated in Ref. [24]. Note that the parameter $r$ in Eq. (13) cannot be directly substituted by $r_0$, because for $r = r_0$, $K_{II}^{\theta\theta}$ becomes singular. Therefore, $K_{II}^{\theta\theta}$ is computed from Eq. (13) at a point very close to the notch tip where $r \to r_0$. 


4.2. The V-notch maximum tangential stress (VMTS) criterion

In recent years, the classical maximum tangential stress (MTS) criterion originally proposed by Erdogan and Sih [26] for predicting mixed mode I/II brittle fracture in cracked components has been extended to notched brittle domains for mixed mode I/II and pure mode II loadings by Torabi and his co-researchers [24,27–37].

According to the V-notch MTS (VMTS) criterion, the first and the second derivatives of the tangential stress $\sigma_{\theta\theta}$ with respect to $\theta$ should be zero and negative, respectively (see for instance Ref. [24]). The first hypothesis of the VMTS criterion proposes that brittle fracture initiates from a point on the notch border and propagates radially along a direction perpendicular to the maximum tangential stress. The direction $\theta$ corresponding to this point is called the fracture initiation angle (FIA) $\theta_0$. The second hypothesis also suggests that brittle fracture in a blunt V-notched member occurs when the tangential stress $\sigma_{\theta\theta}$ along $\theta_0$ and at a critical distance $r_{c,V}$ attains a critical value $\sigma_c$, called the critical stress. The parameter $r_{c,V}$ is the critical distance for blunt V-notches measured from the origin of the polar coordinate system (see Fig. 8) and not from the notch tip.

Therefore, $r_{c,V}$ is not a fixed material property and depends upon the notch geometry. It has been shown in several references that the critical distance for notches can be computed for mode I loading conditions and successfully utilized for the entire domain of mixed mode I/II loading from pure mode I to pure mode II [27–31]. Moreover, in two most recent research papers [34,35], it has been reported that assuming the real portion of the notch critical distance ($r_c$ in Fig. 8 that lies on material) equal to the critical distance for sharp crack results in very good mixed mode notch fracture toughness predictions. Therefore, the notch critical distance for blunt V-notches can be simply considered to be equal to $r_{c,V} = r_0 + r_c = r_0 + \frac{1}{2\pi} \left( \frac{K_{IC}}{\sigma_c} \right)^2$ [34,35,38–43]. The parameter $K_{IC}$ is the plane-strain fracture toughness of material.

The critical distance $r_c = \frac{1}{2\pi} \left( \frac{K_{IC}}{\sigma_c} \right)^2$ has also been utilized successfully in Ref. [27] for predicting mixed mode brittle fracture in sharp V-shaped notches.
The mathematical interpretation of the VMTS criterion is presented herein. According to the first hypothesis of the VMTS criterion, we have:

$$\frac{\partial \sigma_{a0}(r, \theta)}{\partial \theta} = 0 \quad (14.a)$$

$$\frac{\partial^2 \sigma_{a0}(r, \theta)}{\partial \theta^2} < 0 \quad (14.b)$$

By substituting the tangential component of the stress field from Eq. (10) into Eq. (14.a) and replacing $r$ and $\theta$ by $r_{c,V}$ and $\theta_0$, one can derive an equation for determining the FIA $\theta_0$ in terms of $K_{II}^{V,R}$ and $K_{II}^{V,\rho}$ as:

$$K_{II}^{V,\rho} \left[ L (-RS \sin S \theta_0 - RS \sin R \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^\rho M (-\chi_{d_1} V \cos W \theta_0 - \chi_{c_1} V \sin V \theta_0) \right]$$

$$+ \frac{K_{II}^{V,R}}{2\pi (r_{c,V})} \left[ N (TU \cos U \theta_0 + \chi_{d_2} T^2 \cos T \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^Q O (\chi_{d_2} X \cos Y \theta_0 - \chi_{c_2} X \cos X \theta_0) \right] = 0 \quad (15)$$

Note that the entire symbols used in Eq. (15) are presented in Table 5.

Under pure mode I loading, $K_{II}^{V,R}$ is zero and Eq. (15) reduces to

$$L (-RS \sin S \theta_0 - RS \sin R \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^\rho M (-\chi_{d_1} V \cos W \theta_0 - \chi_{c_1} V \sin V \theta_0) = 0 \quad (16)$$

The trivial root of Eq. (16) is $\theta_0 = 0$ because of symmetry in geometry and loading conditions. Under pure mode II loading, $K_{II}^{V,\rho}$ is zero and Eq. (15) can be reduced to

$$N (TU \cos U \theta_0 + \chi_{d_2} T^2 \cos T \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^Q O (\chi_{d_2} X \cos Y \theta_0 - \chi_{c_2} X \cos X \theta_0) = 0 \quad (17)$$

Solving Eq. (17) gives $\theta_0 = \theta_{III}$ where $\theta_{III}$ is the mode II FIA. Note that the roots of Eq. (17) should satisfy Eq. (14.b). It has been reported in several references that a negative value of the mode II FIA satisfies the equation [24,28–30].

The second hypothesis of the VMTS criterion proposes that brittle fracture takes place when the tangential stress along $\theta = \theta_0$ and at the distance $r = r_{c,V}$ from the notch reference frame origin attains the critical stress $\sigma_c$. Thus, at the fracture onset, the tangential stress component of Eq. (10) can be written as:

$$\sigma_c = \frac{K_{II}^{V,\rho}}{2\pi (r_{c,V})^\rho} \left[ L (R \cos S \theta_0 + \chi_{d_1} S \cos R \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^\rho M (\chi_{d_1} V \cos W \theta_0 + \chi_{c_1} \cos V \theta_0) \right]$$

$$+ \frac{K_{II}^{V,R}}{2\pi (r_{c,V})^\rho} \left[ N (T \sin U \theta_0 + \chi_{d_2} T \sin T \theta_0) + \left( \frac{r_{c,V}}{r_0} \right)^Q O (\chi_{d_2} X \sin Y \theta_0 - \chi_{c_2} \sin X \theta_0) \right] \quad (18)$$

For a reduced case when only mode I loading exists, the following equation is valid:

$$\theta_0 = 0 \quad K_{II}^{V,\rho} = K_{II}^{V,R} \quad K_{II}^{V,\rho} = 0 \quad (19)$$
Fig. 9 depicts a typical fracture curve. As seen in Fig. 9, the curve characterizes the onset of brittle fracture, while the areas under and over the curve represent the safe zone and the fracture zone, respectively.

The parameter $K^V/Ic$ is called the mode I notch fracture toughness which can be determined either experimentally or theoretically by means of some appropriate fracture criteria [24,27–30]. $K^V/Ic$ is not a fixed material property because it depends also upon the notch angle and the notch radius. Substituting Eq. (19) into Eq. (18) results in

$$\sigma_c = \frac{K^V/Ic}{\sqrt{2\pi(r_c)^3}} \left[ L(R + \chi_b S) + H^2M(\chi_d V + \chi_c) \right]$$  \hspace{1cm} (20)

Substituting Eq. (20) into Eq. (18) and dividing both sides of the resulted equation by $K^V/Ic$ gives

$$\frac{K^V/Ic}{K^V/Ic} \left[ L(R \cos \phi_0 + \chi_b \cos R \phi_0) + (H)^2M(\chi_d V \cos \phi_0 + \chi_c \cos \phi_0) \right]$$

$$+ \frac{K^V/Ic}{K^V/Ic} \left( r_c (S-U) \right) \left[ N(T \sin \phi_0 + \chi_d T \sin T \phi_0) + (H)^20(\chi_d X \sin \phi_0 - \chi_c \sin X \phi_0) \right]$$

$$= \left[ L(R + \chi_b S) + H^2M(\chi_d V + \chi_c) \right]$$  \hspace{1cm} (21)

Eqs. (15) (divided by $K^V/Ic$) and (21) are the governing equations of the VMTS criterion that make a system of linear equations in terms of the NSIFs in which $K^V/Ic$ and $K^V/Ic$ are unknown. To solve the system, the FIA $\phi_0$ is required. It is trivial that $\phi_0$ varies from 0 to $\phi_{0ll}$ when the mode of loading turns gradually from pure mode I toward pure mode II. Hence, the FIA domain of variation is divided into several increments and for each value of $\phi_0$, the system is solved and a specific point gives $\sigma_c$. By connecting the entire points to each other, a mixed mode fracture curve is achieved. Fig. 9 depicts a typical fracture curve. As seen in Fig. 9, the curve characterizes the onset of brittle fracture, while the areas under and over the curve represent the safe zone and the fracture zone, respectively.

It is worth noting that the parameter $\sigma_c$ is a material property which is commonly considered to be equal to the tensile strength for brittle and quasi-brittle materials (see e.g. Refs. [27–31,44–47]).

![Fig. 9](image_url)
In addition to the notch fracture toughness, the other important parameter in mixed mode fracture analysis of notched components is the FIA ($\theta_0$). This parameter sometimes plays a vital role in determining the overall damage in notched structures. To achieve the curves of FIA, a parameter called the notch mode mixity parameter ($M_{V}^c$) is defined as [24]:

$$M_{V}^c = \frac{2}{\pi} \tan^{-1} \left( \frac{K_{I}^{V,\rho}}{K_{II}^{V,\rho}} \right)$$

(22)

The value of $M_{V}^c$ varies from zero (for pure mode II) to one (for pure mode I). Now, by extracting the ratio $K_{I}^{V,\rho}/K_{II}^{V,\rho}$ from Eq. (15) and substituting it into Eq. (22), we have

$$M_{V}^c = \frac{2}{\pi} \tan^{-1} \left( -\frac{N\left(TU \cos U\theta_0 + \chi_{b_2}T^2 \cos T\theta_0\right) + \left(\frac{rs}{T}\right)^2 \left(\frac{1}{C_6}\right) M(-\chi_{d_1}VW \cos W\theta_0 - \chi_{c_1}V \sin V\theta_0) + \left(\frac{rs}{T}\right)^2 \left(\frac{1}{C_6}\right) M(-\chi_{d_1}VW \cos W\theta_0 - \chi_{c_1}V \sin V\theta_0) \right)$$

(23)

With the aim to plot the curves of FIA, the steps below should be followed one-by-one:

1. Choose an arbitrary value for $M_{V}^c$ between 0 and 1.
2. Substitute $M_{V}^c$ into Eq. (23) and solve it for $\theta_0$.
3. Repeat the steps 1 and 2 for other values of $M_{V}^c$.
4. Draw the parameter $\theta_0$ versus $M_{V}^c$.

A typical FIA curve is shown in Fig. 10 for a blunt V-notch.

It should be noted that by considering the notch coordinate system shown in Fig. 7, the sign of the FIA ($\theta_0$) and the mode II NSIFs ($K_{II}^{V,\rho}$) are negative, but the absolute values are presented in Figs. 9 and 10, and also in the forthcoming figures.

4.3. The V-notch mean-stress (VMS) criterion

During recent three years, the mean-stress (MS) failure concept, which is a well-known brittle fracture criterion for notched components under pure mode I loading, has been extended to notched domains subjected to mixed mode I/II and pure mode II loadings by which brittle fracture in U-notches [32,48], key-hole notches [33,36,37] and V-notches with end holes (VO-notches) [34,35] has been successfully predicted. In this research, this is the first time that the MS criterion is formulated for blunt V-notches subjected to mixed mode I/II loading.

The V-notch mean-stress (VMS) criterion proposes that brittle fracture occurs for a V-notched member when the average of the tangential stress over a specified critical distance ahead of the notch edge reaches to the critical stress. The concepts of the VMTS and VMS criteria are very similar except that for VMS criterion, the average tangential stress distribution is used in the mathematical formulations instead of the tangential stress at a specified point. To start the VMS formulations, a closed-form expression should first be obtained for the average tangential stress distribution over a specified critical distance. Fig. 11 shows the critical distances of the VMS criterion ($d_{c,V}$) which are measured from the coordinate origin and from the notch tip, respectively. It is clear from Fig. 11 that $d_{c,V} = d_c + r_0$.

Eq. (24) gives the average stress over the critical distance $d_c$:

$$\overline{\sigma_{\theta_0}} = \frac{1}{d_c} \int_{r=r_0}^{r=d_c} \sigma_{\theta_0} dr$$

(24)

Substituting $\sigma_{\theta_0}$ from Eq. (10) into Eq. (24) and integrating gives

$$\overline{\sigma_{\theta_0}}(\theta_0) = \frac{K_{I}^{V,\rho}}{\sqrt{2\pi d_c}} \left[ \left( R \cos S\theta_0 + \chi_{b_2} \cos R\theta_0 \right) A_1 + M \left( \chi_{d_1} V \cos W \theta_0 + \chi_{c_1} \cos V \theta_0 \right) A_2 \right]$$

$$+ \frac{K_{II}^{V,\rho}}{\sqrt{2\pi d_c}} \left[ \left( T \sin U\theta_0 + \chi_{d_2} \sin T\theta_0 \right) A_3 + O \left( \chi_{d_2} X \sin Y \theta_0 + \chi_{c_2} \sin X \theta_0 \right) A_4 \right]$$

(25)

where

$$A_1 = -\frac{1}{1 - S} \left( d_{c,V}^{1-S} - r_0^{1-S} \right)$$

$$A_2 = \frac{1}{(1 - S + P)r_0} \left( d_{c,V}^{1-S+P} - r_0^{1-S+P} \right)$$

$$A_3 = \frac{1}{1 - U} \left( d_{c,V}^{1-U} - r_0^{1-U} \right)$$

$$A_4 = \frac{1}{(1 - U + Q)r_0} \left( d_{c,V}^{1-U+Q} - r_0^{1-U+Q} \right)$$

(26)
Herein, the procedure of obtaining the fracture curves and the curves of FIA for the VMTS criterion is similarly followed for the VMS criterion except that $\sigma_{\infty}$ in VMTS formulations is replaced with $\overline{\sigma}_m$. To reach to a maximum, we should have

$$\frac{\partial \overline{\sigma}_w(\theta)}{\partial \theta} = 0 \rightarrow \theta_0 = \overline{\theta}_0$$

(27)

The angle $\overline{\theta}_0$ is the FIA for the VMS criterion. Substituting Eq. (25) into Eq. (27) results in

$$\frac{\partial \overline{\sigma}_w(\overline{\theta})}{\partial \overline{\theta}} = \frac{K_{I}^{\rho}}{\sqrt{2\pi d_c}} \left[ L(-R \sin \overline{\theta}_0 - R \chi_{b1} \sin R \overline{\theta}_0) A_1 + M(-\chi_{d1} V W \sin W \overline{\theta}_0 - \chi_{c1} V \sin V \overline{\theta}_0) A_2 \right]$$

$$+ \frac{K_{II}^{\rho}}{\sqrt{2\pi d_c}} \left[ N(T U \cos U \overline{\theta}_0 + \chi_{d2} T^2 \cos T \overline{\theta}_0) A_3 + O(\chi_{d2} X Y \cos Y \overline{\theta}_0 - \chi_{c2} X \cos X \overline{\theta}_0) A_4 \right] = 0$$

(28)

The FIA is equal to zero under mode I loading because of symmetry in geometry and loading conditions. For mode II loading, $K_{I}^{\rho}$ becomes zero and thus, Eq. (28) is reduced to

$$N(T U \cos U \overline{\theta}_0 + \chi_{d2} T^2 \cos T \overline{\theta}_0) A_3 + O(\chi_{d2} X Y \cos Y \overline{\theta}_0 - \chi_{c2} X \cos X \overline{\theta}_0) A_4 = 0$$

(29)

The root of Eq. (29) is the mode II FIA (herein after denoted by $\overline{\theta}_{0ii}$) predicted by the VMS criterion.

According to the VMS criterion, the average stress $\overline{\sigma}_m$ should attain the critical stress $\sigma_c$ at brittle fracture instance. Therefore

$$\sigma_c = \frac{K_{I}^{\rho}}{\sqrt{2\pi d_c}} \left[ L(R \cos \overline{\theta}_0 + \chi_{b1} S \cos R \overline{\theta}_0) A_1 + M(\chi_{d1} V \cos W \overline{\theta}_0 + \chi_{c1} \cos V \overline{\theta}_0) A_2 \right]$$

$$+ \frac{K_{II}^{\rho}}{\sqrt{2\pi d_c}} \left[ N(T \sin U \overline{\theta}_0 + \chi_{d2} T \sin T \overline{\theta}_0) A_3 + O(\chi_{d2} X \sin Y \overline{\theta}_0 + \chi_{c2} \sin X \overline{\theta}_0) A_4 \right]$$

(30)

Fig. 10. A typical FIA curve for a blunt V-notch.

Fig. 11. The critical distances of the VMS criterion.
It is evident that Eq. (30) is valid over the entire mixed mode I/II domain. As a reduced case, it should be valid in pure mode I loading conditions for which Eq. (19) is satisfied. Thus, we have

$$\sigma_c = \frac{K_{II}^p}{\sqrt{2\pi d_c}} [L(R + S\chi_{b1})A_1 + M(\chi_{d1}V + \chi_{c1})A_2]$$

(31)

Substituting Eq. (31) into Eq. (30) and dividing both sides of the resulted equation by $K_{II}^p$, Eq. (32) is obtained as follows

$$\frac{K_{II}^p}{K_{I}^p} [L(R \cos \theta_0 + \chi_{b1} \cos \theta_0 A_1 + M(\chi_{d1} V \cos \theta_0 + \chi_{c1} \cos \theta_0) A_2] + \frac{K_{II}^p}{K_{I}^p} [(N/T \sin \theta_0 + \chi_{b2} T \sin \theta_0 A_3 + O(\chi_{d2} X \sin \theta_0 + \chi_{c2} \sin X \theta_0) A_4] = [L(R + S\chi_{b1})A_1 + M(\chi_{d1} V + \chi_{c1})A_2]$$

(32)

Eqs. (28) (divided by $K_{II}^p$) and (32) are the governing equations of the VMS criterion that make again a system of linear equations in terms of the NSIFs in which $K_{II}^p/K_{I}^p$ and $K_{II}^p/K_{II}^p$ are unknown. Similar to the VMTS criterion, the FIA domain of variation is divided into several increments and for each value of $\overline{\theta}$, the system is solved and a particular point ($K_{II}^p/K_{I}^p$, $K_{II}^p/K_{II}^p$) is obtained. By connecting the entire points to each other, a mixed mode fracture curve is achieved for the VMS criterion.

Similar to VMTS criterion, the mode mixity parameter over $\overline{\theta}$ can also be defined for the VMS criterion by utilizing Eq. (33) as follows:

$$\overline{M}_{\overline{\theta}} = \frac{2}{\pi} \tan^{-1} \left( - \frac{K_{II}^p}{K_{I}^p} \frac{A_1}{A_3} \right)$$

(33)

Extracting the ratio $K_{II}^p/K_{II}^p$ from Eq. (28) and substituting it into Eq. (33) results in the final expression of $\overline{M}_{\overline{\theta}}$ as:

$$\overline{M}_{\overline{\theta}} = \frac{2}{\pi} \tan^{-1} \left( - \frac{N(T U \cos \theta_0 + \chi_{b2} T^2 \cos \theta_0) + O(\chi_{d2} X Y \cos \theta_0 - \chi_{c2} X \cos \theta_0) \overline{\theta}} {L(-R S \sin \theta_0 - R S \chi_{b1} \sin \theta_0 A_3) + M(-Z W \sin \theta_0 - \chi_{d1} V \sin \theta_0) A_4} \right)$$

(34)

To plot the FIA curves for the VMS criterion, the procedure described after Eq. (23) can be followed except that in the step 2, Eq. (23) should be replaced with Eq. (34).

4.4. Combination of the VMTS and VMS criteria with the EMC

As can be seen in the governing equations of both VMTS and VMS criteria, the only unknown parameters in drawing the failure curves are the critical distances. As previously mentioned, it was assumed in the present research that the critical distances of sharp cracks are valid also for blunt V-notches. Therefore, the expressions for the critical distances utilized in drawing the failure curves are:

$$r_{cV} = r_0 + \frac{K_{II}}{\sigma_c} ; \quad d_{cV} = 0 + \frac{K_{II}}{\sigma_c}$$

(35)

Now, it is the time to combine the EMC with the two brittle fracture criteria (i.e. the VMTS and VMS criteria) to make them capable of being utilized for predicting crack initiation from the notch border in V-notched ductile components under mixed mode loading. For this purpose, one just needs to replace $\sigma_c$ in Eq. (35) with $\sigma_f$ from Eq. (9). Meanwhile, $K_{II}$ may be replaced with $K_{c}$ for thin notched components (like Al 7075-T6 sheets investigated in the present study). Taking into account the mechanical properties of Al 7075-T6 presented in Table 2, the values of the parameters $\sigma_f$, $r_c$, and $d_c$ are computed from Eqs. (9) and (35) to be equal to about 1845 MPa, 0.117 mm, and 0.467 mm, respectively.

In the next section, it is attempted to predict the experimental results provided in Section 2 on mixed mode ductile failure of V-notched Al 7075-T6 thin sheets by means of the two brittle fracture criteria in conjunction with the EMC. The combined criteria are known herein after as the VMTS-EMC and the VMS-EMC criteria.

5. Results and discussion

In order to predict the experimental results reported in Section 2 by means of the VMTS-EMC and VMS-EMC criteria, the experimental critical loads should first be converted to the corresponding critical NSIFs. To do this, a finite element (FE) model should first be created for each V-notched test specimen. Then, the experimentally obtained critical load (see Table 3) is applied to the model and the tangential and in-plane shear stress distributions are determined on the notch bisector line. The critical mode I and mode II NSIFs are finally computed by using Eqs. (12) and (13), respectively. Fig. 12 represents a FE
model for a V-notched Al 7075-T6 specimen. As seen in Fig. 12, refined meshes are used at the notch border vicinity because of high stress gradient.

As seen in Fig. 9, the critical NSIFs $K_{Ic}^{V}$ and $K_{IIc}^{V}$ in a fracture curve are divided by the mode I notch fracture toughness (NFT) $K_{Ic}^{NFT}$. To compare the experimental results with the theoretical ones, it is necessary to divide the experimentally obtained critical NSIFs calculated by the procedure above (i.e. from Eqs. (12) and (13)) by the average value of the experimental $K_{Ic}^{V}$. In order to determine the average experimental $K_{Ic}^{V}$, the average of the three mode I critical loads (see the last column in Table 3 for $\beta = 0$) is applied to the FE model of the specimen and the corresponding tangential stress at the notch tip is computed. Then, the computed value is substituted into Eq. (12) instead of $r_{hh}(r_0, 0)$ and the critical $K_{Ic}^{V}$ is computed. Table 6 presents the average experimental values of $K_{Ic}^{V}$ for the V-notched Al 7075-T6 specimens.

Figs. 13 and 14 show the fracture curves and the curves of FIA for the V-notched Al 7075-T6 specimens of different notch angles and various notch radii, respectively together with the experimental results.

From Fig. 13, the following results could qualitatively be obtained:

- Both the VMTS-EMC and VMS-EMC criteria provide generally good predictions to the experimental results of the mixed mode I/II notch fracture toughness for different notch angles, notch radii, and mode mixity ratios.
- For the entire values of the notch angle and notch radius, the fracture curve of the VMS-EMC criterion locates under that of the VMTS-EMC criterion meaning that the VMS-EMC criterion is more conservative than the VMTS-EMC criterion.
- For the entire notch geometries, the distance between the fracture curves of the two criteria increases, as the contribution of mode II loading enhances. The maximum difference is recognized for pure mode II loading (see the vertical axis). This trend implies that in designing V-notched ductile components subjected to mode I-dominant mixed mode I/II loading conditions, any of the two criteria can be arbitrarily chosen, while for mode II-dominant loadings, the failure criterion should be chosen accurately.
- For a specific notch angle, it is clearly seen that the distance between the fracture curves of the two criteria decreases, as the notch radius increases. For the notch radius of 4 mm, the two curves become very close together and, it can be expected that for the notch radii greater than about 6 mm (maybe), the curves lie on each other meaning that both criteria provide identical predictions to the mixed mode notch fracture toughness. Obviously, there is no need in this case to consider the critical distances in theoretical failure predictions and hence, the stress at the notch border can be taken into account as the governing fracture parameter. Thus, fracture prediction can be performed by utilizing the stress concentration factor (SCF) instead of the notch stress intensity factors (NSIFs).

Table 6presents the average experimental values of $K_{Ic}^{V}$ for the V-notched Al 7075-T6 specimens.

Two main results can also be qualitatively achieved from Fig. 14 as follows:

- In general, both the VMTS-EMC and VMS-EMC criteria provide very good predictions to the experimental results of the fracture initiation angle (FIA) for various notch angles, notch radii, and mode mixity ratios.
Average experimental values of $K_{\text{eff}}^V$ for the V-notched Al 7075-T6 specimens.

<table>
<thead>
<tr>
<th>2x = 30 (deg.)</th>
<th>2x = 60 (deg.)</th>
<th>2x = 90 (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{av}}$ (N)</td>
<td>$K_{\text{eff}}^V$ (MPa m$^{1/2}$)</td>
<td>$K_{\text{eff}}^V$ (MPa m$^{1/2}$)</td>
</tr>
<tr>
<td>$\rho = 1$ (mm)</td>
<td>$\rho = 2$ (mm)</td>
<td>$\rho = 4$ (mm)</td>
</tr>
<tr>
<td>27,587</td>
<td>75.4</td>
<td>28,375</td>
</tr>
</tbody>
</table>

- Although the predictions of the VMS-EMC criterion are somewhat greater than those of the VMTS-EMC criterion, the predictions of both criteria are very close together meaning that any of the criteria can be arbitrarily chosen in engineering design purposes.

To compare quantitatively the experimental results of the notch fracture toughness with the theoretical predictions under mixed mode I/II loading conditions, a dimensionless parameter, called the effective relative notch fracture toughness (ERNFT) $K_{\text{eff}}^V$, is defined here as:

$$K_{\text{eff}}^V = \sqrt{\frac{(K_{\nu,q}^V)^2}{K_{\text{IC}}} + \frac{(K_{\nu,q}^V - A_3)^2}{K_{\text{IC}}^2}}$$

It is worth noting that making $K_{\text{eff}}^V$ non-dimensional, various reference lengths could be used. However, the ratio $A_3/A_1$ was arbitrarily selected herein. The theoretical values of the ERNFT together with the mean values of the experimental ERNFTs are presented in Table 7 for various V-notch geometries. The discrepancies between the experimental and theoretical results are also included in Table 7.

Table 7 clearly shows that the total average discrepancies between the theoretical and average experimental results for the VMTS-EMC and VMS-EMC criteria are approximately 9% and 6.5%, respectively with majority of the average discrepancies less than 10% demonstrating the effectiveness of both criteria in mixed mode I/II NFT prediction for ductile V-notched components. Similar quantitative analysis was also performed for the FIA and it was found that the accuracies of both criteria in predicting the test results are always more than 90% showing that both criteria are also successful in FIA prediction.

As mentioned in Section 2, large plastic deformations were observed during the fracture tests of V-notched Al 7075-T6 specimens around the notch border at crack initiation instance. In order to confirm the macroscopic experimental observations, some elastic–plastic FE analyses were carried out in this research to determine the plastic zone around the notch border at crack initiation instance. The true stress–strain curve of Al 7075-T6 represented in Fig. 1 was used in the analyses. Meanwhile, the average experimental failure load was applied to each FE model. Fig. 15 depicts a sample plastic zone around the V-notch border corresponding to the specimen of $2x = 60^\circ$, $\beta = 30^\circ$ and $\rho = 2$ mm. As shown in Fig. 15, about 9 mm of the ligament encounters plastic deformations meaning approximately 70% of the ligament. For the other notch geometries, similar results were achieved. It was found from the elastic–plastic FE analyses that the percentage of the ligament occupied by the plastic deformations at the onset of crack initiation varies between 70% and 75% depending on the notch geometry and orientation. Accordingly, the elastic–plastic FE analyses confirm the experimental observations which both prove the large-scale yielding (LSY) failure regime for the Al 7075-T6 specimens subjected to mixed mode loading.

It is worth noting that engineers are strongly interested in employing simple, rapid and accurate failure criteria in design of engineering components and structures. As is well-known, applying traditional ductile failure criteria to cracked and notched components (like J-integral, CTOD and CTOA) is complicated and time-consuming because of employing elastic–plastic FE analysis. For example, to determine the stress distributions around the notch in the present notched Al 7075-T6 sheets, about 4.5 h were consumed for each FE analysis using a normal personal computer. Due to the above-mentioned disadvantages of the traditional ductile failure criteria, it was tried in this study to predict the notch fracture toughness and fracture initiation angle of ductile components containing blunt V-notches by using simple mixed mode brittle fracture criteria requiring only the linear-elastic stress analysis. By using the VMTS-EMC and VMS-EMC criteria, there is no need to conduct elastic–plastic analysis for predicting mixed mode failure of notched ductile components encountering large-scale yielding.

It is necessary to highlight that the size of plastic region in the standard stress–strain curve (i.e. the level of material ductility) does not influence the procedure of determining the tensile strength of the equivalent material. It only affects the magnitude of the tensile strength. Trivially, more ductility of material will result in greater tensile strength. The two theoretical failure models developed in the present research do not depend on the plastic zone size around the notch and therefore, they may fundamentally be able to predict the notch fracture toughness (NFT) in the entire ductile failure regimes including moderate-scale, large-scale and gross-yielding regimes. The experiments are carried out herein under large-scale yielding (LSY) conditions because the main goal of this investigation is to measure the plane-stress NFT instead of the plane-strain one.

At the end of this section, the authors would like to present briefly an applied procedure for determining the load-carrying capacity for a V-notched ductile component that fails by large-scale yielding under mixed mode I/II loading. For
Fig. 13. The fracture curves for the V-notched Al 7075-T6 specimens of different notch angles and various notch radii together with the experimental results.
(b) $2\alpha = 60$ (deg.)
Fig. 13 (continued)
the sake of brevity, the procedure is presented just for the VMTS-EMC criterion. For the other criterion, similar procedure can simply be presented and used. The procedure is as follows:

1. Create a FE model for the notched component considering refined meshes at the notch neighborhood and assign the elastic modulus and Poisson’s ratio of ductile material to the model. Note that the mesh-independency of the FE model should be checked prior to the next step.

![Diagram of Fracture Initiation Angle vs. Notch Mode Mixity Parameter](image-url)

Fig. 14. The curves of FIA for the V-notched Al 7075-T6 specimens of different notch angles and various notch radii together with the experimental results.
2. Apply a load to the model equal to unity; perform linear-elastic analysis and compute the values of the NSIFs $K_V^{\rho}$ and $K_I^{\rho}$ using Eqs. (12) and (13), respectively.

3. Determine the ratio $K_V^{\rho}/K_I^{\rho}$ and draw a line starting from the point $(0,0)$ of the corresponding fracture curve and having a slope equal to the ratio.

**Fig. 14 (continued)**
4. Determine the coordinates of the intercept of the line and the fracture curve (i.e. the critical $K_V^{Ic}$ and $K_V^{IIc}$ values).

5. Multiply the two coordinates by $K_V^{Ic}$ obtained from Eq. (20) and determine the critical values of $K_V^{I}$ and $K_V^{II}$. Note that in Eq. (20), $\sigma_c = \sigma_c^f$ and $r_{cV} = r_0 + 1/2\pi (K_c/\sigma_c)^2$ should be used.
Determine the load-carrying capacity of the ductile V-notched component \( P_{cr} \) by using simply

\[
P_{cr} = \frac{(K_{Ic}^{VP})_{\text{step 5}}}{(K_{Ic}^{VP})_{\text{step 2}}}
\]

### 6. Conclusions

For the first time, ductile failure was studied experimentally and theoretically on Al 7075-T6 thin sheets weakened by blunt V-notches and subjected to mixed mode I/II loading. In the experimental part of this study, numerous experimental data were provided on the load-carrying capacity and the fracture initiation angle of V-notched Al thin sheets for different notch geometries and various mode mixity ratios. Experimental observations indicated that crack initiates from the notch border by large-scale yielding and grows rapidly till sudden fracture. Such failure behavior of notched Al sheets was
attributed to the small strain interval between the ultimate and the final rupture points in the standard tensile stress–strain curve of Al 7075-T6. The Equivalent Material Concept (EMC) was utilized in conjunction with the VMTS and VMS mixed mode brittle fracture criteria to predict the experimental results. It was found that both VMTS–EMC and VMS–EMC criteria could predict well the notch fracture toughness of Al 7075-T6 sheets as well as the fracture initiation angle. By using such criteria, engineers do not require to perform elastic–plastic analysis for estimating mixed mode I/II failure of notched ductile components encountering large-scale yielding.

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