On the Ability of the Equivalent Material Concept in Predicting Ductile Failure of U-Notches under Moderate- and Large-Scale Yielding Conditions

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NOMENCLATURE

\(d_c\) — critical distance of the mean-stress criterion measured from the notch tip;
\(d_{c,U}\) — critical distance of the mean-stress criterion measured from the coordinate origin;
\(E\) — elastic modulus;
\(K\) — strain-hardening coefficient;
\(K_{1,U,\rho}\) — mode I notch stress intensity factor (NSIF) for a U-notch;
\(K_{1c}\) — mode I notch fracture toughness for a U-notch;
\(K_c\) — fracture toughness of material;
\(K_{lc}\) — plane-strain fracture toughness of material;
\(n\) — strain-hardening exponent;
\(PS\) — point-stress;
\(r_c\) — critical distance of the PS criterion measured from the notch tip;
\(r_{c,U}\) — critical distance of the PS criterion measured from the coordinate origin;
\(r_0\) — distance between the coordinate origin and the notch tip;
SED — strain energy density;
\(\varepsilon_i^*\) — strain at crack initiation for the equivalent material;
\(\varepsilon_p\) — true plastic strain;
\(\varepsilon_{u,\text{true}}\) — engineering plastic strain at maximum load;
\(\varepsilon_u\) — true plastic strain at maximum load;
\(\varepsilon_y\) — true plastic strain at yield point;
\(\rho\) — notch radius;
\(\sigma\) — true stress;
\(\sigma_c\) — critical stress;
\(\sigma_f\) — tensile strength of the equivalent material;
\(\sigma_{00}\) — tangential stress;
\(\sigma_{00}\) — mean value of tangential stress;
\(\sigma_u\) — ultimate tensile strength;
\(\sigma_y\) — yield strength.

1. INTRODUCTION

Mechanically, brittle and ductile engineering materials exhibit different behaviors depending on their inherent characteristics. In uniaxial tension, brittle mate-
rials show a fracture surface perpendicular to the direction of the applied tensile stress. A completely linear load-displacement (or stress-strain) curve with no plastic deformation is typically recorded for brittle materials during a standard tensile test. Brittle materials fail normally by brittle fracture which takes place suddenly. Most of the characteristics of brittle materials are seen also in quasi-brittle materials except that small plastic deformations are usually recognized in quasi-brittle materials at fracture. Different from brittle and quasi-brittle materials, ductile materials show a fracture surface perpendicular to the direction of the maximum shear stress. Moderate or large amount of plasticity is seen in the stress-strain curve of ductile materials depending on their microstructure. Moreover, necking is usually recognized during tensile tests in ductile materials after which final rupture occurs.

In the presence of notches which are inevitable in design of engineering components and structures, stresses are concentrated at the notch neighborhood making this region vulnerable to failure. When notches are introduced in brittle and quasi-brittle materials, crack may initiate from the notch border without plastic deformations around the notch and fracture happens suddenly just after crack initiation, because the crack propagation is such a rapid phenomenon that it cannot be captured by naked eye. Conversely, dealing with notches introduced in ductile materials, crack initiates from the notch border by moderate- or large-scale yielding (i.e. significant plasticity) and the crack grows rapidly or slowly depending on the material characteristics.

Engineering structures are typically made of ductile materials such as metallic materials (steel, aluminium and titanium alloys etc.), polymers and polymer-based composites etc., and most of them contain notches of various shapes, especially V-, U- and O-shaped notches. Therefore, appropriate failure criteria are essentially needed for predicting the onset of crack initiation from the notch border at which the notched ductile member shows its full load-carrying capacity.

As mentioned above, significant plastic deformations exist at failure instance around notches in ductile materials and hence, the linear elastic notch fracture mechanics principles are no longer valid in failure assessments. As a result, the fracture toughness of notched ductile components is commonly evaluated by means of some well-known approaches such as the critical J-integral, the notch tip opening displacement and the resistance curve (R-curve) etc. [1] which all are complicated, relatively costly and time-consuming approaches. In an interesting work, the well-known theory of critical distances, which is basically a criterion for predicting brittle fracture in notched domains, has been extended by Susmel and Taylor [2] to ductile components obeying power-law elastic-plastic behavior by which they could estimate the load-bearing capacity of V-, U- and O-notched ductile steel plates loaded under pure mode I by tension and bending. In Ref. [2], it has been stated that it was not possible to justify the good accuracy of the theory of critical distances in predicting the experimental results under large-scale yielding (LSY) conditions around notches.

Torabi [3], however, could justify the use of the linear elastic notch fracture mechanics principles in predicting the crack initiation from the notch tip in the ductile steel plates reported in Ref. [2] by proposing the equivalent material concept (EMC). By the equivalent material concept, he could equate a ductile material having valid fracture toughness $K_{IC}$ value with a virtual brittle material having the same elastic modulus and the same fracture toughness, but various tensile strength. The experimental results reported in Ref. [2] have been well estimated by using the equivalent material concept in conjunction with the point-stress and the mean-stress brittle fracture criteria [3–5]. To check the suitability of the equivalent material concept in real engineering applications, Torabi [6] has also predicted successfully the tensile load-carrying capacity of several ductile steel bolts containing V-shaped threads using the combined EMC-MS criterion.

In this research, the aim was to examine the suitability of the EMC-PS and EMC-MS criteria in predicting the onset of tensile crack initiation from the U-notch border in ductile materials under moderate- and large-scale yielding regimes. For this purpose, first, the equivalent material concept and the closed-form expressions of the mode I notch fracture toughness (NFT) for the point-stress and mean-stress criteria were formulated. Then, the two notch fracture toughness expressions were modified by the equivalent material concept parameters and finally, two closed-form notch fracture toughness expressions were developed for the EMC-PS and EMC-MS criteria which might be capable of predicting the notch fracture toughness of U-notches in ductile materials. To verify the two combined criteria in both moderate- and large-scale yielding regimes, several rectangular thin plates weakened by a central bean-shaped slit with two U-shaped ends and made of aluminum alloys Al 7075-T6 and Al 6061-T6 were tested under tension. Experimental observations indicated that Al 7075-T6 plates fail by moderate-scale yielding while Al 6061-T6 plates by large-scale yield-
ing. It was found that the EMC-MS criterion could predict successfully the experimental results of both materials for various notch radii. Moreover, the EMC-PS criterion was found to be suitable for moderate and large notch radii, especially for Al 6061-T6 material.

2. DESCRIPTION OF THE EQUIVALENT MATERIAL CONCEPT

As elaborated also in Refs. [3–6], the equivalent material concept equates a ductile material having valid fracture toughness value with a virtual brittle material having the same elastic modulus and fracture toughness, but various tensile strength. By considering the same values of the tensile strain energy density needed for both real ductile and virtual brittle materials for the crack initiation to occur, the tensile strength of the equivalent material can be determined. The mathematical formulations of the equivalent material concept are presented below.

A typical stress-strain curve for a ductile material is shown in Fig. 1. A simple power-law expression can be written for the true stress-strain relationship in the plastic zone for a ductile material:

\[ \sigma = K \varepsilon_p^n, \]  

where \( \sigma, \varepsilon_p, K, \) and \( n \) are the true stress, the true plastic strain, the strain-hardening coefficient, and the strain-hardening exponent, respectively. The total strain energy density consisting of the elastic and plastic components can be written as

\[ (\text{SED})_{\text{tot}} = (\text{SED})_e + (\text{SED})_p = \frac{1}{2} \sigma_y \varepsilon_y + \int \sigma_d \varepsilon_p, \] 

where \( \sigma_y, \varepsilon_y \) and \( \varepsilon_p \) are the yield strength, the elastic stress at yield point and the true plastic strain at yield point, respectively. Substituting \( \varepsilon_y = \sigma_y / E \) and Eq. (1) into Eq. (2) results in

\[ (\text{SED})_{\text{tot}} = \frac{\sigma_y^2}{2E} + \frac{K}{2E} \varepsilon_p^n \varepsilon_p. \] 

By integrating the second term in Eq. (3), we have

\[ (\text{SED})_{\text{tot}} = \frac{\sigma_y^2}{2E} + \frac{K}{n+1} \left[ (\varepsilon_p)^{n+1} - (\varepsilon_p^y)^{n+1} \right]. \]  

If \( \varepsilon_p^y \) is considered to be equal to 0.002 (obtained from 0.2% offset yield strength), then

\[ (\text{SED})_{\text{tot}} = \frac{\sigma_y^2}{2E} + \frac{K}{n+1} \left[ (\varepsilon_p^{n+1}) - 0.002^{n+1} \right]. \]  

Equation (5) gives the total strain energy density till any point in the plastic region. In order to calculate the total strain energy density associated with the onset of crack initiation (i.e. the area under the standard stress-strain curve from beginning of loading to the peak point, so-called necking; see Fig. 1), one should substitute \( \varepsilon_p \) in Eq. (5) with \( \varepsilon_u \), i.e. the true plastic strain at peak point, which could simply be computed by the expression \( \varepsilon_u = \varepsilon_{u, \text{true}} = \ln(1 + \varepsilon_p) \), where \( \varepsilon_u \) is the engineering plastic strain at peak point. Therefore

\[ (\text{SED})_{\text{necking}} = \frac{\sigma_y^2}{2E} + \frac{K}{n+1} \left[ (\varepsilon_{u, \text{true}})^{n+1} - 0.002^{n+1} \right]. \]  

A typical uniaxial stress-strain curve for the equivalent material is schematically represented in Fig. 2. Here the parameters \( \varepsilon_p^f \) and \( \sigma_f \) are the strain at crack initiation (i.e., the final fracture due to the brittleness of material) and the tensile strength of the equivalent material, respectively. The strain energy density absorbed by the equivalent material till final fracture is therefore

\[ (\text{SED})_{\text{EM}} = \frac{\sigma_f^2}{2E}. \]  

According to the equivalent material concept, strain energy density values for both the real ductile and the virtual brittle materials should be identical (i.e. Eqs. (6) and (7) are identical). Thus

\[ \frac{\sigma_f^2}{2E} = \frac{\sigma_y^2}{2E} + \frac{K}{n+1} \left[ (\varepsilon_{u, \text{true}})^{n+1} - 0.002^{n+1} \right]. \]  

![Fig. 1. A stress-strain curve for a typical ductile material.](image)

![Fig. 2. A typical uniaxial stress-strain curve for the equivalent material.](image)
Ultimately, the tensile strength of the equivalent material $\sigma_t^*$ can be extracted from Eq. (8) as follows:

$$\sigma_t^* = \sqrt{\gamma^2 + \frac{2EK}{n+1} \left[ \varepsilon_{\text{true}}^{n+1} - 0.002^{n+1} \right]} \ldots \ldots (9)$$

The parameter $\sigma_t^*$ presented in Eq. (9) can be used together with the material fracture toughness ($K_f$ or $K_c$), as the two necessary inputs of different brittle fracture criteria, for predicting the crack initiation from the notch in ductile members subjected to tension (i.e., pure mode I loading).

In the next section, two well-known brittle fracture criteria, namely the point-stress and the mean-stress criteria are described and two expressions are achieved for predicting the mode I notch fracture toughness of U-shaped notches. By using the equivalent material concept in conjunction with the point-stress and mean-stress criteria, two closed-form notch fracture toughness expressions are proposed to predict the tensile load-carrying capacity of U-notched ductile components encountering moderate-scale and large-scale yielding failure regimes.

### 3. DUCTILE FAILURE PREDICTIONS

#### 3.1. Linear-Elastic Stress Distribution around a U-Notch under Mode I Loading

A U-notch is shown in Fig. 3 together with its polar and Cartesian coordinate systems. As seen in Fig. 3, the origin of the coordinates locates at the distance $r_0 = \rho/2$ behind the notch tip on the notch bisector line [7, 8]. The parameter $\rho$ is the notch tip radius.

The first ones who derived the elastic stress field equations for blunt cracks have been probably Creager and Paris [7]. Lazzarin and Tovo [8] have also determined the elastic stress distributions in the vicinity of cracks and notches. Their expressions coincide well with those presented in Ref. [7]. By assuming that a U-notch is geometrically a blunt crack, the tangential stress on the U-notch bisector line ($\sigma_{00}(r)$) can be written in terms of $r$ (i.e. the distance from the origin of the polar coordinate system; see Fig. 3) as [7, 8]:

$$\sigma_{00}(r) = \frac{K_{1,U,\rho}}{\sqrt{2\pi r}} \left[ 1 + \frac{r}{2r} \right],$$

where $K_{1,U,\rho}$ is the mode I notch stress intensity factor (NSIF).

#### 3.2. Point-Stress Criterion

The point-stress criterion states that brittle fracture takes place for a notched member when the tangential stress at a specified critical distance ahead of the notch tip attains the critical stress. Figure 4a shows a U-notch including the critical distances of the point-stress criterion.

The parameters $r_c$ and $r_{c,U}$ in Fig. 4a are the critical distances of the point-stress criterion measured from the notch tip and from the coordinate origin, respectively. According to the point-stress criterion, the following conditions should be satisfied at the onset of mode I brittle fracture: (i) the tangential stress at the critical distance should be equal to the critical stress $\sigma_c$ which is normally assumed to be a material property; (ii) the NSIF $K_{1,U,\rho}$ would attain its critical value

![Fig. 3. A U-notch together with its coordinate systems.](image)

![Fig. 4. U-shaped notches and the critical distances of the PS (a) and the MS (b) criteria.](image)
$K_{lc\rho}^U$, called the mode I notch facture toughness; and (iii) the fracture initiation angle is expected to be equal to zero because of symmetry in geometry and loading. Hence, the three conditions above can be formulated as follows:

$$\sigma_{00}(r_c, U) = \sigma_c, \quad K_{lc\rho}^U = K_{lc\rho}^U. \quad (11)$$

Substituting Eq. (11) into Eq. (10) results in

$$\sigma_c = \frac{K_{lc\rho}^U}{\sqrt{2\pi r_c}} \left[ 1 + \frac{\rho}{2r_c} \right]. \quad (12)$$

As seen in Fig. 4a, the following expression is valid:

$$r_{c,U} = \frac{\rho}{2} + r_c. \quad (13)$$

Substituting Eq. (13) into Eq. (12) and extracting, the notch fracture toughness $K_{lc\rho}^U$ would be

$$K_{lc\rho}^U = \frac{\sqrt{\pi(\rho + 2r_c)}\sigma_c}{1 + \rho/(\rho + 2r_c)}. \quad (14)$$

The sole unknown parameters in Eq. (14) are $r_c$ and $\sigma_c$, which both are assumed to be the material properties. Once these critical values are known, the notch fracture toughness for U-notches can simply be computed. Critical distance $r_c$ for blunt V- and U-notches can well be assumed to be equal to that for a sharp crack [9–18], i.e.

$$r_c = \frac{1}{2\pi} \left( \frac{K_{lc}}{\sigma_c} \right)^2,$$

where $K_{lc}$ and $\sigma_c$ are the plane-strain fracture toughness and the tensile strength of brittle material, respectively.

### 3.3. Mean-Stress Criterion

According to the mean-stress criterion, brittle fracture occurs when the average of the tangential stress over a specified critical distance ahead of the notch tip reaches to the material critical stress. Figure 4b depicts a U-notch including the critical distances of the mean-stress criterion. The parameters $d_c$ and $d_{c,U}$ are the critical distances of the mean-stress criterion measured from the notch tip and from the coordinate origin, respectively. The average stress over the critical distance $d_c(\sigma_{00})$ can be written as [12]:

$$\overline{\sigma}_{00} = \frac{d_{c,U}}{d_c} \int_{\rho/2}^{r_{c,U}} \sigma_{00}(r) dr. \quad (15)$$

The idea of stress averaging has been previously proposed by Sewelyn [19] who referred to a work by Novozhilov [20]. According to the mean-stress criterion, $\sigma_{00}$ should attain the critical stress $\sigma_c$ at fracture instance. Hence, substituting $\sigma_{00}$ from Eq. (10) into Eq. (15) and considering that the expression $\overline{\sigma}_{00} = \sigma_c$ is valid at fracture instance, we have

$$\frac{1}{d_c} \int_{\rho/2}^{r_{c,U}} \sigma_{00}(r) dr = \sigma_c. \quad (16)$$

Integrating Eq. (16) and considering that $K_{lc\rho}^U$ is equal to $K_{lc\rho}^U$ at brittle fracture instance, we have

$$K_{lc\rho}^U \left( \frac{2\sqrt{d_{c,U}} - \rho}{\sqrt{d_{c,U}}} \right) = d_c \sigma_c. \quad (17)$$

Noting that the expression $d_{c,U} = \rho/2 + d_c$ is valid (see Fig. 4b), the mode I notch fracture toughness can be extracted from Eq. (17) as:

$$K_{lc\rho}^U = \frac{\sqrt{2\pi \sigma_c d_c}}{2(\rho/2 + d_c)}, \quad (18)$$

The critical distance $d_c$ can be assumed to be equal to

$$d_c = \frac{2}{\pi} \left( \frac{K_{lc}}{\sigma_c} \right)^2$$

for a sharp crack, as previously proposed and strongly demonstrated [9–18].

In the next subsection, the equivalent material concept is employed in conjunction with the point-stress and the mean-stress brittle fracture criteria for developing two closed-form expressions for the notch fracture toughness of ductile U-notched members that fail by considerable plastic deformations around the notch tip.

### 3.4. The Notch Fracture Toughness Predictions by the EMC-PS and EMC-MS Criteria

As stated in Sect. 2, a ductile material can be replaced according to the equivalent material concept with a virtual brittle material with the same elastic modulus and fracture toughness, but with different tensile strength denoted by $\sigma_T^*$ (see Eq. (9)). Thus, the notch fracture toughness expressions for the EMC-PS and the EMC-MS failure criteria can be simply resulted by replacing $\sigma_c$ with $\sigma_T^*$ in Eqs. (14) and (18), respectively. Consequently, the notch fracture toughnesses are:

$$K_{lc\rho}^U = \frac{\sqrt{\pi(\rho + 2r_c)}\sigma_T^*}{1 + \rho/(\rho + 2r_c)}, \quad r_c = \frac{1}{2\pi} \left( \frac{K_{lc}}{\sigma_T^*} \right)^2, \quad (19)$$

$$K_{lc\rho}^U = \frac{\sqrt{2\pi \sigma_T^* d_c}}{2(\rho/2 + d_c)}, \quad d_c = \frac{2}{\pi} \left( \frac{K_{lc}}{\sigma_T^*} \right)^2. \quad (20)$$
It is necessary to highlight that for thin components, the plain-strain fracture toughness $K_{ic}$ may be replaced with the fracture toughness $K_c$. In order to check the suitability of the EMC-PS and EMC-MS criteria for moderate- and large-scale yielding, several tensile fracture tests were carried out on U-notched rectangular plates made of two various ductile materials. The next section presents the details of the experiments.

4. EXPERIMENTAL PROGRAM

In order to verify the EMC-PS and EMC-MS criteria for both moderate- and large-scale yielding failure regimes, several new mode I fracture tests were conducted on U-notched specimens made of two various ductile materials. Details are presented below.

4.1. Material

The materials selected were two aluminum alloys Al 7075-T6 and Al 6061-T6 which exhibit moderate and large plastic deformations under tension, respectively. The materials were provided in the form of plates of 2 and 4 mm thick, respectively. Three standard tests were performed to determine the material properties.

They were the tensile test according to ASTM E8 [21], the Poisson ratio test according to ASTM E132-04 [22], and the fracture toughness test in accordance with ASTM B646-12 [23]. The engineering and true stress-strain curves for the tested materials are shown in Fig. 5. The mechanical properties of Al 7075-T6 and Al 6061-T6 are also presented in Table 1. Considering the values presented in Table 1, the values of the parameters ($\sigma_f$, $r_c$, $d_c$) are calculated from Eqs. (9), (19), and (20) to be equal to (1845 MPa, 0.117 mm, 0.467 mm) and (1066 MPa, 0.2 mm, 0.8 mm) for Al 7075-T6 and Al 6061-T6, respectively.

4.2. Specimen

The specimen was a rectangular plate weakened by a central horizontal bean-shaped slit with two U-shaped ends. A distributed tensile load is monotonically applied to the specimen by the test apparatus and hence, the two U-ends are subjected to pure mode I loading (i.e., pure opening). Figure 6 depicts schematically the U-notched specimen including its geometric parameters.

The parameters $\rho$, $2a$, $L$, $W$, and $P$ in Fig. 6 are the notch radius, twice the notch length (i.e. the total bean-shaped slit length), the specimen length, the specimen

| Table 1. The mechanical properties of Al 7075-T6 and Al 6061-T6 |
|----------------|----------------|----------------|
| Material property | Al 7075-T6 | Al 6061-T6 |
| Elastic modulus $E$, GPa | 71 | 67 |
| Poisson’s ratio | 0.33 | 0.33 |
| Tensile yield strength, MPa | 521 | 276 |
| Ultimate tensile strength, MPa | 583 | 292 |
| Elongation at break, % | 5.8 | 11 |
| Engineering strain at maximum load | 0.047 | 0.034 |
| True fracture stress, MPa | 610 | 299 |
| Fracture toughness $K_c$, MPa $\times m^{1/2}$ | 50 | 38 |
| Strain-hardening coefficient, MPa | 698 | 314 |
| Strain-hardening exponent | 0.046 | 0.021 |
width, and the remotely applied tensile load, respectively. The specimen dimensions considered in the fracture experiments were: \( \rho = 0.5, 1.0, \) and \( 2.0 \) mm, \( 2a = 25 \) mm, \( L = 160 \) mm and \( W' = 50 \) mm. The thickness was also equal to 2 and \( 4 \) mm for the Al 7075-T6 and Al 6061-T6 specimens, respectively. In order to check the repeatability of the experiments, three tests were carried out for each of the notch radius and totally, 18 fracture tests were conducted in the present experimental program (9 tests for each material).

For fabricating the test samples, first, Al 7075-T6 plate of \( 2 \) mm thick and Al 6061-T6 plate of \( 4 \) mm thick were provided. Then, the sketch of the specimens was created by drawing software and given to a high-precision 2D water jet cutting machine. The specimens were ultimately cut from the aluminum plates. The tests were conducted under displacement-control conditions with a speed of \( 1 \) mm/min providing monotonic loading conditions.

### 4.3. Experimental Results

Figure 7 represents sample load-displacement curves resulted from testing the U-notched specimens.

As seen in Fig. 7, fundamentally different behaviors were obtained for Al 7075-T6 and Al 6061-T6 specimens. For Al 7075-T6 specimen, a clear but small nonlinear portion is seen in the curve between the end of the proportional limit and the peak demonstrating considerable plastic deformations around the notch tip before the crack initiation. It is essential to notice that this amount of plastic deformations could not be resulted from a small-scale yielding (SSY) regime. Since in the SSY, no clear nonlinear portion is normally realized in the load-displacement curve. In other words, the SSY curve is usually very similar to that of the ideally brittle fracture. It is also seen in the curve of Al 7075-T6 that the load drops suddenly to zero just the load reaches to the peak meaning that the crack propagation takes place so rapidly which could not be detected by naked eye. For Al 6061-T6, however, a relatively large nonlinear portion is seen in the load-displacement curve suggesting that the ligament encounters large-scale yielding conditions. Different from the curve of Al 7075-T6, the load decreases gradually from the peak (i.e. the onset of crack initiation from the notch tip) to zero in Al 6061-T6 curve meaning that the crack propagation is such a slow and stable phenomenon that it could be detected by the naked eye.

In order to measure experimentally the size of plastic zone around the U-notch at crack initiation instance, several movies were provided during loading. Experimental observations indicated that permanent deformations could be detected during loading by the naked eye. To capture approximately the plastic zone size at the onset of crack initiation, the slow motion of the movie was utilized. Since for Al 6061-T6 the crack initiation and propagation phases are stable, the crack initiation instance could precisely be detected also with-

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**Fig. 6.** The U-notched specimen including its geometric parameters.

**Fig. 7.** Sample load-displacement curves resulted from testing the U-notched specimens: Al 7075-T6 (a), Al 6061-T6 (b).
out slow motion. It was found from the movie analyzer software that about 20% and 80% of the ligament experiences plastic deformations at failure for Al 7075-T6 and Al 6061-T6 specimens, respectively. Consequently, Al 7075-T6 specimens suggest moderate-scale yielding failure regime while Al 6061-T6 ones propose large-scale yielding.

The experimentally obtained critical loads (associated with the crack initiation instance) are presented in Tables 3 and 4 for the U-notched Al 7075-T6 and Al 6061-T6 specimens, respectively. Note that $P_i$ ($i = 1, 2, 3$) denotes each failure load in the repeated tests and $P_{av}$ is the average of the three failure loads. As seen in Tables 3 and 4, for all of the notch radii, the repeated test results are very close together demonstrating the precise specimen fabrication, high test performance and especially the ductility of the failure (since considerable scatter is usually seen in brittle fracture experiments).

In the next section, the experimental results presented in Tables 3 and 4 are used to verify the EMC-PS and EMC-MS criteria.

5. RESULTS AND DISCUSSION

In order to compare the theoretical results of the EMC-PS and EMC-MS criteria with the experimental results presented in Tables 2 and 3, the critical loads should be converted to the corresponding values of the

<table>
<thead>
<tr>
<th>$\rho$, mm</th>
<th>$P_{1}$, N</th>
<th>$P_{2}$, N</th>
<th>$P_{3}$, N</th>
<th>$P_{av}$, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>25 424</td>
<td>24 982</td>
<td>25 700</td>
<td>25 369</td>
</tr>
<tr>
<td>1.0</td>
<td>27 668</td>
<td>27 083</td>
<td>28 043</td>
<td>27 598</td>
</tr>
<tr>
<td>2.0</td>
<td>30 236</td>
<td>30 475</td>
<td>30 041</td>
<td>30 250</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$\rho$, mm</th>
<th>$P_{1}$, N</th>
<th>$P_{2}$, N</th>
<th>$P_{3}$, N</th>
<th>$P_{av}$, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>29 459</td>
<td>29 191</td>
<td>29 338</td>
<td>29 329</td>
</tr>
<tr>
<td>1.0</td>
<td>31 181</td>
<td>30 283</td>
<td>31 001</td>
<td>30 822</td>
</tr>
<tr>
<td>2.0</td>
<td>33 068</td>
<td>33 300</td>
<td>33 366</td>
<td>33 245</td>
</tr>
</tbody>
</table>

Fig. 8. A U-notched specimen meshed in finite element software.
Table 4. Theoretical values of the mode I notch fracture toughness together with the mean experimental notch fracture toughnesses for Al 7075-T6 specimens including discrepancies

<table>
<thead>
<tr>
<th>ρ, mm</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
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</thead>
<tbody>
<tr>
<td>Mean experimental notch fracture toughness</td>
<td>73.1</td>
<td>75.8</td>
<td>85.1</td>
</tr>
<tr>
<td>Notch fracture toughness for ECM-PS criterion</td>
<td>52.7</td>
<td>63.4</td>
<td>81.5</td>
</tr>
<tr>
<td>Notch fracture toughness for ECM-MS criterion</td>
<td>61.9</td>
<td>71.4</td>
<td>88.6</td>
</tr>
<tr>
<td>Discrepancy for ECM-PS criterion, %</td>
<td>28</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>Total discrepancy 17.3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discrepancy for ECM-MS criterion, %</td>
<td>15.3</td>
<td>8.4</td>
<td>4.0</td>
</tr>
<tr>
<td>Total discrepancy 9.2%</td>
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</table>

Notch fracture toughness $K_{lc}^{U,\rho}$. A finite element (FE) model should be created for each U-notched specimen. Then, the critical load presented in Tables 2 and 3 is applied to the FE model and the linear-elastic tensile stress at the notch tip is calculated. The computed stress is finally substituted into Eq. (21) instead of $\sigma_{\theta\theta}(\rho/2)$ and the value of the experimental critical NSIF (i.e., the notch fracture toughness) is computed

$$K_{lc}^{U,\rho} = \frac{\sqrt{\pi \rho} \sigma_{\theta\theta}(\rho/2)}{2}. \quad (21)$$

Now, the experimental notch fracture toughnesses can easily be compared with the theoretical ones resulted from Eqs. (19) and (20) for the ECM-PS and ECM-MS criteria, respectively. A U-notched specimen meshed in FE software is represented in Fig. 8. The meshes are refined at the notch tip vicinity due to the high level of stress gradient.

Tables 4 and 5 present the theoretical values of the mode I notch fracture toughness $K_{lc}^{U,\rho}$ corresponding to the ECM-PS and ECM-MS criteria together with the mean experimental notch fracture toughnesses for Al 7075-T6 and Al 6061-T6 specimens, respectively.

It is seen in Table 4 that the mean discrepancies between the theoretical and the mean experimental results are equal to 17% and 9.2% for the ECM-PS and ECM-MS criteria, respectively, meaning that in general, the ECM-MS criterion was successful in predicting the notch fracture toughness of Al 7075-T6 specimens while the ECM-PS criterion was not. Moreover, according to Table 5, both the ECM-PS and ECM-MS criteria are successful on Al 6061-T6 specimens. It can easily be found in Table 4 that for the notch radii 0.5 and 1.0 mm, the ECM-MS criterion predicts the experimental results of Al 7075-T6 much better than the ECM-PS criterion. For the notch radius of 2 mm, identical accuracies are obtained for the two criteria. The values presented in Table 5 for Al 6061-T6 indicate that for the notch radius of 0.5 mm, the ECM-MS criterion is more accurate than the ECM-PS criterion, while for the other notch radii, the experimental results locate between the two theoretical results (below

![Fig. 9](image-url) Sample plastic zones around the U-notch tip at crack initiation instance corresponding to Al 7075-T6 specimen of ρ = 1 mm (a) and Al 6061-T6 specimen of ρ = 2 mm (b).
the EMC-MS and over the EMC-PS) with almost identical distances from the two criteria.

As can be seen in Tables 4 and 5, for the entire values of notch radius, the EMC-MS criterion provides larger estimates than the EMC-PS criterion. It has been reported in some references that for relatively small critical distances, like those in the present study, the mean stress criterion estimates the notch fracture toughness significantly higher than the point stress criterion \([9, 14, 15]\). This is because, as the critical distance decreases, the tensile stress at the critical distance \(r_c\) becomes far from the average stress over the critical distance \(d_c\) resulting in significantly different predictions.

As previously mentioned, the experimental observations indicated that almost 20% and 80% of the ligament experience plastic deformations at the onset of crack initiation from the notch tip for Al 7075-T6 and Al 6061-T6 specimens, respectively. With the aim to confirm the experimental observations, the tensile tests on U-notched specimens were simulated and the elastic-plastic FE analysis was carried out. Note that the true stress-strain curve of materials presented in Fig. 5 was utilized in the analysis and the mean experimental failure load was applied to the FE model. Figure 9 represents sample plastic zones around the U-notch at crack initiation instance associated with Al 7075-T6 specimen of \(\rho = 1\) mm and Al 6061-T6 specimen of \(\rho = 2\) mm. Note that the plastic zones were obtained on the basis of the von Mises stress distributions. As seen in Fig. 9, 2.3 mm (about 18%) and 9.3 mm (about 75%) of the ligament encounters plastic deformations in Al 7075-T6 and Al 6061-T6 specimens, respectively which are very close to the percents obtained from the experimental observations. For the other notch radii, similar results were achieved. The sizes of the plastic zones obtained prove the moderate-scale yielding and large-scale yielding failure regimes for the tested Al 7075-T6 and Al 6061-T6 specimens, respectively.

For engineering design purposes, simple, accurate and rapid failure criteria are interested and essentially needed. Dealing with ductile failure prediction of cracked and notched components, elastic-plastic analysis are usually carried out and criteria like J-integral, CTOD and CTOA etc. are utilized. Two important disadvantages of elastic-plastic analysis are that they are time-consuming and relatively complicated. To avoid elastic-plastic analysis, it was tried in the present study to estimate the load-carrying capacity of ductile components weakened by U-notches by using two stress-based brittle fracture criteria requiring only the linear-elastic stress analysis. By using the EMC-PS and EMC-MS criteria, there is no need to conduct elastic-plastic analysis for predicting the tensile load-carrying capacity of U-notched ductile components that fail in the moderate-scale and large-scale yielding regimes. It is useful to note that the equivalent material concept is capable of being simply used in conjunction with the other brittle fracture criteria in the context of notch mechanics, like the strain energy density \([24–30]\) and the cohesive crack model \([31–34]\) etc., for predicting the crack initiation from the notch border in ductile materials. This may be a topic for future investigations.

6. CONCLUSIONS

By using the equivalent material concept, a ductile material was equated with a virtual brittle material allowing one to employ brittle fracture criteria for predicting ductile failure of U-notched components encountering considerable plastic deformations around the notch border. The equivalent material concept was used in conjunction with the point stress and the mean stress criteria to predict the mode I notch fracture toughness for U-shaped notches. To check the suitability of the EMC-PS and EMC-MS criteria in both moderate- and large-scale yielding (i.e. moderate-scale and large-scale yielding) failure regimes, fracture experiments were performed on U-notched rectangular plates made of aluminum alloys Al 7075-T6 and Al 6061-T6 for various notch radii. It was resulted from both the experimental observations and the numerical finite element simulations that about 20% and 80% of the ligament experiences plastic deformations at failure for Al 7075-T6 and Al 6061-T6 specimens, respectively demonstrating moderate-scale yielding failure for Al 7075-T6 and large-scale yielding failure for Al 6061-T6 specimens. Comparison of the theoretical and experimental results indicated that while the EMC-MS criterion was successful for both materials and the entire notch radii, the EMC-PS criterion was not successful for small notch radii, particularly for Al 7075-T6 specimens.

REFERENCES

3. Torabi, A.R., Estimation of Tensile Load-Bearing Capacity of Ductile Metallic Materials Weakened by a


