A proposed mathematical model for exergy analysis of an infrared (IR) drying process

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Abstract: In this paper, a new mathematical model was proposed for exergy analysis of an IR drying process. The exergy balance equation was derived based on the IR heating principle and accordingly was applied for determining the exergetic parameters, i.e., exergy destruction and exergy efficiency. A simple heat and mass transfer model was taken into account to ascertain the applicability of the presented model to actual IR drying process at different drying conditions. An illustrative example was numerically solved for a typical food during IR drying process. This model can be offered as an adaptable framework to scrutinise the economical and environmental concerns associated with the application of the IR drying method, to compare the exergetic performance of the IR drying process with the other available drying techniques, to optimise the design of IR drying facilities and their elements and to recognise the suitable applications and optimal configurations for IR drying equipments.

Keywords: exergy analysis; exergy efficiency; exergy destruction; infrared (IR) drying process; mathematical model.


Biographical notes: Mortaza Aghbashlo is an Assistant Professor at the Faculty of Agricultural Engineering and Technology, University of Tehran, Iran. He received his BSc at Agricultural Machinery Engineering Department of Tabriz University in 2006 and his MSc and PhD at the University of Tehran in 2008 and 2012, respectively. He received University of Tehran’s award for Excellence in Research in 2008 and 2012. His research interest is concerned primarily with energy and exergy analyses of food processing facilities and renewable energy systems.

1 Introduction

Drying process is one of the most extensively employed unit operations in many industries to ensure product stability, to facilitate product handling, to reduce product
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bulk and volume, to discount storage cost, to reduce physiochemical changes during storage and to prevent microbiological activity and growth in the case of biological materials (Aghbashlo et al., 2013). Commonly, the well-known convective or conductive drying techniques using the heated medium or surface are applied for drying of the wet products. However, these methods suffer from several drawbacks such as high energy consumption, long drying time and low quality end-product. The problems associated with these drying techniques can be remarkably overcome if other heating techniques such as microwave (MW) and infrared (IR) heating modes are applied for drying purposes. It is well-documented that the processing time and energy demand of the drying process can be significantly reduced by applying the IR heater. It can be attributed to the direct penetrating of the electromagnetic wave radiated by the IR source into the product and subsequent efficacious absorption of its energy by molecules of the product without loss to the ambient (Nimmol et al., 2007). However, energy utilisation of the drying process is well-known to be a problematic issue of drying technology. This is because of the fact that the latent heat of water evaporation is remarkably high and the materials to be dried absorb merely a small fraction of the supplied energy by drying medium or IR heater. This is why there is an increasing demand for innovative engineering approaches and new drying techniques to save the energy and reduce the manufacturing costs in drying technology.

Although, energy analysis, based on the first law of thermodynamics, has been routinely used for analysing different energy conversion processes in the manufacturing industry, but it is unable to distinguish the quality of different energy (Nazghelichi et al., 2011). To overcome the drawbacks of the energy analysis, a relatively new version of thermodynamic analysis, namely exergy analysis, is applied to the process analysis within the last years, which gives a more realistic picture of the process. Simply speaking, exergy analysis refers to the thermodynamic analysis of the energy system using the integrated fundamentals of conservation of the mass and energy along with the second law of thermodynamics. Therefore, exergy analysis can provide an alternative and promising means of diagnosing the positions, kinds and quantities of wastes and losses. As well, it can ascertain whether or not and by how much it is feasible to develop more efficacious thermal equipments by minimising the sources of available inefficiencies (Aghbashlo et al., 2013).

In recent years, a considerable amount of theoretical and experimental works have been published on the exergy analysis of drying processes and systems such as fixed-bed experimental or industrial-scale hot air dryer (Dincer and Sahin, 2004; Ozgener and Ozgener, 2006; Colak and Hepbasli, 2007; Cay et al., 2007, 2009, 2010a, 2010b; Aghbashlo et al., 2008, 2009; Ozgener and Ozgener, 2009a; Coskun et al., 2009; Erbay and Icier, 2009b; Hancioglu et al., 2010; Dincer, 2011; Erbay and Icier, 2011; Colak et al., 2013), fluidised bed drying (Syahrul et al., 2002; Syahrul et al., 2003; Nazghelichi et al., 2010; Khanali et al., 2013), spray drying (Aghbashlo et al., 2012a, 2012b; Erbay and Koca, 2012a, 2012b), heat pump drying with various heating source (Kuzgunkaya and Hepbasli, 2007a, 2007b; Colak et al., 2008; Erbay and Icier, 2009a; Gungor et al., 2011; Ganjehsarabi et al., 2014), solar drying (Akpinar and Sarsilmaz, 2004; Celma and Cuadros, 2009; Chowdhury et al., 2011), greenhouse drying (Tiwari et al., 2009; Ozgener and Ozgener, 2009b), freeze drying (Bruttini et al., 2001; Liapis and Bruttini, 2008; Liu et al., 2008; Fissore et al., 2014) and vacuum drying (Dikmen et al., 2012). The conclusions of these studies have revealed that the exergy analysis is becoming very useful tool in optimising and designing the drying processes and facilities.
Although a number of researches have been carried out on the IR drying process (Wang, 2002; Ranjan et al., 2002; Sharma et al., 2005; Doymaz, 2012), the focus of those researches was mainly on its feasibility, modelling and simulation. Nonetheless, for conceptual and comprehensive analysis of the IR drying process with respect to the energy issue, innovative engineering tools such as exergy analysis must be applied. As such, the current survey is directed at the presentation of a new mathematical model for exergy analysis of the IR drying process. An illustrative example using a simplified heat and mass transfer model was applied to acquire the required data for exergy analysis of the IR drying process of a typical foodstuff. To the best of the author’s knowledge, this research is the first work presenting the exergetic modelling of the IR drying process. This model will permit designers and engineers to determine the optimum design and operating parameters of the industrial-scale IR dryers including heater (emitter) type and location, heater (emitter) temperature, drying air flow rate and other drying variables to obtain high-quality dried product together with high exergy efficiency and sustainability index.

2 Theoretical consideration

2.1 Radiation, reflection, transmission, emission and absorption of IR energy

Thermal (IR) radiation can be incidental on the wet product, it can transmit through the product, and it can be also reflected and absorbed by the product being heated or dried (Figure 1). It should be mentioned that all bodies at a temperature above absolute zero emit the IR energy.

Figure 1 Schematic illustration of energy balance for a piece of the wet material underlying the IR radiation

1) Incident IR energy
2) Absorbed component of the IR energy
3) Sum of reflected component and emitted IR energy
4) Transmitted component of the IR energy
By energy balance on a piece of the product receiving the IR energy, the magnitude of absorbed IR energy by product can be expressed as:

\[ IR_{abs} = IR_{inc} - IR_{ref} - IR_{emit} - IR_{tran} \]  

(1)

Generally, the absorbed component of the IR energy or IR transfer rate to the product can be approximated by taking into account the dryer as a three-surface enclosure (infrared emitter, drying chamber and wet material) with a radiating network (Meeso et al., 2007).

\[ IR_{abs} = \frac{1 - \varepsilon_R^1 + \sigma(T_R^1 - T_P^1)}{A_R^1\varepsilon_R^1 + 1/(1/A_R^1 F_{IR-R-P}) + 1/(1/A_R^1 F_{IR-DC}) + (1/A_R^1 F_{P-DC})} + \frac{1 - \varepsilon_P}{A_R^1\varepsilon_P} \]

2.2 Exergetic balance equation

Figure 2 manifests a schematic illustration of drying chamber via IR radiator as a heating source with input and output terms.

**Figure 2** Schematic illustration of an IR drying process

According to Figures 1 and 2, the exergy balance equation for the IR drying process on known control volume can be written as follows:

\[ \frac{m_f (ex_{P,4} - ex_{P,2})}{\Delta t} = \dot{E}_{x_{s,4}} - \dot{E}_{x_{s,1}} + \dot{E}_{X_{IR,inc}} - \dot{E}_{X_{IR,ref}} - \dot{E}_{X_{IR,emit}} = -\dot{E}_{X_{IR,tran}} + \dot{E}_{x_{n}} + \dot{E}_{X_{tgt}} - \dot{E}_{x_{ds}} \]  

(3)

or

\[ \frac{m_f (ex_{P,4} - ex_{P,2})}{\Delta t} = \dot{E}_{x_{s,4}} - \dot{E}_{x_{s,1}} + \dot{E}_{x_{abs}} - \dot{E}_{x_{n}} - \dot{E}_{x_{ds}} \]  

(4)

The specific exergy of the product at the inlet and outlet conditions can be determined as follows:
The exergy transfer rates due to the inlet dry air and outlet moist air can be specified using the following equation (Dincer and Sahin, 2004):

$$\dot{E}_a = m_a \left[ \left( C_a + \omega_a C_r \right) (T_a - T_o) - T_o \ln \frac{T_a}{T_o} \right]$$

(5)

Based on the mass conversation principle, the following equation can be written for inlet dry air and outlet moist air:

$$m_{a,1} = m_{a,3} = m_a$$

(7)

The humidity ratio of vented air from drying chamber can be obtained using the water balance equation for drying process.

$$\omega_{a,1} m_{a,1} + \omega_{a,2} m_{a,2} = \omega_{a,3} m_{a,3} + \omega_{a,4} m_{a,4}$$

(8)

The infrared heater can be assumed as a non-black body with an emissivity factor of $\varepsilon_{ir}$. Thus, its exergy flux $E_{exir}$ at a known temperature can be calculated as follows (Wright et al., 2002):

$$E_{exir} = \sigma T_{ir}^4 \left( \epsilon_{ir} + \frac{1}{3} \left( \frac{T_o}{T_{ir}} \right)^4 - \frac{4}{3} \epsilon_{ir}^{1/4} \left( \frac{T_o}{T_{ir}} \right) \right).$$

(9)

Moreover, the energy flux $E_{enir}$ of an infrared emitter can be obtained as follows:

$$E_{enir} = \epsilon_{ir} \sigma T_{ir}^4$$

(10)

It is often convenient to relate the exergy value of a system to its energy value during performing an exergy analysis. This can be carried out by defining a quality factor $\beta$ for the energy quantity as the ratio of the exergy value to the energy value. The quality factor of an IR radiation can be therefore determined as

$$\beta = 1 + \frac{1}{3} \left( \frac{T_o}{T_{ir}} \right)^4 \epsilon_{ir}^{-1} - \frac{4}{3} \epsilon_{ir}^{-0.25} \left( \frac{T_o}{T_{ir}} \right)$$

(11)

Therefore,

$$\dot{E}_{exir} = \beta \times IR_{abs}$$

(12)

The rate of exergy utilised for drying of the wet product can be determined as follows:
A proposed mathematical model for exergy analysis

\[
\dot{E}_{x, e} = \left(1 - \frac{T_e}{T_p}\right) \dot{Q}_{e}
\]

(13)

The rate of heat transfer due to the moisture evaporation can be calculated as follows:

\[
\dot{Q}_{e} = (\dot{m}_{w,e})_e h_{fg}
\]

(14)

The mass flow rate of the evaporated water can be:

\[
(\dot{m}_{w,e})_e = \dot{m}_{w,2} - \dot{m}_{w,4}
\]

(15)

Furthermore, the exergy rate due to the heat loss via drying chamber can be identified as

\[
\dot{E}_{x, l} = \left(1 - \frac{T_e}{T_{DC}}\right) \dot{Q}_{l}
\]

(16)

The heat loss to the ambient can be computed as follows:

\[
\dot{Q}_{l} = U A_{DC} (T_{DC} - T_0)
\]

(17)

The exergy efficiency of IR drying process (\(\eta_{x,e}\)) can be obtained by dividing the invested exergy for water evaporation to the exergy supplied by inlet air plus the IR exergy absorbed by product:

\[
\eta_{x,e} = \frac{\dot{E}_{x,e}}{\dot{E}_{x,1} + \dot{E}_{x,abs}} \times 100
\]

(18)

2.3 Heat and mass transfer model for IR drying process

In this study, a lumped parameter simplified heat and mass transfer model was applied to compile the required data for exergy analysis of the IR drying process of a typical foodstuff. For this purpose, the simple mathematical model developed by Lu et al. (1999) for microwave drying process was modified according to the IR drying principles:

\[
\rho_p C_p V_p \frac{dT_p}{dt} = IR_{abs} - \left[ \bar{h}_e h_{fg} A_p (\bar{M}_p - M_{\infty}) + \bar{h}_p A_p (\bar{T}_p - T_0) \right]
\]

(19)

The average temperature of product was then obtained as follows:

\[
\left( T_p \right)_{\text{avg}} = \left( T_p \right)_i + \frac{1}{(\rho_p) (C_p) (V_p)} \left\{ (IR_{abs}) - \left[ (\bar{h}_e) (h_{fg}) (A_p) ((\bar{M}_p)_i - M_{\infty}) \right. \right.

\left. \left. + (\bar{h}_p) (A_p) ((\bar{T}_p)_i - T_0) \right] \Delta t \right\}
\]

(20)

The density of material at the inlet and outlet conditions was identified using following equation (Choi and Okos, 1986):
\[
\frac{1}{(\rho_r)_i} = \sum_{i=1}^{n} \left( \frac{(X)_i}{(\rho)_i} \right) \tag{21}
\]

The specific heat of product at the inlet and outlet conditions was computed by Choi and Okos's proposed approach (Choi and Okos, 1986):

\[
(C_p)_i = \sum_{i=1}^{n} (X)_i (C_i) \tag{22}
\]

The wet material was assumed to be spherical with uniform shrinkage during the drying process. Thus, the following simple mathematical model for apparent shrinkage reported by Michailidis et al. (2008) was applied for approximating the volume, equivalent diameter and surface area of a typical food during IR drying process.

\[
(V_p)_i = \left( \frac{(MC_p)_i + 0.8}{MC_o + 0.8} \right) V_o \tag{23}
\]

Additionally, the density of the material was determined as follows:

\[
(\rho_r)_i = \frac{(m_p)_i}{(V_p)_i} \tag{24}
\]

Moreover, for laminar flow, the Nusselt number \( \overline{(Nu)} \) and accordingly \( \overline{N}_r \) for all Prandtl numbers was obtained as follows (Zheng et al., 2008):

\[
\overline{(Nu)} = \frac{\overline{N}_r}{(\overline{D}_{ct})_i} (\overline{D}_{ct})_i \rho_v \left( 0.4 + \frac{\text{Re}_{ct}^{1/2} + 0.06 \text{Re}_{ct}^{1/3}}{\Pr^{2/3}} \right) \Pr^{2/3} \left( \frac{\mu_a}{(\mu_r)_i} \right)^{1/4} \tag{25}
\]

where \( \mu_a \) and \( \mu_r \) are the dynamic viscosity of air at the inlet air and material temperatures, respectively.

As well, Prandtl number \( \Pr \) and Reynolds number \( \text{Re}_{ct} \) can be calculated as

\[
Pr = \frac{\nu}{\alpha_a} = \frac{C_v \mu_a}{k_a} \tag{26}
\]

\[
\text{Re}_{ct} = \frac{\rho_a v_a (\overline{D}_{ct})_i}{\mu_a} \tag{27}
\]

The heat and mass transfer analogy was used for computing the surface mass transfer coefficient as follows:

\[
\left( \frac{\overline{N}_r}{\overline{N}_r} \right)_i = \frac{\rho_v C_o (Le)^{2/3}}{(D_{ct})_i} \tag{28}
\]

\[
(Le)_i = \frac{d_a}{(D_{ct})_i} \tag{29}
\]
Finally, the temperature of outlet air from drying chamber was calculated using a simple energy balance as follows:

\[
\dot{m}_{a,1} C_{a,1} (T_{a,1} - T_{\text{env}}) + \dot{m}_{T} (A_{p}) \left( \bar{T}_{T} - T_{\text{env}} \right)
= U A_{DC} \left( (T_{DC})_i - T_{\text{env}} \right) + \dot{m}_{a,3} C_{a,3} \left( (T_{a,3})_i - T_{\text{env}} \right)
\]  

(30)

The temperature of drying chamber was computed as follows:

\[
(T_{DC})_i = \frac{T_{a,3} + (T_{a,3})_i}{2}
\]

(31)

The heat and mass transfer model solution and exergetic calculation of the IR drying process were carried out using a subroutine developed in the MATLAB computer program (MathWorks Inc., Natick, MA).

Table 1 indicates the data and equations applied in the heat and mass transfer modelling of the IR drying process of a typical foodstuff.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Value or equation</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial water fraction of fresh food</td>
<td>$X_w$</td>
<td>0.75</td>
<td>–</td>
<td>Assumed</td>
</tr>
<tr>
<td>Initial protein fraction of fresh food</td>
<td>$X_{\text{pro}}$</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial fat fraction of fresh food</td>
<td>$X_{\text{fat}}$</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial carbohydrate fraction of fresh food</td>
<td>$X_{\text{carbo}}$</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial fibre fraction of fresh food</td>
<td>$X_{\text{fibre}}$</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial ash fraction of fresh food</td>
<td>$X_{\text{ash}}$</td>
<td>0.05</td>
<td></td>
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<tr>
<td>Specific heat of water component</td>
<td>$C_w$</td>
<td>4.180</td>
<td>kJ/kg K</td>
<td>Singh and Heldman (2001)</td>
</tr>
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<td>Specific heat of protein component</td>
<td>$C_{\text{pro}}$</td>
<td>2.037</td>
<td></td>
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<tr>
<td>Specific heat of fat component</td>
<td>$C_{\text{fat}}$</td>
<td>2.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific heat of carbohydrate component</td>
<td>$C_{\text{carbo}}$</td>
<td>1.594</td>
<td></td>
<td></td>
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<tr>
<td>Specific heat of fibre component</td>
<td>$C_{\text{fibre}}$</td>
<td>1.888</td>
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<tr>
<td>Specific heat of ash component</td>
<td>$C_{\text{ash}}$</td>
<td>1.137</td>
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</tbody>
</table>
Table 1  Data and equations applied in the heat and mass transfer modelling of the IR drying process of a typical foodstuff (continued)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Value or equation</th>
<th>Unit</th>
<th>Reference</th>
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<tbody>
<tr>
<td>Density of water component</td>
<td>$\rho_w$</td>
<td>1000</td>
<td>kg/m$^3$</td>
<td>Singh and Heldman (2001)</td>
</tr>
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<td>Density of protein component</td>
<td>$\rho_{pro}$</td>
<td>1316</td>
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<td>Density of fat component</td>
<td>$\rho_{fat}$</td>
<td>915</td>
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<td>Density of carbohydrate component</td>
<td>$\rho_{carbo}$</td>
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<td>Density of fibre component</td>
<td>$\rho_{fibre}$</td>
<td>1302</td>
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<tr>
<td>Density of ash component</td>
<td>$\rho_{ash}$</td>
<td>2353</td>
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<tr>
<td>Effective moisture diffusivity of a typical food</td>
<td>$D_{eff}$</td>
<td>$0.06 \exp \left( -\frac{60000}{8.314 \times T_p} \right)$</td>
<td>m$^2$/s</td>
<td>Assumed</td>
</tr>
<tr>
<td>Initial equivalent diameter of material</td>
<td>$D_{eq}$</td>
<td>0.05</td>
<td>m</td>
<td>Assumed</td>
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<td>Temperature of fresh product</td>
<td>$T_p$,2</td>
<td>298.15</td>
<td>K</td>
<td>Assumed</td>
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<tr>
<td>Drying chamber cross sectional area</td>
<td>ADC</td>
<td>0.32</td>
<td>m$^2$</td>
<td>Assumed</td>
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<tr>
<td>Overall heat transfer coefficient of drying chamber wall</td>
<td>$U$</td>
<td>0.0079</td>
<td>kW/m$^2$ K</td>
<td>Assumed</td>
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<td>Infrared-material view factor</td>
<td>$F_{IR-P}$</td>
<td>0.25</td>
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<td>Meeso et al. (2007)</td>
</tr>
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<td>Infrared -drying chamber view factor</td>
<td>$F_{IR-DC}$</td>
<td>0.75</td>
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<tr>
<td>Material -drying chamber view factor</td>
<td>$F_{P-DC}$</td>
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<td>Material emissivity</td>
<td>$\varepsilon_p$</td>
<td>0.72</td>
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<td>Assumed</td>
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<tr>
<td>Inlet air temperature</td>
<td>$T_{a,1}$</td>
<td>293.15</td>
<td>K</td>
<td>Assumed</td>
</tr>
<tr>
<td>Dead state temperature</td>
<td>$T_0$</td>
<td>293.15</td>
<td>K</td>
<td>Assumed</td>
</tr>
<tr>
<td>Ambient temperature</td>
<td>$T_{env}$</td>
<td>293.15</td>
<td>K</td>
<td>Assumed</td>
</tr>
<tr>
<td>Inlet air pressure</td>
<td>$P_{a,1}$</td>
<td>101.3</td>
<td>kPa</td>
<td>Assumed</td>
</tr>
<tr>
<td>Dead state pressure</td>
<td>$P_0$</td>
<td>101.3</td>
<td>kPa</td>
<td>Assumed</td>
</tr>
<tr>
<td>Density of air</td>
<td>$\rho_a$</td>
<td>1.147</td>
<td>kg/m$^3$</td>
<td>Aghbashlo et al. (2008)</td>
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<tr>
<td>Humidity ratio of inlet air</td>
<td>$\omega_{a,1}$</td>
<td>0.01</td>
<td>kg water/kg dry air</td>
<td>Assumed</td>
</tr>
</tbody>
</table>
Table 1  Data and equations applied in the heat and mass transfer modelling of the IR drying process of a typical foodstuff (continued)

<table>
<thead>
<tr>
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<th>Symbols</th>
<th>Value or equation</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Humidity ratio of dead state</td>
<td>( \omega_0 )</td>
<td>0.009</td>
<td>kg water/kg dry air</td>
<td>Assumed</td>
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<td>Specific heat of dry air</td>
<td>( C_d )</td>
<td>1.872</td>
<td>kJ/kg K</td>
<td>Dincer and Sahin (2004)</td>
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<td>Specific heat of water vapour</td>
<td>( C_v )</td>
<td>1.004</td>
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<tr>
<td>Dry air constant</td>
<td>( R_a )</td>
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<td>Vapour constant</td>
<td>( R_v )</td>
<td>0.4615</td>
<td></td>
<td></td>
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<td>Latent heat of water vaporisation</td>
<td>( h_{fg} )</td>
<td>(-2 \times 10^{-5} T_e^6 + 1.71 \times 10^{-2} T_e^2 + 7.674 \times 10^{-5} T_e + 3702.9)</td>
<td>kJ/kg</td>
<td>Fitted</td>
</tr>
<tr>
<td>Thermal diffusivity of air</td>
<td>( \alpha_d )</td>
<td>(-1 \times 10^{-14} T_e^6 + 10^{-10} T_e^2 + 9 \times 10^{-8} T_e - 10^{-3})</td>
<td>m²/s</td>
<td>Fitted</td>
</tr>
<tr>
<td>Thermal conductivity of air</td>
<td>( k_a )</td>
<td>(7 \times 10^{-11} T_e^6 - 3 \times 10^{-11} T_e^2 + 9 \times 10^{-3} T_e + 8 \times 10^{-7})</td>
<td>kW/m K</td>
<td>Fitted</td>
</tr>
<tr>
<td>Kinematic viscosity of air</td>
<td>( \nu_a )</td>
<td>(-10^{-14} T_e^6 + 9 \times 10^{-11} T_e^2 + 4 \times 10^{-8} T_e - 5 \times 10^{-6})</td>
<td>m²/s</td>
<td>Fitted</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>( Pr )</td>
<td>(-4 \times 10^{-10} T_e^6 + 2 \times 10^{-16} T_e^4 - 10^{-22} T_e^4 - 5 \times 10^{-10} T_e^6 + 10^{-8} T_e^2 - 7 \times 10^{-7} T_e + 0.8455)</td>
<td></td>
<td>Fitted</td>
</tr>
</tbody>
</table>

All temperatures are based on Kelvin.

Table 2 shows the important parameters of the IR drying process and their simulation values applied in this study.

Table 2  Important parameters of the IR drying process and their simulation values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>IR source temperature</td>
<td>( T_{IR} )</td>
<td>1073.15, 1173.15, 1273.15, 1373.15, and 1473.15</td>
<td>K</td>
</tr>
<tr>
<td>IR source active surface area</td>
<td>( A_{IR} )</td>
<td>0.04, 0.06, 0.08, 0.10, and 0.12</td>
<td>m²</td>
</tr>
<tr>
<td>IR emissivity factor</td>
<td>( \varepsilon_{IR} )</td>
<td>0.75, 0.80, 0.85, 0.90, and 0.95</td>
<td>–</td>
</tr>
<tr>
<td>Product mass</td>
<td>( m_p )</td>
<td>1, 5, 10, 15, and 20</td>
<td>kg</td>
</tr>
<tr>
<td>Air mass flow rate</td>
<td>( m_a )</td>
<td>0.036, 0.183, 0.367, 0.734, and 1.101</td>
<td>kg/s</td>
</tr>
</tbody>
</table>

3 Results and discussion

Figure 3 indicates the variation of exergy efficiency of the IR drying process cs. inlet air mass flow rate at various IR source temperatures. The exergy efficiency decreased
logarithmically as the inlet air mass flow rate increased. This occurred because of the increasing inlet air mass flow rate vented the major portion of the absorbed IR exergy to the ambient air without useful application for water evaporation. In other words, enhancing the air mass flow rate augmented the cooling effect at the surface of the product due to an increase in the convective heat transfer coefficient and, thus, dramatically lowered the exergy efficiency. Furthermore, increasing the IR source temperature almost linearly increased the exergy efficiency (Figure 4). This may be because, an increase in the IR source temperature led to a rapid increase in temperature of the material to be dried, which in turn enhanced the vapour pressure inside material and accordingly accelerated the moisture diffusion towards the surface. Thus, the exergy efficiency of process increased due to an increase in the amount of exergy invested for moisture removal.

**Figure 3**  The variation of exergy efficiency of the IR drying process with inlet air mass flow rate at different IR source temperatures

![Figure 3](image)

**Figure 4**  The variation of exergy efficiency of the IR drying process with IR source temperature at different levels of IR source active surface area

![Figure 4](image)

The variation of exergy efficiency of the IR drying process as a function of product mass at various IR source emissivity factors is shown in Figure 5. The exergy efficiency diminished exponentially with increasing the product mass. It could be attributed to the fact that the increasing product mass increased the amount of product to be warmed up
and accordingly exhausted the majority of absorbed IR exergy to the ambient air, instead of efficient usage in the drying process. On the other hand, increasing the product mass drastically increased the surface of the product exposed to the IR radiation and retarded the product heating up and mass diffusion. This in turn reduced the amount of exergy invested for moisture evaporation and thus decreased exergy efficiency, based on equation (18).

**Figure 5** The variation of exergy efficiency of the IR drying process as a function of product mass at different IR source emissivity factors

Figure 6 illustrates the variation of exergy efficiency of the IR drying process vs. IR source active surface area at various IR source emissivity factors. Obviously, increasing the IR source active surface area increased the exergy efficiency in a non-linear manner. This occurred because of the magnitude of absorbed IR exergy and subsequent exergy utilisation for moisture evaporation is proportional to the IR emitting surface area, according to equation (2).

**Figure 6** The variation of exergy efficiency of the IR drying process with IR source active surface area at different levels of IR source emissivity factor

The variation of exergy efficiency of the IR drying process as a function of the IR source emissivity factor at different levels of IR source temperatures is represented in Figure 7.
Clearly, increasing the IR source emissivity factor slightly increased the exergy efficiency, because of an increase in the amount of absorbed IR exergy by material to be dried, according to equations (2) and (12).

Figure 7  The variation of exergy efficiency of the IR drying process as a function of IR source emissivity factor at different levels of IR source temperature

Figure 8 displays the variation of exergy destruction of the IR drying process vs. inlet air mass flow rate at various IR source temperatures. It was expected that an increase in the inlet air mass flow rate would enhance the exergy destruction due to the higher heat and mass transfer rates. However, the exergy destruction did not considerably change with the variation of inlet air mass flow rate. It is well-understood that the heat and mass transfer phenomena are the most important contributors in the total entropy generation and corresponding exergy destruction during thermal processing of the wet solids. Nevertheless, increasing the inlet air mass flow rate reduced the contribution of mass transfer on the total exergy destruction by venting the major part of absorbed exergy to the ambient air, as previously discussed.

Figure 8  The variation of exergy destruction of the IR drying process with air mass flow rate at different IR source temperatures

Figure 9 exhibits the variation of exergy destruction of the IR drying process against IR source temperature at different levels of the IR source surface area. Generally, increasing
the IR emitter temperature non-linearly increased the exergy destruction due to the intensive heat and mass transfer.

Figure 9  The variation of exergy destruction of the IR drying process with IR source temperature at different levels of the IR source active surface area

The variation of exergy destruction of the IR drying process as a function of the product mass at various IR source emissivity factors is expressed in Figure 10. Generally, increasing the product mass sharply enhanced the exergy destruction at lower product mass, while its additional augmentation did not remarkably affect the exergy destruction. This occurred due to the fact that the increasing product mass elevated the entropy generation as consequence of mass transfer to a certain level, which in turn increased the exergy destruction. However, further increment of the product mass enhanced the quantity of product to be heated and accordingly vented the major part of absorbed IR exergy to the ambient air without a predominant modification in the entropy generation due to the mass transfer.

Figure 10  The variation of exergy destruction of the IR drying process against product mass at various levels of IR source emissivity factor

Figure 11 depicts the variation of exergy destruction of the IR drying process vs. IR source surface area at different levels of the IR source emissivity factor. The exergy destruction indicated a linear increase with the IR source active surface area, noteworthy
due to an increase in the quantity of absorbed IR exergy and subsequent severe heat and mass transfer.

**Figure 11** The variation of exergy destruction of the IR drying process with the IR source active surface area at different IR source emissivity factors

Eventually, Figure 12 describes the variation of exergy destruction of the IR drying process vs. IR source emissivity factor at various IR source temperatures. There was an almost linear relationship between exergy destruction and IR source emissivity factor. Obviously, increasing the IR emissivity factor increased the quantity of absorbed IR exergy by the product exposed to the IR radiation and, thus, led to the higher exergy destruction due to the intensive heat and mass transfer.

**Figure 12** The variation of exergy destruction of the IR drying process with IR source temperature at different levels of IR source emissivity factor

It could be concluded that the exergy destruction of the IR drying process was remarkably high in the most of drying conditions, indicating that the IR drying process is extremely entropy-generative and exergy-destructive process. It seems that the absorption of electromagnetic wave by the exposed material and subsequent conversion of the absorbed thermal radiation to the heat energy destructed the majority of the provided IR exergy into the drying chamber. Additionally, it could be argued that at a typical IR drying process, rapid increasing of the product temperature intensified the heat and mass
transfer rates and therefore augmented the rate of exergy destruction. Therefore, conscious and planned attempts should be devoted to reduce the exergy destruction for increasing the sustainability of the IR drying process. It is hoped that the mathematical model presented here can be employed not only to improve the design of IR dryers but also to obtain more meaningful information into their operations and efficiencies. The present study has much more practical importance, because it assists the engineers and plant managers to discern the best configurations and suitable applications and to find the optimal drying conditions for the IR drying technology. Additionally, it can be employed to answer the financial and environmental issues related to the IR drying technique.

4 Conclusions

A new mathematical model was presented to determine the exergy efficiency and exergy destruction of the IR drying process based on a simple heat and mass transfer model. Numerical simulations were conducted to examine the validity of this model at various drying conditions. It was found that the exergetic performance assessment of the IR drying process can be accurately performed using the presented methodology. The current survey indicated that the exergy destruction of the IR drying process is very high, mainly due to the absorption of the infrared radiation by the exposed material and subsequent transformation of the absorbed electromagnetic wave to the heat energy along with the intensive heat and mass transfer parameters. However, further experimental and theoretical studies should be carried out to applying this model for actual drying conditions. Finally, the presented model has an unique capability to provide a greater understanding of the IR drying process and it can be applied to the design and optimisation of the IR drying process of various wet products.

References


A proposed mathematical model for exergy analysis


**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Surface area, $m^2$</td>
</tr>
<tr>
<td>$C$</td>
<td>Specific heat of product, kJ/kg K</td>
</tr>
<tr>
<td>$D_{eff}$</td>
<td>Effective moisture diffusivity of water, $m^2/s$</td>
</tr>
<tr>
<td>$ex$</td>
<td>Specific exergy, kJ/kg</td>
</tr>
<tr>
<td>$En$</td>
<td>Energy irradiance or flux, W/ $m^2$</td>
</tr>
<tr>
<td>$Ex$</td>
<td>Exergy irradiance or flux, W/ $m^2$</td>
</tr>
<tr>
<td>$\dot{E}x$</td>
<td>Exergy rate, kJ/s</td>
</tr>
<tr>
<td>$F$</td>
<td>Shape factor, –</td>
</tr>
<tr>
<td>$\bar{h}_w$</td>
<td>Mass transfer coefficient, m/s</td>
</tr>
<tr>
<td>$\bar{h}_r$</td>
<td>Heat transfer coefficient, W/$m^2$ K</td>
</tr>
<tr>
<td>$IR$</td>
<td>Infrared radiation energy, kJ/s</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of air, kW/$m$ K</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter, m</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass, kg</td>
</tr>
<tr>
<td>$M$</td>
<td>Moisture concentration, kg water/$m^3$</td>
</tr>
<tr>
<td>$MC$</td>
<td>Moisture content, kg water/kg dry matter</td>
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<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate, kg/s</td>
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<tr>
<td>$P$</td>
<td>Air pressure, kPa</td>
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<tr>
<td>$\dot{Q}$</td>
<td>Heat transfer rate, kJ/s</td>
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<td>$R$</td>
<td>Constant, kJ/kg K</td>
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<td>$t$</td>
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<tr>
<td>$T$</td>
<td>Temperature, K</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coefficient, kW/$m^2$ K</td>
</tr>
<tr>
<td>$v$</td>
<td>Air velocity, m/s</td>
</tr>
</tbody>
</table>
A proposed mathematical model for exergy analysis

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Volume, m³</td>
</tr>
<tr>
<td>X</td>
<td>Mass fraction, –</td>
</tr>
</tbody>
</table>

**Dimensionless numbers**
- Le: Lewis number
- Nu: Nusselt number
- Pr: Prandtl number
- Re: Reynolds number

**Greeks**
- ∆t: Simulation time step, s
- α: Thermal diffusivity, m²/s
- β: Energy quality factor, –
- ε: Emissivity factor, –
- λ: Latent heat of water vaporisation, kJ/kg
- η: Exergy efficiency, –
- μ: Dynamic viscosity of air, Pa s
- ρ: Density, kg/m³
- σ: Stephan–Boltzman constant, W/m² K⁴
- ν: Kinematic viscosity, m²/s
- φ: Hydraulic diameter, m
- ω: Humidity ratio of air, kg water/kg dry air

**Subscripts**
- 0: Dead state
- 1: Wet product
- 2: Inlet air
- 3: Dried product
- 4: Outlet air
- a: Air
- abs: Absorbed
- amb: Ambient
- ash: Ash
- carbo: Carbohydrate
- d: Dry matter
- DC: Drying chamber
- des: Destruction
- emit: Emitted
- ev: Evaporation
- ex: Exergy
- eq: Equivalent
- fat: Fat
- fibre: Fibre
<table>
<thead>
<tr>
<th>Character</th>
<th>Meaning</th>
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<tr>
<td>i</td>
<td>Component</td>
</tr>
<tr>
<td>Inc</td>
<td>Incident</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared source</td>
</tr>
<tr>
<td>l</td>
<td>Loss</td>
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<tr>
<td>o</td>
<td>Initial state</td>
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<td>pro</td>
<td>Protein</td>
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<tr>
<td>P</td>
<td>Product</td>
</tr>
<tr>
<td>ref</td>
<td>Reflected</td>
</tr>
<tr>
<td>t</td>
<td>Drying time</td>
</tr>
<tr>
<td>tran</td>
<td>Transmitted</td>
</tr>
<tr>
<td>v</td>
<td>Vapour</td>
</tr>
<tr>
<td>w</td>
<td>Water</td>
</tr>
</tbody>
</table>