Viscous Fingering of Thixotropic Fluids: 
 a Linear Stability Analysis

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The displacement of a thixotropic fluid by an immiscible Newtonian fluid is investigated theoretically in a rectangular Hele-Shaw cell. Assuming that the thixotropic fluid of interest obeys the purely-viscous Moore model, use is made of the generalized Darcy’s law for finding the basic-flow. Having imposed infinitesimally-small normal-mode perturbations to the basic-flow, an analytical solution is found for calculating the growth rate and wavenumber of unstable modes. The analytical solution enables us to easily investigate the effects of the Moore’s model thixotropy parameters on the viscous fingering phenomenon in the Hele-Shaw cell. Based on the results obtained in the present work, it is concluded that thixotropy has a stabilizing effect on the Saffman-Taylor instability.

Key Words: Viscous fingering / Saffman-Taylor instability / thixotropic fluid / Moore model / viscosity ratio

1. INTRODUCTION

To increase the rate of crude oil production, most oil reservoirs rely on secondary-oil-recovery (SOR) operation, among others. In this operation a suitable fluid such as water is injected into a reservoir to maintain oil pressure, and also to sweep the oil left in the reservoir’s porous rocks. In practice, however, the operation is marred by the fact that the interface between the two fluids becomes unstable after a short while. The instability exhibits itself by the rise of a large number of fingers of the displaced fluid which penetrate into the displaced fluid—the so-called viscous fingering phenomenon. Under severe conditions, this unwanted phenomenon may lead to an early breakthrough of the displacing fluid to the production well thereby leaving large patches of oil effectively unrecoverable in the reservoir. In addition to the oil industry, this unwanted phenomenon is also encountered in plastic industry (say, in gas-assisted injection molding of polymeric melts). For reasons like these, the viscous fingering phenomenon has been the subject of extensive studies in the past [see Ref. 2 for a review]. Through such studies, it is well established that the main cause of instability is the sharp viscosity contrast between the injected fluid and the crude oil, although the slight density difference between the two fluids may also play a role.

For ease of analysis, earlier attempts to understand this phenomenon have focused on Newtonian-Newtonian (N-N) displacement. One can mention, for example, the pioneering work of Hill in this area. The phenomenon was rediscovered by Saffman and Taylor in a later but more comprehensive work. Their work was deemed to be so significant that nowadays the viscous fingering phenomenon is often referred to as the Saffman-Taylor (ST) instability. It has been demonstrated that when a less viscous fluid drives a more viscous one in a rectangular Hele-Shaw cell, certain modes grow and compete dynamically eventually leading to a single finger of constant width. The width of the single finger formed this way has been found to decrease with an increase in the capillary number $Dm U/s_{12}$, where $Dm$ is the viscosity difference, $U$ is the interface velocity, and $s_{12}$ is the surface tension at the interface. Further contributions made by Chouke et al. and MacLean and Saffman in this area [see Ref. 6 and 7].

In contrast to ordinary oils, waxy crude oils exhibit marked non-Newtonian behavior at temperatures slightly lower than their wax-appearance-temperature (WAT). The non-Newtonian behavior of waxy crude oils demonstrate itself by a yield stress, a shear-dependent viscosity, some degree of elasticity, and also a time-dependent viscosity. The effect of yield stress, shear-thinning, and elasticity on the ST instability has already been investigated in previous studies. In contrast, the effect of a fluid’s thixotropy on the
ST instability remains virtually unexplored. Having said this, it should be conceded that in a stimulating work, Pritchard and Pearson\(^{12}\) have shown that when a shear-thinning thixotropic fluid is used to displace a thixotropic fluid, it is theoretically shown that their prediction is indeed true when a Newtonian fluid is used to displace a thixotropic fluid obeying Moore model\(^{14}\) in a rectangular Hele-Shaw cell. Because of the purely-viscous nature of the Moore model, the generalized Darcy’s law is used for obtaining the basic-flow. We then proceed with imposing infinitesimally-small, normal-mode perturbations to the basic-flow and examine the growth rate of disturbances. Analytical results are then presented addressing the effects of fluid’s thixotropic parameters on the instability picture of the flow. We also try to discuss the significance of the results obtained in this work. The work is concluded by highlighting its major findings.

2. MATHEMATICAL FORMULATIONS

We consider pressure-driven flow of two incompressible, immiscible fluids in a rectangular Hele-Shaw cell, as shown in Fig. 1. The cell is seen to consist of two horizontal parallel plates separated by a small distance h apart. We adopt a Cartesian frame of reference with its origin located at the average position of the interface separating the fluids (see Fig. 1). In this figure, x denotes the direction of the flow, y is parallel to the interface and z is perpendicular the plates. Denoting the displacing and displaced fluids by subscripts 1 and 2, respectively, we assume that under steady-state conditions both fluids are moving with an average velocity of \(U\), which is also the velocity of the interface. Although we are primarily interested in the case in which the displacing fluid is Newtonian while the displaced fluid is thixotropic, we start with the more general case in which both fluids are allowed to be thixotropic. For ease of analysis, the gravitational effects are neglected.

Since the gap distance, \(h\), is much smaller than the length, \(L\), and the width, \(W\), of the Hele-Shaw cell, we rely on the lubrication approximation to neglect the velocity component in the \(z\)-direction. With the same token, the velocity gradients in the \(x\)- and \(y\)-directions are omitted as compared to that in the \(z\)-direction. Also, based on the assumption that the flow is occurring under creeping conditions, we ignore all inertial terms from the equations of motion. Under these conditions, we rely on the generalized Darcy’s law to relate the pressure field to the gap-averaged velocity vector; that is:\(^{12}\)

\[
\nabla \cdot \mathbf{v} = 0 \quad (1)
\]

\[
\nabla p = -\frac{\eta(\dot{\gamma}, t)}{k} \mathbf{v} \quad (2)
\]

where \(p(x,y,t)\) is the pressure, \(\mathbf{v}\) is the gap-averaged velocity vector, \(k\) is the permeability of the Hele-Shaw cell, and \(\eta\) is the variable viscosity of the fluid. It is to be noted that for Newtonian fluids, the permeability is related to the gap size by 4,

\[
k = h^{2/12} \quad (3)
\]

2.1 Moore Model

For Eq. 2 to be of any good, it is required that the constitutive equation of the fluid is given. For purely-viscous fluids the shear stress \(\tau\) is related to the shear rate \(\dot{\gamma}\) by the generalized Newtonian relationship; that is,

\[
\tau = \eta(\dot{\gamma}, t) \dot{\gamma} \quad (4)
\]

where \(\dot{\gamma}\) is a generalized deformation rate defined by the relationship \(\dot{\gamma} = \sqrt{2D_{ij}d_{ij}}\) and the viscosity has been allowed to be shear- and time-dependent. In structural-based thixotropic models, the viscosity function \(\eta(\dot{\gamma}, t)\) is related to a structural parameter, \(\lambda\), through the following relationship,

\[
\eta(\dot{\gamma}, t) = \eta_c + \lambda(\eta_0 - \eta_c) \quad (5)
\]

where the fully built-up structure is represented by \(\lambda=1\) and the fully broken-down structure by \(\lambda=0\). The viscosities corresponding to these limiting cases are denoted by the zero-shear viscosity \(\eta_0\) and the infinite-shear viscosity \(\eta_c\), respectively. This equation reduces to the Newtonian fluid model as a special case by simply setting \(\eta_0 = \eta_c\). To complete the model, we need an appropriate kinetic equation for the
time-evolution of the structural parameter. In the Moore model, the structural parameter satisfies the following kinetic equation\(^4\),

\[
d\lambda_e/dt = a(1 - \lambda) - b\gamma \lambda_e
\]

(6)

In this equation, the coefficients “a” and “b” are thixotropic parameters controlling the rate of structure build-up and structure break-down, respectively. The ratio b/a has the dimension of time, and so it can be interpreted as the characteristic time of the fluid. In the present work, this ratio is called the thixotropy ratio and is referred to by the letter “r”. The thixotropy ratio, r, is an important material property for Moore fluids. For example, in simple shear (i.e., where \(\gamma = \gamma_0\)) the fluid becomes progressively more shear-thinning. Moreover, for r \(\leq 1\) the equilibrium viscosity approaches the zero-shear viscosity. On the other hand, for large values of “r”, the equilibrium viscosity becomes virtually equal to the infinite-shear viscosity. In other words, at these two extremes, the behavior of a Moore fluid can hardly be differentiated from that of Newtonian fluids.

This also means that the thixotropic effects of a Moore fluid can be better observed at intermediate values of “r”, say, for \(0.01 < r < 1\).

The thixotropy ratio is not the only parameter influencing the response of Moore fluids. If we integrate Eq. 6 in steady shear (say, starting from an initial value of \(\lambda_e\) to an equilibrium \(\lambda_e\)) we end up with the relationship: \(\lambda(t) = \lambda_e - (\lambda_e - \lambda_0)e^{-t/(\Lambda a)}\) revealing that the rate of structural decay is controlled by the ratio \(\Lambda = 1/(a + r\gamma_0)\). This ratio, which is called the decay time, is obviously a measure of the degree of thixotropy for Moore fluids. As can be seen in Fig. 3, Moore fluids having large \(\Lambda\) can be regarded as strongly-thixotropic because they need a long time to forget their initial configuration before reaching their equilibrium state.

Unfortunately, \(\Lambda\) is not a true material property because it is shear-dependent. But, as noted by Derksen\(^5\), for Moore model we can define a characteristic shear rate equal to \(\gamma_c = 1/\lambda\) at which the steady-state viscosity of the Moore fluid is exactly equal to \((\eta_0 + \eta_\infty)/2\). At this particular shear rate the decay time is just equal to 1/2a. Thus, by merely decreasing “a” we can increase the decay time and intensify the thixotropic behavior of the Moore fluid. Alternatively, for a given “a” we can increase the thixotropic behavior of the Moore fluid by simply reducing “r” for any given shear rate.

### 2.2 Dimensionless Governing Equations

By substituting Eq. 5 into Eq. 6 we obtain,

\[
d\eta/dt + a(1 + r\gamma_0)\eta = a(\eta_0 + r\gamma_\infty)\eta_e
\]

(8)

Based on Darcy’s formulation, we can estimate the shear rate at each section using the gap-averaged velocity instead of the local velocity; that is 12),

**Fig. 2.** The effect of thixotropy ratio, r, on the dimensionless equilibrium viscosity.

**Fig. 3.** The effect of decay time, \(\Lambda\), on the time-evolution of the structural parameter.
\[
\dot{\gamma} = \alpha \frac{|v|}{h} \tag{9}
\]

where \(\alpha\) is a constant which is equal to 3 for Newtonian fluids. By inserting Eq. 9 into Eq. 8, we obtain,

\[
\frac{dn}{dt} + a(1 + \alpha \frac{|v|}{h}) \eta = a \left( \eta_0 + \alpha \frac{|v|}{h} \eta \right) \tag{10}
\]

Equations 1, 2 and 10 are the equations governing the flow of a Moore fluid being displaced in a rectangular Hele-Shaw cell. To make these equations dimensionless, we substitute,

\[
x^* = \frac{(x,y)}{W}, \quad V^* = W V, \quad \nu^* = \frac{v}{U}, \quad p^* = \frac{k}{\eta_{2,0} U W} p.
\]

\[
\iota^* = \frac{-1 - \eta^*}{\eta_{2,0}}, \quad \sigma^* = \frac{a W}{U}, \quad \Lambda^* = \frac{1}{2a}, \quad \tau^* = \frac{\alpha U}{W} \tag{11}
\]

Having dropped the asterisks “*” from the dimensionless parameters, the non-dimensional form of the governing equations becomes,

\[
\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y} = 0 \tag{12}
\]

\[
\frac{\partial p_j}{\partial x} = -\eta_j u_j \tag{13}
\]

\[
\frac{\partial p_j}{\partial y} = -\eta_j v_j \tag{14}
\]

\[
\frac{dn_j}{dt} + a_j \left(1 + e^{-1} \tau_1 u_j^2 + v_j^2 \right) \eta_j = a_j \left( S_{1,0} + e^{-1} \tau_1 u_j^2 + v_j^2 S_{1,\infty} \right) \tag{15}
\]

where \(j = 1, 2\). In Eq. 15 we have,

\[
S_{1,0} = \frac{\eta_{1,0}}{\eta_{2,0}}, \quad S_{1,\infty} = \frac{\eta_{1,\infty}}{\eta_{2,0}}, \quad S_{2,0} = 1, \quad S_{2,\infty} = \frac{\eta_{2,\infty}}{\eta_{2,0}}, \quad \varepsilon = \frac{h}{W} \tag{16}
\]

where \(\varepsilon\) is the aspect ratio of the Hele-Shaw cell. Under steady-state conditions the basic-flow velocity, pressure, and viscosity fields can be obtained for each phase from Eqs. 12-15 as,

\[
\bar{u}_j = 1 \tag{17}
\]

\[
\bar{v}_j = 0 \tag{18}
\]

\[
\bar{p}_j = -\bar{\eta}_j \tag{19}
\]

\[
\bar{\eta}_1 = \frac{S_{1,0} + e^{-1} \tau_1 S_{1,\infty}}{1 + e^{-1} \tau_1}, \quad \bar{\eta}_2 = \frac{1 + e^{-1} \tau_1 S_2}{1 + e^{-1} \tau_2} \tag{20}
\]

At the interface, from Eq. 19 we obtain,

\[
\tilde{p}_j = -\bar{\tilde{\eta}}_j x + c_j \tag{21}
\]

where tilde sign (\(\tilde{}\)) above parameters refers to basic-flow quantities. In Eq. 21, the constants of integration can be related to each other by the relationship,

\[
c_1 - c_2 = \kappa \Gamma \tag{22}
\]

where \(\kappa\) is the curvature in the \((x,z)\) plane, and \(\Gamma\) is the dimensionless surface tension,

\[
\Gamma = \frac{\kappa \sigma_{12}}{W^2 \eta_{2,0} U} \tag{23}
\]

where \(\sigma_{12}\) is the interfacial tension between the two fluids.

### 3. Linear Stability Analysis

Knowing the basic-flow, we can now proceed with slightly perturbing them using normal modes. For ease of analysis, we rely on two-dimensional perturbations for this purpose. The perturbed interface, pressure, velocity components, and viscosity field can then be written as,

\[
\phi(x,y,t) = \delta e^{ikY} e^{\sigma t} \tag{24}
\]

\[
p_j'(x,y,t) = \tilde{p}_j(x) + \delta \Lambda_j(x) e^{ikY} e^{\sigma t} \tag{25}
\]

\[
u_j'(x,y,t) = 1 + \delta B_j(x) e^{ikY} e^{\sigma t} \tag{26}
\]

\[
u_j'(x,y,t) = \delta \phi_j(x) e^{ikY} e^{\sigma t} \tag{27}
\]

\[
\eta_j'(x,y,t) = \bar{\eta}_j + \delta \phi_j(x) e^{ikY} e^{\sigma t} \tag{28}
\]

where \(K\) is the wavenumber and \(\sigma\) is the growth rate of the perturbations. By assuming that \(0 < \delta < 1\) the amplitude of the disturbance imposed on the interface (see Eq. 24), and also the amplitude of all other pertinent parameters, as represented by \(A(x), B(x), C(x),\) and \(D(x)\) in Eqs. 25-28, are forced to be infinitesimally small. Substituting Eqs. 26 and 27 into the continuity equation (Eq. 12), we obtain,

\[
C_j = \frac{i}{K} \frac{dB_j}{dx} \tag{29}
\]

Now, substituting Eqs. 25 to 28 into the momentum equations (i.e., Eqs. 13 and 14) and neglecting the nonlinear terms, we deduce,
A_j(x) = -\frac{\tilde{\eta}_j dB_j}{K^2 dx} \quad (30)

D_j = \frac{\tilde{\eta}_j d^2 B_j}{K^2 dx^2} - \tilde{\eta}_j B_j \quad (31)

Hence from Eq. 15 we obtain,

\frac{d^2 B_j}{dx^2} - f_j K^2 B_j = 0 \quad (32)

where,

f_j = \frac{\sigma + a_j \left(1 + e^{-r_j S_j} \tilde{\eta}_j^{1/2}\right)}{\sigma + a_j \left(1 + e^{-r_j}\right)} \quad (33)

Equation 32 can easily be solved for B_j(x) so that we have,

B_j(x) = M_j \exp (-f_j^{1/2} K x) + N_j \exp (f_j^{1/2} K x) \quad (34)

To obtain the coefficients, M and N, we resort to appropriate boundary conditions. Since the perturbations should tend to zero as x \to \pm \infty, it can be concluded that M_1 and N_2 are both equal to zero. The velocity along the interface can be obtained by taking the time-derivative of Eq. 24 and also from Eq. 26 at x = 0. So, we have,

B_j(0) = \sigma \quad (35)

Thus we obtain,

B_j(x) = \sigma \exp \left((-1)^{j+1} f_j^{1/2} K x\right) \quad (36)

Having found B_j(x), we can obtain A_j(x) from Eq. 30, so that we have,

A_j(x) = \frac{\sigma}{K} (-1)^{j} \tilde{\eta}_j f_j^{1/2} \exp \left((-1)^{j+1} f_j^{1/2} K x\right) \quad (37)

Similarly, C_j(x) and D_j(x) can be obtained from their respective equations. In perturbed state, the pressure jump at the interface can be related to the curvature in the (x,y) and (x,z) planes as,

p'_1 - p'_2 = \Gamma (\kappa_{xz} + \kappa_{xy}) \quad (38)

Having neglected nonlinear terms, the curvature in the (x,y) plane, \kappa_{xy}, can be related to \phi by,

\kappa_{xy} \approx -\phi' (y) \quad (39)

Substituting Eq. 39 and Eq. 25 into Eq. 38, at the interface we can write,

\tilde{p}_1(\phi) - \tilde{p}_2(\phi) - \Gamma \left(K^2 \delta c K y e^{\alpha \tau} + \kappa_{xz} \right) = (A_2(0) - A_1(0)) \delta c K y e^{\alpha \tau} \quad (40)

Using Eqs. 21 and 22, Eq. 40 becomes,

\tilde{\eta}_2 - \tilde{\eta}_1 - K^2 = A_2(0) - A_1(0) \quad (41)

And, finally from Eq. 37, we obtain,

\tilde{\eta}_2 - \tilde{\eta}_1 - K^2 = \frac{\sigma}{K} \left[\tilde{\eta}_1 f_1^{1/2} + \tilde{\eta}_2 f_2^{1/2}\right] \quad (42)

This equation is the major contribution of this paper. It gives the growth rate, \sigma, as a function of the wavenumber K for a Moore fluid displacing another Moore fluid. From this equation it can be concluded that for the instability to occur it is necessary that \tilde{\eta}_2 > \tilde{\eta}_1. As expected, for Newtonian/Newtonian displacement this equation reduces to the following well-known dispersion equation [see Ref. 4],

1 - S_1 - K^2 = \frac{\sigma}{K} \left[S_1 + 1\right] \quad (43)

In the present work, with an eye on the displacement of waxy crude oils by water, we use Eq. 42 for the case in which a Newtonian fluid displaces a Moore fluid in a rectangular Hele-Shaw cell. For this special case, this equation becomes,

\tilde{\eta}_2 - S_1 - K^2 = \frac{\sigma}{K} \left[S_1 + \tilde{\eta}_2 \sqrt{f_2^2}\right] \quad (44)

where S_1 = \eta_{1}/\eta_{2,0} is the dimensionless viscosity of the displacing Newtonian fluid. In the next section, we use this equation to investigate the effect of the viscosity ratio of the Moore fluid S_1 = \eta_{1}/\eta_{2,0} and the thixotropy ratio \tau_1 (these two parameters enter the dispersion equation through f_1 in Eq. 33 and \tilde{\eta}_1 in Eq. 20), and also the characteristic decay time \Lambda, (which enters the dispersion equation through f_2 in Eq. 33), on the growth rate of the unstable modes.

4. RESULTS AND DISCUSSIONS

As mentioned above, our main objective in the present work is to investigate the effect of the viscosity ratio, S_2, the thixotropy ratio, \tau_2, and the decay time, \Lambda, on the ST instability when a Moore fluid is to be displaced by a Newtonian fluid in a rectangular Hele-Shaw cell. Equation 44, being an analytical expression, should enable us to easily achieve these objectives. We are going to present typical results only [see Ref. 16 for more results]. To that end, we set the aspect ratio of the Hele-Shaw cell at a typically small
value of $\varepsilon = 0.01$ and the dimensionless surface tension at $\Gamma = 0.0001$. Based on Eq. 44, for the flow to become unstable, the viscosity of the displacing fluid must be less than the basic-flow viscosity of the displaced fluid (i.e., $S_1 < \bar{\eta}_2$). To guarantee this, the dimensionless viscosity of the displacing Newtonian fluid is set at $S_1 = 0.01$.

Figure 4 shows the effect of the viscosity ratio of the Moore fluid, $S_2$, on the growth rate of the unstable modes at two different thixotropy ratios. This figure also includes the Newtonian-Newtonian (N-N) displacement which is recovered by simply setting $S_2 = 1$. As can be seen in this figure, the flow becomes progressively more unstable when $S_2$ is increased. In other words, by increasing $S_2$, the maximum growth rate and the corresponding wavenumber are increased and the spectrum of the unstable modes becomes broader (see Fig. 4). An increase in the thixotropy ratio from 0.01 to 0.1 is seen not to affect the overall picture although it is seen to have a stabilizing effect on the flow for any given $S_2 < 1$. Because the ST instability is driven by the viscosity difference between the displacing and displaced fluids, it is expected that the flow becomes more unstable by an increase in $\bar{\eta}_2$. On the other hand, based on Eq. 20 the basic-flow viscosity of the Moore fluid ($\bar{\eta}_2$) is increased at any given $r_2$ when $S_2$ is increased (see Fig. 5). Therefore, it is not surprising that by an increase in $S_2$ the flow becomes more unstable, as can be seen in Fig. 4.

Figure 6 shows the effect of the thixotropy ratio, $r_2$, on the instability characteristics of the flow. As can be seen in this figure, $r_2$ has a stabilizing effect on the flow; that is, the maximum growth rate decreases when $r_2$ is increased. To explain the stabilizing effect of thixotropy ratio on the ST stability, it should be noted that based on Eq. 20 at any given $S_2$ the basic-flow viscosity of the Moore fluid, $\bar{\eta}_2$, decreases when $r_2$ is increased (see Fig. 7). Thus, the viscosity difference increases by a drop in $r_2$, and so we should see a destabilizing effect when $r_2$ is decreased, as can be seen in Fig. 6. It is also interesting to note that for $r_2 < 1$ the instability curve approaches a limiting value labelled as N-N in Fig. 6. This is due to the fact that for very small values of $r_2$ the basic-flow viscosity of the Moore fluid ($\bar{\eta}_2$) becomes approximately equal to its zero-shear viscosity (see Fig. 2). On the other hand, for $r_2 > 1$, the ST instability fades out for $S_2 = 0.01$; because now $\bar{\eta}_2$ becomes approximately equal to its infinite-shear viscosity (i.e., $\bar{\eta}_2 \approx S_2 = S_1$) and therefore, there would be no interfacial ST instability essentially.

Figure 8 shows the effect of the decay time, $\Lambda_2$, on the growth rate of unstable perturbations for a given thixotropy ratio, $r_2$, and viscosity ratio, $S_2$. As can be seen in this figure, an increase in $\Lambda_2$ has a stabilizing effect on the viscous fingering phenomenon. Since an increase in $\Lambda_2$ means an increase in the fluid’s thixotropy (see Fig. 3), this means that thixotropy has a stabilizing effect on the ST instability in a rectangular Hele-Shaw cell, as previously conjectured by Pearson and Tardy.\(^{13}\) Interestingly, the spectrum of unstable modes are seen to remain the same for all $\Lambda_2$ values. This is
not surprising realizing the fact that the basic-flow viscosity difference, i.e., $\eta_2 - S_1$, remains unaltered for all plots in Fig. 8, because according to Eq. 20, $\eta_2$ is not a function of $a_2$ and consequently $\Lambda_2$.

Finally, as mentioned earlier, our main objective in the present work has been to investigate the effect of a fluid’s thixotropicity on the ST instability. But, we have also investigated the effect of the “$h$” and $\Gamma$ on the characteristics of the ST instability. Such results can be used for further verifying the analytical solution as the effect of these parameters has long been established in the literature. Figure 9 shows such results. As can be seen in this figure, the aspect
ratio and the dimensionless surface tension have strong effects on the ST instability of Moore fluids. In fact, by an increase in the gap size and/or the dimensionless surface tension, the flow becomes more stable, as expected.

5. CONCLUDING REMARKS

In the present work, we have addressed two-phase viscous fingering phenomenon in a rectangular Hele-Shaw cell when a Newtonian fluid displaces an immiscible thixotropic fluid of the Moore type. We have adopted the Darcy equation to establish the basic-flow solution and the unstable modes. We have shown that for a thixotropic fluid obeying Moore model, it is possible to obtain an analytical solution for the growth rate of the unstable modes in a rectangular Hele-Shaw cell. The closed-form solution obtained in this work has enabled us to investigate the effect of fluid’s thixotropy on the ST instability. Based on the results obtained in the present work, it is concluded that thixotropy has a stabilizing effect on the viscous fingering phenomenon.

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