Experimental investigation of two-phase water–oil flow pressure drop in inclined pipes

Pedram Hanafizadeh *, Amir Karimi, Alireza Taklifi, Alireza Hojati

Center of Excellence in Design and Optimization of Energy Systems, School of Mechanical Engineering, College of Engineering, University of Tehran, P.O. Box: 11155-4563, Tehran, Iran

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A B S T R A C T

In this experimental work the effect of inclination on the pressure gradient in two phase oil–water flow is investigated. The experiments were performed in a 6 m long, 20 mm inner diameter and inclinable acrylic pipe using oil (3 mPa s viscosity and 830 kg/m³ density) and water (1 mPa s viscosity and 990 kg/m³ density) as test fluids. Pressure gradients between inlet and outlet of flow in pipe were measured for inclination angles of 0°, ±5°, ±15°, ±30° and ±45° with respect to the horizontal plane. The experimental results were compared with Homogeneous and Two-Fluid models. It was observed that in high mixture velocities, where dispersed flow prevails, there is a peak pressure gradient which is related to phase inversion. It was also found that, phase inversion appears at higher inlet water cut values in inclinations of –30° and –45° compared with other inclinations. However the two-fluid model and homogeneous model both over-predicted the pressure drop, but two-fluid model predicted the pressure drop with less average deviation. Several correlations for effective mixture viscosity in a homogeneous model were considered and the results were compared with experimental results. Acceptable agreement was seen between the computed and measured data.

The experimental two-phase friction factors were compared with the friction factors for single phase flow of oil and water, at the same velocities as the two phase mixture and it was found that the experimental friction factors were less than the predicted friction factors of single phase flow.

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1. Introduction

In many chemical and oil industries, oil–water two-phase flows are important. So understanding the behavior of these kinds of flows is necessary. For instance, oil extraction from oil wells is often accompanied by a high water fraction which increases during the producing life of well [1], also in many cases in order to enhance oil transportation, water is injected into the pipe and so oil and water are transported together in pipelines that may experience various degrees of inclination respecting to the horizontal direction. For this reason, knowing two-phase liquid–liquid flow behavior in various pipe angles is important in designs related to the petroleum industry. Despite knowing this matter, two phase liquid–liquid flow is considerably less studied compared with two phase gas–liquid flow and theoretical models presented for analysis of pressure drop of two-phase liquid–liquid flow and these studies marked by considerable limitations [2]. Generally, pressure drop in oil–water pipelines is one of the most important design parameters and presenting an appropriate theoretical model for predicting pressure drop of liquid–liquid flow is needed for many applications.

Very early studies of oil–water flows were accomplished in decades 1950 and 1960. Charles et al. [3] and Russell et al. [4] discovered that adding water to oil decreases pressure drop. After an interval, the interest for liquid–liquid flows grew up due to increased applications of this kind of flow in various industries. Flow patterns are known to be important for analysis of pressure drop in two-phase oil–water flows by many researchers [5,6]. Even if oil properties are close to water such that oil viscosity is only 2 or 3 times more than of water’s viscosity, there would be no reliable model for predicting pressure drop [7,8] due to its highly nonlinear behavior. One of the matters that makes the pressure drop prediction more complicated is phase inversion phenomena. The phase inversion is a behavior occurs when with a small change in the operating conditions; the continuous and dispersed phase of flow spontaneously inverts [9]. The volume fraction in which, the phase inversion occurs is called a phase inversion point. According to previous studies, pressure drop considerably increases at phase inversion point [10,11]. Angeli and Hewitt [11] found that homogeneous model with viscosity calculated from the Brinkman model [12] is able to predict pressure drop at phase...
inversion point but with high uncertainty. Poesio et al. [13] developed two fluid model based on the assumption that one of the phases is dispersed, but did not verify their model at a phase inversion point.

Despite the large number of studies currently available in the literature of pressure drop in horizontal and vertical pipes, very few works have tried to predict the pressure drop and other properties of two phase flow at phase inversion point. Also, most works done for two-phase liquid–liquid flow are related to pipes with diameters greater than 20 mm and less is done for the pipes with smaller diameters. Most of the articles that have studied two-phase flows in smaller pipes are done within recent years [14–17]. In this article, we have measured pressure drop for different inclination angles of a 6 meter long pipe. The inner diameter of the pipe was 20 mm and it is completely made of acrylic to be transparent. Various correlations are used for calculating effective viscosity in a homogeneous model to realize which one can predict pressure drop more precisely. Also, we have compared experimental pressure drop results with two-fluid model for separated flow presented by Taitel and Dukler [18] and two fluid model presented by Poesio et al. [13]. We have focused on dispersed flow pattern to observe and investigate effects of the phase inversion phenomenon on pressure drop, mixture viscosity and friction factor to find which model predicts these properties better at phase inversion point. One of the reasons that the small pipe observations was conducted in the current work is that in small pipes the dispersed flow pattern could be observed in laboratory scales with lower flow rates. So using a small pipe test is a suitable choice for the study of phase inversion in dispersed flows and according to Xiao-Xuan [1] results could be generalized for different pipe diameters to some extents.

2. Experimental setup

The experiments were performed in a multiphase flow facility with a pipe which is capable of having different inclination angles with respect to the horizontal direction. A schematic sketch of this test facility is given in Fig. 1. The test section has a 6 m acrylic pipe with an inner diameter of 20 mm and outer diameter of 30 mm. The test pipe could be inclined from 45° in a downward direction to 45° in upward direction. Working fluids for this experiment were water and oil, which kept in two separate storage tanks. Water and oil pumped by positive displacement pumps to test the line and were stored in a gravity separator tank in which oil and water separate due to density differences. Superstation tank is placed in a height well above water and oil tank. After returning the separated oil to its respective storage tank, the remained water

\begin{center}
\begin{tabular}{ll}
\text{Nomenclature} & \\
\hline
\text{dp/dz} & \text{pressure gradient (Pa/m)} \\
\text{USO and USW} & \text{oil and water superficial velocity (m/s)} \\
\text{UO and UW} & \text{oil and water in-situ velocity (m/s)} \\
\text{AO and AW} & \text{area occupied by oil and water (m²)} \\
\text{Do and Dw} & \text{hydraulic diameter of oil and water (m)} \\
\text{So and SW} & \text{wall wetted perimeter for oil and water (m)} \\
\text{s_i} & \text{interfacial periphery (m)} \\
\text{\tau_{oil-water}} & \text{interfacial oil–water shear stress (N/m²)} \\
\text{Um} & \text{mixture velocity (m/s)} \\
\text{ReO and ReW} & \text{oil and water Reynolds number} \\
\text{\muO and \muW} & \text{oil and water viscosity (Pa s)} \\
\text{\rhoO and \rhoW} & \text{oil and water density (kg/m³)} \\
\text{\mu_m} & \text{mixture viscosity in homogeneous model (Pa s)} \\
\text{\rho_m} & \text{mixture density in homogeneous model (kg/m³)} \\
\text{\mu_c and \mu_d} & \text{continuous and dispersed viscosity (Pa s)} \\
\text{H_O and H_W} & \text{oil and water hold-up} \\
\end{tabular}
\end{center}
was discharged to the sewage and fresh feed water was used for the next tests.

Water used in these experiments had a density of 980 kg/m$^3$ and viscosity of 1 mPa s in temperature of 20 °C. The oil used had a density of 830 kg/m$^3$ and viscosity of 3 mPa s in temperature of 20 °C. The oil–water interfacial tension was measured as 0.032 N/m. Pressure drop was measured by a pair of calibrated Indumart PTF106 pressure transmitters with an accuracy of 0.3% of full scale. Pressure transmitters were placed by distance of 4 m from each other along the pipe. The temperature of the two-phase flow was measured during the experiments by a thermocouple. The temperature data were used to consider the variations of oil viscosity and interfacial tension during tests.

The range of oil superficial velocity was from 0.29 m/s to 2.35 m/s and for water the superficial velocity was from 0.5 m/s to 3.21 m/s. So the inlet oil cut was from 8% to 80%. Pressure gradient was measured in pipe for inclination angles of 0°, ±5°, ±15°, ±30° and ±45° and each measurement was repeated 3 times and average pressure drop was considered as final pressure drop.

The main uncertainties in the current experiments were introduced by flowmeters and pressure transmitters. However, the uncertainties related to thermocouples and inclination angle adjustment devices could be added to the overall uncertainty of these experiments. The flowmeters were located after the pumps and the maximum flow rates of both oil and water was 90 L/min and after calibration of flowmeters for each fluid they were found to have an uncertainty of 0.09 L/min. The uncertainties of the data being collected from pressure transmitters were up to 0.3%. The measurement uncertainties are listed in the form of standard deviation in Table 1. According to this table the overall experimental uncertainties of the measured two phase pressure gradients were estimated up to 2.11%

### 3. Pressure drop modeling

Correlations used to predict the two-phase liquid–liquid flow pressure drop are classified in two categories, first category consists of experimental correlations presented based on the results of experimental works and the second category consists of correlations that assume the flow is occurring in a specific flow pattern. Each of these categories has its own advantages and limitations.

#### 3.1. Flow pattern independent correlations

These correlations do not consider flow pattern for pressure drop calculations and they are specifically for some experimental conditions, therefore they might be unreliable [19] for all kinds of flow conditions. The most important experimental correlations are for Hoogendorn [20] and Lockhart and Martinelli [21] which were developed for two-phase gas-liquid flow. Thereafter, Charles and Lilleleht [22] presented a correlation for calculation of pressure drop for separated flow based on the parameters introduced in Lockhart & Martinelli correlation. Meanwhile, Baker et al. [23] presented a correlation for prediction of pressure drop for separated flow. Theissing [24] presented a correlation which is applicable both for gas–liquid flow and liquid–liquid flow and it is as follows:

\[
\frac{\Delta p}{\Delta z} = \left[ \frac{\Delta p}{\Delta z \text{gas}} \left( \frac{M_o}{M_l} \right) ^{1/n} \right] ^{1/e} + \left[ \frac{\Delta p}{\Delta z \text{water}} \left( \frac{M_w}{M_l} \right) ^{1/e} \right] ^{1/e} \tag{1}
\]

\[
e = 3 - 2 \left( \frac{2 \sqrt{\rho_o/\rho_w}}{1 + \rho_o/\rho_w} \right)^{0.7/n} \tag{2}
\]

\[
n = \frac{n_1 + (1/X)^{0.2}}{1 + (1/X)^{0.2}} \tag{3}
\]

\[
n_1 = \frac{\ln \left( \frac{\Delta p}{\Delta z \text{gas}} / \Delta p}{\Delta p}{\Delta z \text{water}} \right)}{\ln (M_o/M_l)} \quad n_2 = \frac{\ln \left( \frac{\Delta p}{\Delta z \text{water}} / \Delta p}{\Delta p}{\Delta z \text{gas}} \right)}{\ln (M_w/M_l)} \tag{4}
\]

where $\rho_o$, $\rho_w$ are the densities of the oil and the water, respectively, $M_o$ and $M_w$ are the mass flow rates of the oil and the water, $(\Delta p/\Delta z)_{\text{gas}}$ and $(\Delta p/\Delta z)_{\text{water}}$ are the pressure gradients when the oil or the water flows alone at $M_o$ or $M_w$, respectively, and $(\Delta p/\Delta z)_{\text{total}}$ and $(\Delta p/\Delta z)_{\text{water}}$ are the pressure gradients if the oil or the water flows alone at the total mixture mass rate of flow ($M_o + M_w$).

Due to ease of use of this correlation and since it can be used for liquid–liquid flow; Theissing Correlation has been used extensively during recent years for modeling. But one of the limitations of Theissing Correlation is that it can be used only in horizontal pipes. Empirical correlations do not consider fluid flow pattern, so the realization of the matter that which phase is continuous and which phase is dispersed is not possible and effect of phase inversion on pressure gradient could not be observed, which is an important aspect for analysis of pressure gradient in pipelines [11].

### 3.2. Correlations based on flow pattern

Correlations taking into account the flow patterns need to have the values for parameters like two-phase interface shape, interfacial shear stress and wall shear stress of phases. In order to reach to these values experimental works are needed. Correlations based on flow pattern have obtained for separated flow and dispersed flow.

#### 3.2.1. Separated flow

For analysis of separated flow there have been two main approaches. The first approach involves the analytical solution of the Navier–Stokes equations in the laminar flows. However, if any of the phases be turbulent, the exact solution of the Navier–Stokes equations is not possible. In the second approach two phases are represented as two separate regions and two continuous fluids are considered to flow in different layers in a circular pipe according to their density and assumed to be separated by a smooth and flat interface, this approach is named Two-fluid model.

In Two-Fluid Model, momentum equations are presented for each of the phases as:

\[
-A_o \frac{dp}{dz} - \tau_o S_o - \tau_i S_i - \rho_o A_o g \sin \beta = 0 \tag{5}
\]

\[
-A_w \frac{dp}{dz} - \tau_w S_w + \tau_i S_i - \rho_w A_w g \sin \beta = 0 \tag{6}
\]

In which $(\frac{dp}{dz})$ is the pressure gradient along pipe, $A_o$ and $A_w$ are oil and water cross sectional area respectively, $S_o$ and $S_w$ are the wall wetted perimeter for oil and water, $\tau_o$ and $\tau_w$ are the wall shear stresses for the oil and the water, $\tau_i$ is the interfacial oil–water shear stress, $S_i$ is interfacial periphery and $\beta$ is pipe inclination ($\beta > 0$ for upward flow), in above equations it is assumed that oil velocity is higher than water velocity (see Table 2).

#### Table 1

<table>
<thead>
<tr>
<th>Measured parameter</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter</td>
<td>±0.1 mm</td>
</tr>
<tr>
<td>Pressure</td>
<td>±0.3%</td>
</tr>
<tr>
<td>Oil and water flow rates</td>
<td>±0.05 L/min</td>
</tr>
<tr>
<td>Oil and water density</td>
<td>±0.5 kg/m$^3$</td>
</tr>
<tr>
<td>Temperature</td>
<td>±0.1 °C</td>
</tr>
<tr>
<td>Inclination</td>
<td>±0.5°</td>
</tr>
</tbody>
</table>
Table 2
Geometric parameters used in the two-fluid model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water cross sectional area ($A_w$)</td>
<td>$A_w = \frac{4}{\pi} \left( \frac{D_w}{2} \right)^2$</td>
</tr>
<tr>
<td>Oil cross sectional area ($A_o$)</td>
<td>$A_o = A - A_w$</td>
</tr>
<tr>
<td>Interfacial periphery ($S_i$)</td>
<td>$S_i = 2R \cos(\theta)$</td>
</tr>
<tr>
<td>Oil wall wetted perimeter ($S_w$)</td>
<td>$S_w = (\pi - 2\theta)/R$</td>
</tr>
<tr>
<td>Water wall wetted perimeter ($S_W$)</td>
<td>$S_W = (\pi + 2\theta)/R$</td>
</tr>
<tr>
<td>Water superficial velocity ($U_{SW}$)</td>
<td>$U_{SW} = \frac{q_w}{S_w}$</td>
</tr>
<tr>
<td>Oil superficial velocity ($U_{SO}$)</td>
<td>$U_{SO} = \frac{q_o}{S_o}$</td>
</tr>
<tr>
<td>Oil hold-up ($H_o$)</td>
<td>$H_o = \frac{q_o}{q_o + q_w}$</td>
</tr>
<tr>
<td>Water velocity ($U_w$)</td>
<td>$U_w = \frac{q_w}{A_w}$</td>
</tr>
<tr>
<td>Oil velocity ($U_o$)</td>
<td>$U_o = \frac{q_o}{A_o}$</td>
</tr>
</tbody>
</table>

Table 3
The wall shear stresses and friction factors in pipe.

<table>
<thead>
<tr>
<th>Friction factor</th>
<th>laminar flow</th>
<th>turbulent flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil friction factor</td>
<td>$f_o = 64 \frac{D}{L}$</td>
<td>$f_o = 0.046 R e_o^{0.2}$</td>
</tr>
<tr>
<td>Water friction factor</td>
<td>$f_w = \frac{D_w}{U_w}$</td>
<td>$f_w = 0.046 R e_w^{0.2}$</td>
</tr>
<tr>
<td>Oil wall shear stress ($\tau_o$)</td>
<td>$\tau_o = f_o \rho_o U_o^2$</td>
<td>$\tau_o = f_o \rho_o U_o^2$</td>
</tr>
<tr>
<td>Water wall shear stress ($\tau_w$)</td>
<td>$\tau_w = f_w \rho_w U_w^2$</td>
<td>$\tau_w = f_w \rho_w U_w^2$</td>
</tr>
<tr>
<td>Oil Reynolds number ($Re_o$)</td>
<td>$Re_o = \frac{U_o D_o \rho_o}{\mu_o}$</td>
<td>$Re_o = \frac{U_o D_o \rho_o}{\mu_o}$</td>
</tr>
<tr>
<td>Water Reynolds number ($Re_w$)</td>
<td>$Re_w = \frac{U_w D_w \rho_w}{\mu_w}$</td>
<td>$Re_w = \frac{U_w D_w \rho_w}{\mu_w}$</td>
</tr>
</tbody>
</table>

Parameters used in Two Fluid Model are defined as follows:
The wall shear stresses, $\tau_o$ and $\tau_w$ are expressed in terms of the fluid friction factors as Table 3.
Where $D_o$ and $D_w$ are oil and water hydraulic diameters respectively, and defined as follows [25]:

$$D_o = \frac{4A_o}{S_o + S_i}, \quad D_w = \frac{4A_w}{S_w + S_i} \quad \text{for} \quad U_w < U_o$$

$$D_o = \frac{4A_o}{S_o}, \quad D_w = \frac{4A_w}{S_w + S_i} \quad \text{for} \quad U_w > U_o$$

$\tau_o$ is defined as follows:

$$\tau_o = f_o \left( \frac{U_o - U_w}{2} \right) \left( \frac{S_o U_o \rho_o}{\pi \mu_o} \right)^{-0.2}$$

$$\rho_o, \mu_o, \mu_w \begin{cases} U_w > U_o & \\
U_w < U_o \end{cases}$$

From Eqs. (5) and (6) we obtain the combined momentum balance equation:

$$-\tau_o S_o + \tau_w S_i + H_o A_w (\rho_w - \rho_o) g \sin(\theta) = 0$$

The above equation can be solved based on $H_o$ by numerical method to obtain pressure gradient and phase velocities.

3.2.2. Dispersed flow

Homogeneous model is an appropriate model for predicting pressure drop in dispersed flows. In this model, the two liquid phases are assumed as a single phase with average properties of the two phases and possible differences between the two individual phases are neglected.

The homogeneous model for the pressure gradient in liquid–liquid dispersion is often given as:

$$\frac{dp}{dx} = -\frac{f_m \rho_w U_w^2}{2D} - \rho_m g \sin(\theta)$$

$$f_m = 0.312 \left( \frac{\mu_m U_w D}{\mu_w} \right)^{0.25}$$

$$U_m = U_w + U_{hw}$$

$$\rho_m = 2\rho_o + (1 - \alpha)\rho_w$$

where $f_m$ is the mixture friction factor, $\rho_m$ is the mixture density, $U_m$ is the mixture velocity, $\alpha$ is the volume fraction of oil and $D$ is the diameter of the pipe. In homogeneous model, the problem is calculation of mixture viscosity. Many methods have been developed for the calculation of mixture viscosity until today. The simplest equation is as follows (Dukler et al. [26]):

$$\mu_m = 2\mu_o + (1 - \alpha)\mu_w$$

Application of this equation is simple but the problem of that is it fails to predict phase inversion because it is not clear which phase is continuous and which one is dispersed.

Brinkman [12] started from Einstein’s formula for viscosity of suspensions in extreme dilution and obtained following equation for mixture viscosity:

$$\mu_m = \mu_c (1 - \alpha)^{-2.5}$$

In which $\mu_c$ is continuous phase viscosity and $\alpha$ is the volume fraction of dispersed phase. Brinkman’s equation despite to that of Dukler’s has a peak in mixture viscosity that is related to phase inversion.

There is another equation for calculating the mixture viscosity that was presented by Taylor [27]:

$$\mu_m = \mu_c \left[ 1 + 2.5 \alpha \left( \frac{0.4 + \mu_w / \mu_c}{1 + \mu_w / \mu_c} \right) \right]$$

where $\mu_c, \mu_w$ and $\mu_o$ respectively indicate mixture viscosity, continuous phase viscosity and viscosity of dispersed phase and $\alpha$ is the volume fraction of dispersed phase.

Another model which has been developed for estimating pressure drop in dispersed flow besides the homogeneous model was presented by Poesio et al. [13]. This model was obtained based on two fluid model with the difference that in this model it is assumed that dispersed flow prevails and one phase is considered continuous and the other dispersed. Therefore, it is expected that this model predict pressure drop of dispersed flow better than two fluid model. For a fully developed steady state flow, the integral forms of the one dimensional momentum equations for the two phases are given by:

$$-A_o \left( \frac{dp}{dx} \right) - A F_{D_o/w} - \rho_o A_o g \sin(\theta) = 0$$

$$-A_w \left( \frac{dp}{dx} \right) - \tau_w S_w + A F_{D_w/o} - \rho_w A_w g \sin(\theta) = 0$$
In which $A$ and $S_w$ respectively indicate the pipe’s cross sectional area and water wall wetted perimeter, $\tau_w$ is wall shear stress for water, $\theta$ is the pipe inclination angle. In these equations, oil is assumed to have dispersed phase and water continues, otherwise, the following equation is acquired:

$$-A_w \left( \frac{dp}{dz} \right) - AF_{D,W/O} - \rho_w A_w g \sin \theta = 0$$ (21)

$$-A_0 \left( \frac{dp}{dz} \right) - \tau_0 S_0 + AF_{D,W/O} - \rho_0 A_0 g \sin \theta = 0$$ (22)

By equating the pressure drop in the two phases, the following equation is derived:

$$\alpha (\rho_w - \rho_0) A_w g \sin \theta + \tau_w S_w x - F_{D,A/O} A$$

$$= 0 \quad \text{oil in water dispersion}$$ (23)

$$(1 - \alpha) (\rho_0 - \rho_w) A_0 g \sin \theta + \tau_0 S_0 (1 - \alpha) - F_{D,O/A} A$$

$$= 0 \quad \text{water in oil dispersion}$$ (24)

In this model, we assume that dispersed phase has no contact with wall, therefore:

$$S_w = S_0 = \pi D$$ (25)

We assume that $F_D$ is drag force transferred from the continuous phase to solid spherical particles, so we have [28]:

$$F_D = \frac{3}{4} \pi \rho C_d \left( U_d - U_I \right) \frac{U_d - U_I}{d_d}$$ (26)

where $C_d$ is the drag coefficient, $U_d$ is the dispersed phase velocity, $U_I$ is the continuous phase velocity, and $d_d$ is the dispersed phase drop effective diameter. The drag coefficient is (cliff [29]):

$$C_d = \begin{cases} \frac{24}{5} (1 + 0.15 Re^{0.687}) & Re \leq 800 \\ 0.44 & Re > 800 \end{cases}$$ (27)

where the droplet Reynolds number yields:

$$Re = \frac{\rho_d |U_d - U_I| d_d}{\mu_m}$$ (28)

That $\mu_m$ is mixture viscosity.

4. Results and discussion

In this section the results of the experimental tests are discussed. These results contain pressure gradients due to two-phase oil–water flow in pipe for different inclination angles with respect to the horizontal direction. These inclinations angles are $0^\circ$, $\pm 5^\circ$, $\pm 15^\circ$, $\pm 30^\circ$ and $\pm 45^\circ$. A special care was provided to dispersed flow to observe phase inversion phenomenon. The pressure gradients obtained from tests are compared with two-fluid model (presented by Poesio et al. [13]), homogeneous model, two-fluid model (presented by Taitel and Dukler [18]) and Theissing model pressure gradient predictions at phase inversion point. In all experimental runs, the pipe was prewetted with water before the two phase mixture was introduced and then oil injected into the pipe and measurements were taken.

Figs. 2–10 show pressure drop along the pipe in 4 m distance for the defined inclinations of various oil and water superficial velocities. As it could be found from these figures, for higher oil superficial velocities, the pressure drop rises. Although for these high oil superficial velocities for some specific superficial water velocities, the pressure drop suddenly falls. This is the phase inversion point, where the dispersed phase changes to the continuous phase. As it could be seen in Figs. 9 and 10 for downward flows the phase inversion point moves to higher water superficial velocities. This is because the each phase dispersion in another one would become slower for downward flows. It is also obvious in Figs. 2–6 that for some values $U_{so}$ like 1.85 the pressure drop increase in pipe is more significant.

As pressure drop figures indicate, a similar trend is observed in the oil superficial velocity of $U_{so} = 0.26, 0.53, 0.79, 1.06$ m/s (before phase inversion) in all inclinations. This trend has been observed previously in horizontal pipe with the diameter of 15 mm by Wahabi et al. [30] and in horizontal pipe with the diameter of 25.4 mm by Al-Wahibi et al. [31].
A peak occurs in pressure drop in the oil superficial velocity of $U_{so} = 2.38, 1.85$ m/s thereafter pressure drop experiences a significant decrease that is related to phase inversion of converting water in oil dispersion to oil in water dispersion [32]. We can say that one of the reasons of decrease of pressure drop at phase inversion point is that pipe is wetted by oil before phase inversion, but after phase inversion pipe is wetted by water and due to lower viscosity of water, shear stress and so the pressure drop decreases.

At $+15^\circ, +30^\circ$ and $+45^\circ$ pipe inclination, it is observed that oil in water dispersion (right after the peak point or phase inversion) extends to higher inlet oil cut compared to horizontal flows. Lum and Angeli [33] has pointed out this matter in her experiments.

Phase inversion is observed in pipe inclination of $0, \pm 5, \pm 15, \pm 30$ and $\pm 45$ at 30–35% inlet water cut and it is observed at 35–40% in inclination of $-30^\circ$ and $-45^\circ$. This delay of phase inversion at $-30^\circ$ was observed by Mukherjee et al. [34] for another fluid and flow conditions. In addition, it is observed in the experimental work...
of Lum & Angeli [33] that in downward inclinations, water in oil dispersion extends to higher inlet water cut compared to horizontal and it may cause delay in phase inversion. This matter can be observed in more detail in Fig. 11.

A comparison between the measured pressure gradient in this experimental test and theoretical models for horizontal pipe flow is performed and finally it was concluded that Two Fluid Model_dis (which considers flow as dispersed flow and presented by Poesio et al. [13]) has the best agreement with experimental pressure gradient results (Figs. 12–14).

Two Fluid Model_dis predicts experimental pressure gradient better in comparison with Two Fluid Model_st (presented by Taitel and Dukler [18]) for horizontal flow as shown in Fig. 12, because the flow pattern observed in these operational conditions (properties of liquids, geometry of the pipe and superficial velocity of water and oil) is dispersed flow. For this reason, Two Fluid Model_dis which obtained based on dispersion of one of the phases better predicts pressure drop.

The comparisons of two homogenous models in predicting a pressure drop are shown in Fig. 13. Viscosity of one is calculated based on Brinkman and viscosity of the other calculated based on Taylor correlation. It could be found that the homogeneous model with viscosity calculated from the Brinkman model better predicts pressure drop. The comparison between experimental pressure drops and Theissing Model is depicted in Fig. 14. This Theissing Model predicts the pressure drop well, however this correlation is only valid for horizontal flows.

For inclined pipe flows it could be easily seen that the errors of the two-fluid models are more significant (Fig. 15) when the inclination angle increases. For upward flows this deviation is more obvious.

Again, if this comparison for various inclination angles are being performed for homogeneous models as for just 45° it is being depicted in Fig. 16, it could be concluded that this Two-Fluid model-dis still works better than these homogeneous ones.

Fig. 17 shows the comparison of experimental pressure drop with predicted pressure drop in three models including homogenous model (Taylor), Two Fluid Model_dis and Two Fluid Model_st at $U_{so} = 2.38$ (m/s) for horizontal pipe flows. As it is obvious homogenous (Taylor) and Two Fluid_dis are able to predict phase inversion, Because in these two models it is considered one phase is dispersed and other is continues, and the flow pattern observed in this mixture velocity is dispersed, but in Two Fluid Model_st both phases are considered continues so this model is not able to predict phase inversion. It is also obvious that Two Fluid Model_st better predicts experimental pressure drop compared with the homogeneous model before and after phase inversion point. Therefore, it can be concluded that Two Fluid Model_dis can predict pressure drop at phase inversion point better than other models.

Fig. 18 shows the comparison of experimental pressure drop with theoretical models at $U_{so} = 1.06$ m/s, As it is obvious, homogenous (Brinkman) and Theissing models have significant deviation compared with other models and Two Fluid Model_dis has the lowest deviation in spite of the fact that at this mixture velocity
Fig. 15. Comparison between experimental measurements and the two-fluid models for different inclination angles.

Fig. 16. Comparison between experimental measurements and the homogeneous models for different inclination angles.

Fig. 17. Comparison between experimental measurements and models predictions at $U_{so} = 2.38 \text{m/s}$.
both phases are continues ([6]. Table 4 shows the average deviation between pressure gradient data and pressure drop predictions by models.

Fig. 19 shows the comparison between experimental results and model predictions for inclined pipe flows at both \( U_{so} = 1.06 \) and \( U_{so} = 2.38 \) for inclination angles of ±45°. This figure clearly shows that for inclined pipes the models have larger errors. This could be because of difficulties being introduced when someone wants to consider the effect of gravity forces on pressure drop in these models. Many of the models that being developed in this purpose like Theissing are very good but just for horizontal pipe flows. None of them well perform for inclined pipes, however in our study we had a small diameter pipe with small volumetric flows in comparison with some big real oil transportation lines.

The homogeneous models and Theissing’s model offer similar unsatisfactory predictions which significantly disagree with experimental data, the two-fluid (dis) model proposed in this work presented the best results with an overall average deviation of 18%.

Average deviation is calculated by using following equation:

\[
\text{average deviation} = \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{(dp/dz)_{\text{exp}} - (dp/dz)_{\text{model}}}{(dp/dz)_{\text{exp}}} \right)^2 \right)^{1/2} \times 100
\]

(29)

Table 4
Average deviation between pressure gradient data and models predictions.

<table>
<thead>
<tr>
<th>Inclination</th>
<th>Homogeneous (br)</th>
<th>Homogeneous (tay)</th>
<th>TTM (st)</th>
<th>TTM (dis)</th>
<th>Theissing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>56</td>
<td>30</td>
<td>29</td>
<td>19</td>
<td>43</td>
</tr>
<tr>
<td>5° upward</td>
<td>41</td>
<td>21</td>
<td>21</td>
<td>13</td>
<td>–</td>
</tr>
<tr>
<td>10° upward</td>
<td>31</td>
<td>17</td>
<td>17</td>
<td>12</td>
<td>–</td>
</tr>
<tr>
<td>30° upward</td>
<td>21</td>
<td>12</td>
<td>15</td>
<td>11</td>
<td>–</td>
</tr>
<tr>
<td>45° upward</td>
<td>21</td>
<td>12</td>
<td>13</td>
<td>8</td>
<td>–</td>
</tr>
<tr>
<td>5° downward</td>
<td>88</td>
<td>31</td>
<td>33</td>
<td>21</td>
<td>–</td>
</tr>
<tr>
<td>15° downward</td>
<td>95</td>
<td>36</td>
<td>37</td>
<td>23</td>
<td>–</td>
</tr>
<tr>
<td>30° downward</td>
<td>105</td>
<td>43</td>
<td>43</td>
<td>27</td>
<td>–</td>
</tr>
<tr>
<td>45° downward</td>
<td>119</td>
<td>53</td>
<td>51</td>
<td>30</td>
<td>–</td>
</tr>
<tr>
<td>Average</td>
<td>64</td>
<td>28</td>
<td>28</td>
<td>18</td>
<td>43</td>
</tr>
</tbody>
</table>

Fig. 19. Comparison between experimental measurements and models predictions at \( U_{so} = 1.06 \) (1st row) & at \( U_{so} = 2.38 \) (2nd row).

Fig. 20. Friction factor against mixture velocity.
Fig. 20 shows the experimental two-phase friction factors during dispersed flow being plotted against the mixture velocity in order to compare the results with the friction factors being calculated for single-phase flow of oil and water, at the same velocities as the two-phase mixture. It is obvious that experimental two-phase friction factors are well below the ones expected from the single phase data when oil or water is the continuous phase. These results could be due to drag reduction phenomenon occurring in oil-water flows.

If this friction factor is being depicted for inclined tube conditions with ±45° of inclination (Fig. 21), it could be observed that deviation from water or oil properties of friction becomes more considerable. This is because of high nonlinearity that is being introduced in friction factor due to gravitational body forces and slight changes in phase inversion point by changing the inclination.

It can be seen from Fig. 22 experimental friction factors at one point is higher than water friction factors and oil friction factors. This difference in results is related to phase inversion which occurs at these points and it could not be predicted by single phase estimations. In fact, at the phase inversion point, effective viscosity of two-phase flow increases and so friction factor increases.

Some comparisons between the experimental friction factors and analytical predictions are shown in Fig. 20. In this figure the experimental two-phase friction factor \( f_{TP} \) was plotted against the two types of Reynolds numbers. Viscosity of one type is calculated based on Brinkman and viscosity of the other one was calculated based on Taylor equation. The analytical data to compare are the friction factors obtained by three empirical correlations of Haaland, Blasius, and Kays and Crawford. From Fig. 18 it is evident that analytical data are in poor agreement with experimental friction factors because of drag reduction phenomenon in oil-water flows.

Again considering the inclination effect on friction factor and comparing them with theoretical models at Fig. 23 for inclinations of ±45° indicates that for downward flow the models experience much error in comparison with upward flow. This effect could be specifically being noticed in low Reynolds numbers, where the flow pattern may be still as slug. So it could be concluded that these compared theoretical models are poor in bubbly and slug flow regions.

Fig. 24 shows the comparison of experimental effective viscosity with predictions of Taylor and Brinkman equations at oil superficial velocity of \( U_{so} = 1.06 \text{m/s} \). At Fig. 25 these results are compared with the experimental results for \( U_{so} = 2.39 \text{m/s} \) which includes phase inversion effects.

It is obvious from Figs. 19 and 20 that Taylor correlation predicts effective viscosity at both two oil superficial velocities better than other one. Furthermore, at \( U_{so} = 2.39 \text{m/s} \) both Taylor and
Brinkman predict phase inversion. That is because Taylor considered diverse flow mixed viscosity effects in its proposed equation.

5. Conclusion

In this experimental work, effect of pipe inclination on pressure drop of oil–water flow was studied. The results from measurements were compared with theoretical models presented for liquid–liquid flow. It was found that the pressure drop, experiences a similar trend in entire inclination angles at low mixture velocities, but in high mixture velocities where dispersed flow patterns prevail, there is a peak in pressure drop which is related to flow pattern change from water in oil dispersion to oil in water dispersion (Phase Inversion). Phase Inversion occurs in entire inclinations at 30–35% inlet water cut, but in –30° and –45° pipe inclination, phase inversion occurs at 35–40%. The inclination affects the both flow pattern and water hold up and as far as the tests were directed in the manner that the flow pattern being kept as dispersed flow. As the test tube was transparent the flow pattern could be observed to be dispersed. So at ambivalent range where the phase inversion occurred the inclination had an active effect in faster or later occurrence of phase inversion. To this extent it should be also noticed that within the ambivalent range, a variety of factors will determine the exact phase inversion point, like the viscosities of the two fluids, their density deference and the temperature.

Comparison of experimental pressure drop with theoretical models indicates that Two Fluid Model (dis), which is obtained based on dispersed flow, shows a good agreement with experimental measurements than other homogeneous models. Furthermore, this model is able to predict phase inversion. Also, different equations presented for the calculation of mixture viscosity in homogeneous models were studied and it was found that Taylor's equation predictions of mixture viscosity are in good agreement with experimental mixture viscosities.

Experimental friction factors of two-phase oil–water flow were found to be significantly below than the friction factors for single-phase flows of oil or water with the same velocity as of two-phase flow. This indicates drag reduction in two-phase dispersed flows of oil–water compared with single-phase flows of oil or water. The amount of drag reduction was observed to be up to 30% in some cases in comparison to oil single phase flow.

References