Simple physically-based constitutive equations for hot deformation of 2024 and 7075 aluminum alloys

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Abstract: The hot working behaviors of 2024 and 7075 aluminum alloys were studied through constitutive analysis based on a physically-based approach which accounts for the dependence of the elastic modulus and the self-diffusion coefficient of aluminum on temperature. It was demonstrated that the lattice self-diffusion activation energy of aluminum (142 kJ/mol) can be used in the Zener–Hollomon parameter’s formula as the deformation activation energy and the theoretical exponent of 5 can be set in the modified hyperbolic sine law to describe the peak flow stresses. By consideration of physically-based material’s parameters, it was possible to conduct a comparative study on the hot flow stress of 2024 and 7075 aluminum alloys. It was concluded that the used approach in the current work can be considered as a versatile tool in future comparative hot working studies, especially in studies dedicated to alloy development.

Key words: aluminum alloy; hot deformation; constitutive equation; activation energy; diffusion

1 Introduction

The 2xxx and 7xxx aluminum alloys are among the metallic materials with the highest specific strengths which make them suitable for many applications requiring light-weight materials [1–3]. Until the introduction of the 7xxx series, the 2xxx aluminum–copper alloys with copper content ranging from 2% to 6% (mass fraction) were the highest strength aluminum alloys [4–6]. Aluminum alloy 2024, as the first high strength light-weight alloy, gains its high strength by precipitation hardening resulted from the CuAl2 intermetallic compound. The high strength of 7xxx aluminum alloys is obtained by the addition of Zn, Mg, Cr, and Cu and from precipitation hardening [1,7]. Aluminum alloy 7075 with about 6% Zn (mass fraction) is among the most widely used aluminum alloys in light-weight structures.

Hot deformation processing is a suitable shaping method in the industry due to both the possibility of structural refinement (which significantly influences the mechanical response of the alloy) and the increased formability [6,8]. The understanding of the hot working behavior and the constitutive relations describing material flow are two of the prerequisites for the implementation of shaping technology in the industry [9,10].

The well-known Zener–Hollomon parameter \( Z = \dot{\varepsilon} \exp((Q/RT)) \) can be related to flow stress in different ways. The power law description of stress \( (Z = A' \dot{\varepsilon}^n) \) is preferred for relatively low stresses. Conversely, the exponential law \( (Z = A' \exp(\beta\dot{\varepsilon})) \) is suitable for high stresses. Finally, the hyperbolic sine law \( (Z = A'(\sinh(\alpha\sigma))^n) \) can be used for a wide range of temperatures and strain rates [11]. In these equations, \( A' \), \( A'' \), \( A \) (the hyperbolic sine constant), \( n' \), \( n \) (the hyperbolic sine power), \( \beta \) and \( \alpha = \beta/n' \) (the stress multiplier) are constants and \( Q \) is the deformation activation energy. The stress multiplier \( \alpha \) is an adjustable constant which brings \( a\sigma \) into the correct range that gives linear and parallel lines in \( \ln \dot{\varepsilon} \) versus \( \ln(\sinh(\alpha\sigma)) \) plots [11].

Conventionally, \( n \) and \( Q \) are considered to be apparent parameters in the hyperbolic sine law, which makes it impractical to conduct the comparative studies to elucidate the effects of alloying elements or second phases. Recently, it has been shown that when the deformation mechanism is controlled by the glide and...
climb of dislocations, a constant hyperbolic sine power of \( n = 5 \) and self-diffusion activation energy \( (Q_{sd}) \) can be used to describe the appropriate behavior [12]. This can be applied to flow stress data by taking into account the dependences of elastic modulus \((E)\) and self-diffusion coefficient \((D)\) on temperature in the hyperbolic sine law. Accordingly, the unified relation can be expressed as

\[
\dot{\varepsilon} D = B \sinh(\alpha' \varepsilon / E)^\beta
\]  

(1)

where \( D = D_0 \exp[-Q_{sd}/(RT)] \), in which \( D_0 \) is a pre-exponential constant. The constants \( \alpha' \) and \( B \) are the modified stress multiplier and the hyperbolic sine constant, respectively. The consideration of hyperbolic sine power of 5 and self-diffusion activation energy gives a physical and metallurgical meaning to Eq. (1) and also reduces the number of unknown parameters and constant to 2 \((\alpha' \) and \( B \)). The former results in a more reliable constitutive equation and the latter simplifies the constitutive analysis and makes it possible to conduct comparative hot deformation studies.

In the current work, the constitutive behaviors of 2024 and 7075 Al alloys were studied by consideration of physically-based material parameters (Eq. (1)) in order to propose a reliable constitutive equation for hot deformation of each material.

### 2 Experimental

The flow stress data of the 2024 alloy, hot compressed at deformation temperatures between 250 and 500 °C under strain rates of 0.001 to 12.5 s\(^{-1}\), and those of the 7075 alloy, hot compressed at deformation temperatures between 300 and 510 °C under strain rates of 0.001 to 10 s\(^{-1}\), were taken from Refs. [13–20]. Note that the description of flow stress by Eq. (1) is incomplete, because no strain for determination of flow stress is specified. Therefore, characteristic stresses that represent the same deformation or softening mechanism for all flow curves, such as steady-state or peak stress, should be used in this equation. Since the peak stress \((\sigma_P)\) is the most widely accepted one in obtaining the hot working constants [21–24], the values of peak stress were taken with emphasis on the consistency of stress level among different research works.

Since the flow data were taken from the literature and the details of the considered materials and experiments in each research work are different, some other factors such as grain size, texture, and variations in chemical compositions can affect the level of flow stress but the consideration of these parameters is not easy and needs a suitable database, which is not the case for the considered alloys in the current work. Therefore, the following analysis can fairly demonstrate the averaged constitutive behaviors of these materials.

### 3 Results

In Eq. (1), the values of \( D_0 \) and \( Q_{sd} \) can be taken from the Frost and Ashby tables [25]. In these tables, the dependence of the shear modulus \((G)\) on temperature in the form of \( G / G_0 = 1 + \eta(T - 300)/T_M \) is also available. Here, \( G_0 \) is the shear modulus at 300 K, \( T_M \) is the melting temperature of the material, and \( \eta = (T_M/G_0)\sigma G / dT \) shows the temperature dependence of the shear modulus. According to the relation of \( E = 2G(1 + \nu) \), the values of \( E \) can be estimated \((\nu \) is usually taken as 0.3). Using the available data for aluminum (as shown in Table 1), the following expressions can be derived for \( D \) and \( E \):

\[
D = 1.7 \times 10^{-4} \exp[-142000/(RT)]
\]  

(2)

\[
E = 66040(1 - 0.5(T - 300)/933)
\]  

(3)

#### Table 1 Data required for characterizing temperature dependence of lattice self-diffusion coefficient and shear modulus for Al

<table>
<thead>
<tr>
<th>( D_P (m^2s^{-1}) )</th>
<th>( Q_{sd} (kJmol^{-1}) )</th>
<th>( \eta )</th>
<th>( G_0 ) MPa</th>
<th>( T_M ) K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 ( \times 10^{-4} )</td>
<td>142</td>
<td>-0.5</td>
<td>2.54 ( \times 10^4 )</td>
<td>933</td>
</tr>
</tbody>
</table>

It can be deduced that there are only two unknown parameters \((B \) and \( \alpha' \)). In order to find the value of \( \alpha' \), the power and exponential laws were modified as \( \dot{\varepsilon} D = B' (\sigma_P/E)^\beta' \) and \( \dot{\varepsilon} D = B' \exp(\beta' \sigma_P/E) \), respectively. It follows from these expressions that the slope of the plot of \( \ln(\dot{\varepsilon} D) \) against \( \ln(\sigma_P/E) \) and the slope of the plot of \( \ln(\dot{\varepsilon} D) \) against \( \sigma_P/E \) can be used for obtaining the values of \( n' \) and \( \beta' \), respectively. These plots are shown in Figs. 1(a) and (b). The linear regression of these data resulted in the average value of \( \alpha' = \beta' / n' = 682.96 \). According to Eq. (1), the slope of the plot of \( \ln(\dot{\varepsilon} D)^{0.2} \) against \( \sinh(\alpha' \sigma_P/E) \) by fitting a straight line with the intercept of zero \( y = ax \) was used for obtaining the value of \( B^{0.2} = 536.94 \) (Fig. 1(c)). The resultant constitutive equation can be expressed as

\[
\dot{\varepsilon} = ([1.7 \times 10^{-4} \exp[-142000/(RT)])] = 536.94[^{0.2} \sinh(682.96 \times \sigma_P/E)]^{0.5}
\]  

(4)

Therefore, a suitable constitutive equation for describing the hot working behavior of 2024 alloy can be expressed as

\[
Z = \dot{\varepsilon} \exp[142000/(RT)] = 94.63[^{0.2} \sinh(682.96 \sigma_P/E)]^{0.5}
\]  

(5)

Based on Table 1 and the methods described above, the slope of the plot of \( \ln(\dot{\varepsilon} D) \) against \( \ln(\sigma_P/E) \) and the slope of the plot of \( \ln(\dot{\varepsilon} D) \) against \( \sigma_P/E \) were also used for obtaining the values of \( n' \) and \( \beta' \) for the
7075 alloy. The corresponding plots are shown in Figs. 2(a) and (b) and the average value of $\alpha'$ was determined as 652.46. Moreover, according to Eq. (1), the slope of the plot of $(\dot{\varepsilon}/D)^{1/2}$ against $\sinh[\alpha'\sigma_p/E]$ by fitting a straight line with the intercept of zero was used for obtaining the value of $B_{0.2} = 372.6$ for the 7075 alloy. The corresponding plot is shown in Fig. 2(c) and the resultant constitutive equation can be expressed in its simplified form as follows:

$$Z = \dot{\varepsilon}\exp[142000/(RT)] = 65.66^{5}[\sinh(652.46\sigma_p/E)]^{5}$$

(6)

4 Discussion

Based on Eqs. (5) and (6), it seems that the consideration of hyperbolic sine power of 5 and the lattice self-diffusion activation energy of Al as the deformation activation energy works perfectly for the 2024 and 7075 Al alloys. A comparison between the hot flow stresses of these alloys can be made by rewriting Eqs. (5) and (6) in the form shown in Eqs. (7) and (8), respectively:

$$\sigma_p/E = \sinh^{-1}(Z^{0.2}/94.63)/682.96$$

(7)

$$\sigma_p/E = \sinh^{-1}(Z^{0.2}/65.66)/652.46$$

(8)

It can be deduced that, for any given value of $Z$, the right-hand side of Eq. (8) is larger than that of Eq. (7). Therefore, the level of hot flow stress is higher for Eq. (8) and hence for the 7075 alloy. Note that the 7075 alloy has a higher room temperature strength compared to the 2024 alloy [1]. Moreover, the physically-based approach employed in this work makes it possible to verify this fact based on the experimental data. The required plot is shown in Fig. 3(a), which depicts the values of $\sigma_p$ vs $Z_{0.2}$. Since the values of $Z$ were determined based on the lattice self-diffusion activation energy of Al as the deformation activation energy, the values of $Z$ are the same for both alloys under a given deformation condition (deformation temperature and strain rate). Therefore, as can be seen in Fig. 3(a), at each $Z$, the level of flow stress is higher for the 7075 alloy.
The values of $\alpha'$ for 2024 and 7075 aluminum alloys were determined as 682.96 and 652.46, respectively, which are relatively comparable to each other. The average values of $\alpha'=(682.96+652.46)/2=668$ can be considered for further analysis, which makes it possible to compare the hot flow stress of 2024 and 7075 alloys based on the value of the modified hyperbolic sine constant ($B$). Again, according to Eq. (1), the slope of the plot of $(\dot{\varepsilon}D)^{0.2}$ against $\sinh(\alpha'\sigma_p/E)$ by fitting a straight line with the intercept of zero was used for obtaining the values of $B^{0.2}=558.28$ and 349.18 for the 2024 and 7075 alloys, respectively. The corresponding plots are shown in Fig. 3(b) and the resultant constitutive equations can be expressed in their simplified form as follows:

$$Z = \dot{\varepsilon} \exp[142000/(RT)] = \begin{cases} 
98.39^5 \sinh(668\sigma_p/E) & \text{for 2024 alloy} \\
61.54^5 \sinh(668\sigma_p/E) & \text{for 7075 alloy} 
\end{cases} (9)$$

The value of $B^{0.2}=558.28$ for 2024 alloy is considerably higher than the value of 349.18 for 7075 alloy. Therefore, it can be concluded that the level of the hot flow stress of 7075 alloy and probably its creep resistance are higher. This also implies that the used approach in the current work can be considered as a versatile tool in future hot working studies, especially in studies dedicated to alloy development.

5 Conclusions

1) The lattice self-diffusion activation energy of aluminum (142 kJ/mol) can be used as the deformation activation energy for both 2024 and 7075 aluminium alloys in the Zener–Hollomon parameter formula of the form $Z = \dot{\varepsilon} \exp[Q/(RT)]$.

2) The physically-based formula of the form $\dot{\varepsilon}D = B(\sinh(\alpha'\sigma_p/E))^5$ not only results in a reliable constitutive equation, but also significantly simplifies the constitutive analysis by consideration of theoretical exponent of 5 and self-diffusion activation energy, which in turn makes it possible to conduct comparative hot working studies.

3) Based on the results of the comparative study by the proposed physically-based approach, it can be deduced that the level of the hot flow stress of 7075 alloy and probably its creep resistance are higher than those of the 2024 alloy.

References


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基于物理的2024和7075铝合金简化热变形本构方程

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摘要: 通过基于物理方法的本构方程研究2024和7075铝合金的热变形行为。研究温度对铝弹性模量和自扩散系数的影响。研究结果表明, 为描述峰值流变应力, 铝的晶格自扩散激活能(142 kJ/mol)可作为Zener–Hollomon参数方程的变形激活能, 而改进双曲正弦函数中的理论指数可设为5。考虑基于物理的材料参数, 可以对2024和7075铝合金的热流变应力进行对比研究。所采用的方法是一个通用工具, 可用于热加工的对比研究和合金开发。

关键词: 铝合金; 热变形; 本构方程; 激活能; 扩散

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