Engineering Computations
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An efficient method for seismic analysis of structures

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Abstract

Purpose – The purpose of this paper is to present an efficient method for dynamic analysis of structures utilizing a modal analysis with the main purpose of decreasing the computational complexity of the problem. In traditional methods, the solution of initial-value problems (IVPs) using numerical methods like finite difference method leads to step by step and time-consuming recursive solutions.

Design/methodology/approach – The present method is based on converting the IVP into boundary-value problems (BVPs) and utilizing the features of the latter problems in efficient solution of the former ones. Finite difference formulation of BVPs leads to matrices with repetitive tri-diagonal and block tri-diagonal patterns wherein the eigensolution and matrix inversion are obtained using graph products rules. To get advantage of these efficient solutions for IVPs like the dynamic analysis of single DOF systems, IVPs are converted to boundary-value ones using mathematical manipulations. The obtained formulation is then generalized to the multi DOF systems by utilizing modal analysis.

Findings – Applying the method to the modal analysis leads to a simple and efficient formulation. The laborious matrix inversion and eigensolution operations, of computational complexities of $O(n^{2.373})$ and $O(n^3)$, respectively, are converted to a closed-form formulation with summation operations.

Research limitations/implications – No limitation.

Practical implications – Swift analysis has become possible.

Originality/value – Suitability of solving IVPs and modal analysis using conversion and graph product rules is presented and applied to efficient seismic optimal analysis and preliminary design.

Keywords Seismic analysis, Finite difference method, Block tri-diagonal matrices, Boundary-value problems, Graph products, Initial-value problems, Design of structures

Paper type Research paper

1. Introduction

In recent decades powerful computers have smoothed the way for efficient computations. However, because of laborious and time-consuming computations in many algorithms and convergence issues in many iterative methods, optimal analyses are still of great importance in literature. In structural engineering, optimal design of structures is one the main purposes but in many cases such a design is possible only if optimal analysis is adopted (Kaveh et al., 2013b, 2015; Kaveh and Talatahari, 2011; Chen and Feng, 2012; Chan et al., 1998; Fang et al., 2011; Ikeda et al., 1992; Hughes et al., 1977; Ventura et al., 1995; Lucchese, 2004; Thomas, 1979; Rahami et al., 2015; Zingoni, 2009, 2012).

In literature, efficient solutions are usually limited to structures with specific regular patterns. Besides, the methods are mainly applicable to static analysis. Various optimal
solutions of symmetric regular and near-regular structures can be found in Kaveh (2013). The methods employ graph products and group theoretical method for the solution of structural problems. Although the methods were first used in the analysis of regular structures, they became a base for the solution of near-regular and irregular structures later. Analysis of near-regular structures using partitioned stiffness matrices was introduced by Kaveh et al. (2013a). Analysis of near-regular structures convertible to regular ones was performed by Shojaei et al. (2013). Optimal solution of near-regular structures using the force method was obtained in Kaveh et al. (2013a). Methods based on combined graph products and substructuring methods were used in the eigensolution and optimal analysis of irregular structures (Rahami et al., 2014). Different methods of efficient eigensolution can be found in Yueh (2005).

Application of graph products rules (tri-diagonal and block tri-diagonal matrices characteristics) is not limited to the regular and near-regular systems in which a geometrical regularity is present. The graph products rules can be used in any formulation where a kind of repetition exists. One of these formulations is the finite difference method in which the governing differential equation in each point depends on the point and the previous and following points and this pattern is repeated for all points. Consequently, this repetition is seen in the obtained tri-diagonal or block tri-diagonal matrix which can efficiently be solved. A problem with a traditional finite difference method is the computational time which increases by choosing smaller time steps. However, the graph products rules solve the problem so that the reduction of the time steps has a very small effect on the computational load of the solution.

In the paper for simplifying the dynamic analysis of structures, transforming an initial-value problem (IVP) into a boundary-value problem (BVP), the graph product relationships are combined with those of the finite difference method, to obtain much simpler relations for modal analysis of multi DOF systems.

In the seismic analysis of structures, the numerical methods of solving the governing initial-value equation are often performed in step by step and recursive manner (Clough and Penzien, 1993; Chopra, 2007). However, the present method offers closed-form formulas for the efficient seismic analysis of structures using modal analysis.

2. Swift solution of BVPs using a combined finite difference and graph product methods

Finite difference method is a known numerical method for solving BVPs. Like other numerical methods, by increasing the dimension of the problem or reducing the step size (to decrease the error) the amount of computations grows. However, using the solution of tri-diagonal matrices and block tri-diagonal matrices, presented in the references Kaveh (2013) and Yueh (2005) and combining these with finite difference method, the solution of BVPs can be obtained. In the present method, due to finding closed-form formulas, increasing the dimension and step size of the problem lead to additional scalar summation operations. These operations are much less complicated than matrix operations with increased dimension and step size.

In the solution of those problems which leads to ordinary differential equations with boundary conditions, if the coefficients of the equations are constant values, then for second-order equations one obtains numerical tri-diagonal matrices and for fourth-order equations penta-diagonal matrices can be obtained. In case an equation is a differential equation of dimension 2 or 3, the corresponding matrices will be block tri-diagonal matrices. The deformation of beams and plates, buckling of frames and
plates, free vibration and forced vibrations are some examples of this kind which are thoroughly discussed in Kaveh (2013). As an example, in the simplest case when we have an ordinary differential equation of order 2 with some boundary conditions (e.g. deformation of a beam), the final matrix will be in the following form:

\[ Ax = B; \quad A = \begin{bmatrix} b & c \\ a & b & c \\ & \ddots & \ddots \\ & & a & b & c \\ & & & a & b \end{bmatrix}_{n \times n} \]  

(1)

The eigenvalues and eigenvectors of the following matrix can be obtained via Equations (2) and (3), respectively:

\[ \lambda_k = b + 2\sqrt{ac} \cos \frac{k\pi}{n+1}, \quad k = 1, 2, \ldots, n \]  

(2)

\[ v^{(k)} = \begin{pmatrix} v_1^{(k)} \\ v_2^{(k)} \\ \vdots \\ v_n^{(k)} \end{pmatrix}, \quad \bar{v}_j^{(k)} = \begin{pmatrix} \sqrt{\frac{a}{c}} \sin \frac{jk\pi}{n+1} \\ \sin \frac{k\pi}{n+1} \end{pmatrix}, \quad j = 1, 2, \ldots, n \]  

(3)

And also we have:

\[ A^{-1} = V\lambda^{-1}V^t \]  

(4)

Now, using Equations (2)-(4) the components of the inverse of matrix \( A \) are found:

\[ A_{ij}^{-1} = \sum_{k=1}^{n} \left( \frac{\sqrt{a}}{2} \right)^{i-j} \sin \left[ \frac{ik\pi}{n+1} \right] \sin \left[ \frac{jk\pi}{n+1} \right] \frac{1}{\lambda_k} \]  

(5)

where \( i \) and \( j \) indicate row and column, respectively.

In the BVPs with two- or three-dimensional domains, the final matrix obtained from the finite difference method will have the block tri-diagonal form (Kaveh, 2013).

The algorithm for solving BVPs using finite difference method and graph product rules is summarized as follows:

1. Constructing matrix form (i.e. \( Ax = B \)) of the differential equation using finite difference method.
2. Finding eigenpairs of matrix \( A \) using graph products.
3. Inverting matrix \( A \) using Equation (4). For one-dimensional problems this leads to the closed-form solution in Equation (5).
4. Calculating the unknowns via \( x = A^{-1}B \), where \( A^{-1} \) is obtained from the previous step.

Solution of BVPs were obtained using the features of tri-diagonal and block tri-diagonal matrices. In the next section we will study the more complicated IVPs.
3. Analysis of systems with single DOF by transforming an IVP to a BVPs

The solution of IVPs via finite difference method leads to a recursive solution where for obtaining the function value of a specific point, all previous points should be solved in advance. While, in BVPs, as seen previously, by inverting a matrix the function values of all points are gained simultaneously.

Considering the importance of IVPs in structural engineering; especially in the dynamic solution of structures under seismic loads, an efficient method for the analysis of these differential equations is presented. The method works based on converting the IVPs into the BVPs and consequently utilizing the features of BVPs in the efficient solution. Consider the following equation:

\[
\ddot{u}(t) + u(t) = \rho(t); \quad u(0) = u_s, \quad \dot{u}(0) = v_s; \quad t \geq 0
\]

This holds the form of the dynamic equation. Using finite difference method, we will have:

\[
m\ddot{u}(t) + c\dot{u}(t) + k\dot{u}(t) = \rho(t)
\]

\[
\ddot{u}_i = \frac{1}{h^2}(u_{i-1} - 2u_i + u_{i+1}); \quad \dot{u}_i = \frac{1}{2h}(u_{i+1} - u_{i-1}); \quad \frac{1}{h^2}(u_{i-1} - 2u_i + u_{i+1}) + u_i = \rho_i
\]

To find a simple closed-form solution, first this IVP is transformed to a BVP, and then Equation (5) is utilized. This process is explained in the following.

Suppose we want to find function values in the interval \([0, T]\). Consider the \(n\) points used in the finite difference method. Suppose \(u(0) = u_s\) and \(u(T) = u_f\), then we have:

\[
\begin{bmatrix}
  u_2 \\
  u_3 \\
  \vdots \\
  u_{n-2} \\
  u_{n-1} \\
  \end{bmatrix}
\begin{bmatrix}
  F_{n-2}
  \begin{pmatrix}
    1 & -\frac{2}{h^2} & \frac{1}{h^2}
    \\
    -\frac{2}{h^2} & 1 & -\frac{2}{h^2}
    \\
    \frac{1}{h^2} & -\frac{2}{h^2} & 1
    \end{pmatrix}
  
  K_{(n-2) \times (n-2)}
  
  U_{(n-2)}
  \end{bmatrix}
\begin{bmatrix}
  \frac{p_2 - u_s}{h^2} \\
  \frac{p_3}{h^2} \\
  \vdots \\
  \frac{p_{n-2}}{h^2} \\
  \frac{p_{n-1} - u_f}{h^2}
  \end{bmatrix}
\]

Now, the matrix \(K\) has the form as in Equation (1) and therefore it can be inverted using Equation (5). However, the vector \(U\) cannot be found since the value \(u_f\) in the last term of the vector \(P\) is not defined. Now, we want to define \(u_f\):

\[
U = K^{-1}P = DP
\]

\[
u_2 = D_{11} \left( \frac{p_2 - u_s}{h^2} \right) + D_{12} \frac{p_3}{h^2} + D_{13} \frac{p_4}{h^2} + \ldots + D_{1(n-2)} \left( \frac{p_{n-1} - u_f}{h^2} \right)
\]

\[
= a + D_{1(n-2)} \left( \frac{p_{n-1} - u_f}{h^2} \right)
\]

\[
(10)
\]
Defining a virtual node at \( i = 1 \) we can write:

\[
\dot{u}_1 = \frac{1}{2h}(u_2 - u_0) = \dot{u}(0) = v_s \Rightarrow u_0 = u_2 - 2hv_s
\]  

(11)

Substituting Equation (11) in the governing equation at \( i = 1 \), we have:

\[
\frac{1}{h^2}(u_2 - 2hv_s - 2u_1 + u_2) + u_1 = p_1
\]  

(12)

Substituting \( u_2 \) from Equation (10) into Equation (12) results in:

\[
p_{n-1} - \frac{u_f}{h^2} = \frac{p_1 h^2 - u_1 h^2 + 2hv_s + 2u_1 - 2a}{2D_{(n-2)}}
\]  

(13)

All terms in the right hand side of this equation are numerical values and thus the value of right hand of Equation (13) can be inserted in the last term of the vector \( P \) in Equation (8). Therefore, using Equation (9) all function values are calculated simultaneously.

And the value of \( u_f \) is obtained utilizing Equation (13).

Now, considering Equation (7), the general solution for this equation results in a form similar to that of Equation (9):

\[
\begin{bmatrix}
  k - \frac{2m}{k^2} & \frac{m}{k^3} + \frac{c}{2n} \\
  \frac{m}{k^3} - \frac{c}{2n} & k - \frac{2m}{k^2} \\
  \frac{m}{k^3} - \frac{c}{2n} & \frac{m}{k^3} + \frac{c}{2n} \\
  \vdots & \vdots & \ddots & \ddots \\
  \frac{m}{k^3} - \frac{c}{2n} & \frac{m}{k^3} + \frac{c}{2n} \\
  \frac{m}{k^3} - \frac{c}{2n} & k - \frac{2m}{k^2}
\end{bmatrix}
\begin{bmatrix}
  U_n \\
  U_{n-3} \\
  U_{n-4} \\
  \vdots \\
  U_1 \\
  U_0
\end{bmatrix}

\frac{p_2 - \left(\frac{m}{k^3} + \frac{c}{2n}\right)u_s}{K_{(n-2) \times (n-2)}}

\begin{bmatrix}
  u_2 \\
  u_3 \\
  u_4 \\
  \vdots \\
  u_{n-4} \\
  u_{n-3} \\
  u_{n-2} \\
  u_{n-1}
\end{bmatrix}

(14)
and:

\[
p_{n-1} - \left( \frac{m}{h^2} + \frac{c}{2h} \right) u_f = p_1 \left( \frac{k-2n}{\xi} \right) u_x \left( \frac{2n}{\xi} \right) a + 2hv_s \left( \frac{a-n}{\xi} \right)
\]

\[
a = D_{11} \left[ p_2 \left( \frac{m}{h^2} - \frac{c}{2h} \right) u_x \right] + D_{12} p_3 + D_{13} p_4 + \ldots + D_{1(n-3)} p_{(n-2)}
\]

(15)

Now, the only point is obtaining the inverse of matrix \( K \). Matrix \( K \) has the pattern of matrix \( A \) in Equation (1) and therefore \( K^{-1} \) is calculated from Equation (5). Multiplying the vector \( P \) by Equation (5) results in the displacements as:

\[
u_i = \sum_{j=1}^{n-2} \sum_{k=1}^{n-2} \left( \frac{\sqrt{\frac{\pi}{\pi^2 + n^2/4}}}{\sin \left( \frac{k\pi}{n-1} \right)^2} \sin \left( \frac{ik\pi}{n-1} \right) \sin \left( \frac{jk\pi}{n-1} \right) \frac{p_{k+1}}{\lambda_k} \right) ; \quad p_{k+1}
\]

\[
= \begin{cases} 
  p_2 \left( \frac{m}{h^2} - \frac{c}{2h} \right) u_x & k = 1 \\
  p_{k+1} & 1 < k < n-2 \\
  p_{n-1} - \left( \frac{m}{h^2} + \frac{c}{2h} \right) u_f & k = n-2
\end{cases}
\]

(16)

The algorithm for the solution of IVPs using conversion, finite difference method and graph product rules is summarized as follows:

1. constructing matrix form of the problem such in Equation (14) using finite difference method;
2. defining the unknown (last) term in the right side of Equation (14) using Equation (15) that is conversion of the IVP into a BVP;
3. inverting matrix \( K \) in Equation (14) using the closed-form formula in Equation (5); and
4. calculating the components of unknown vector \( U \) in Equation (14) using Equation (16).

4. Seismic analysis of structures using an efficient modal solution

In the previous sections an approach was presented for solving IVPs. Moreover, closed-form formulas were obtained for the eigenpairs of tri-diagonal and block tri-diagonal matrices. Now, we want to utilize the eigenpairs and initial-value solution in the seismic analysis of structures. In the modal analysis, a structure with \( n \) DOF system is decomposed into \( n \) single DOF systems. This decomposition leads to solution of an eigenvalue problem which is laborious for large structures. After the decomposition, \( n \) differential equations should be solved. Because of seismic loads in the right-hand side of these equations, the equations are solved using numerical methods. These numerical methods are recursive methods with step-by-step solutions and again are time-consuming
for large structures. In this section, using the eigenpairs and initial-value solution, we want to ease the two difficult parts of modal analysis. The governing equations in the forced vibration of a structure in the modal analysis can be written as:

\[ M\ddot{x} + C\dot{x} + Kx = P(t), \quad x = \Phi u, \quad \Phi = \begin{bmatrix} \{ \phi \}_1, \ldots, \{ \phi \}_n \end{bmatrix} \]  \hfill (17)

When damping exists, by considering \( C \) as a combination of \( K \) and \( M \), we will have:

\[ C = \alpha K + \beta M \Rightarrow \ddot{u}_i + 2\xi_i\omega_i \dot{u}_i + \omega_i^2 u_i = \varphi_i^1 P(t) \]  \hfill (18)

Equation (18) shows the importance of calculating the eigenpairs.

We consider an \( n \)-story shear building with stiffness and mass of each floor being \( k_i \) and \( m_i \), respectively.

For \( k_i = k \) and \( m_i = m \), the eigenvalues are obtained as follows:

\[ \omega_i = 2\cos \left( \frac{(n-i+1)\pi}{2n+1} \right) \sqrt{\frac{k}{m}} \]  \hfill (19)

Now the eigenvectors are obtained by considering identical heights for the stories:

\[ \varphi_{ij} = \sin \left( \frac{(2i-1)\pi}{2n+1} \right) \frac{2}{\sqrt{m(2n+1)}}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n \]  \hfill (20)

After obtaining closed-form formulas for the eigenpairs, the modal loading is found as:

\[ P_i = \left[ \sin \left( \frac{(2i-1)\pi}{2n+1} \right), \sin \left( \frac{(2i-1)\pi}{2n+1} \right), \ldots, \sin \left( \frac{(n(2i-1)\pi}{2n+1} \right) \right] \frac{2P(t)}{\sqrt{m(2n+1)}} \]  \hfill (21)

When the structure is under seismic motions, we will have:

\[ P(t) = P_{eff}(t) = -M\{1\} \ddot{v}_g(t) \]  \hfill (22)

where \( \{1\} \) is a column of ones and \( \ddot{v}_g(t) \) is the applied acceleration at the base of the structure. Since \( m_i = m \), we will have:

\[ P_i = \left[ \sin \left( \frac{(2i-1)\pi}{2n+1} \right) + \sin \left( \frac{(2i-1)\pi}{2n+1} \right) + \ldots + \sin \left( \frac{(n(2i-1)\pi}{2n+1} \right) \right] \frac{-2m\ddot{v}_g(t)}{\sqrt{m(2n+1)}} \]  \hfill (23)

Calculate the summation of the \( \sin \) with complex analysis, we obtain:

\[ P_i = \alpha \ddot{v}_g(t); \quad \alpha = \left[ \frac{\sin \left( \frac{(2i-1)\pi}{2n+1} \right) + \sin \left( \frac{(n(2i-1)\pi}{2n+1} \right) - \sin \left( \frac{(n+1)(2i-1)\pi}{2n+1} \right)}{2 - 2\cos \left( \frac{(2i-1)\pi}{2n+1} \right)} \right] \frac{-2m}{\sqrt{m(2n+1)}} \]  \hfill (24)

According to Equation (18), for the \( \varphi_i \) mode we will have:

\[ \ddot{u}_i + 2\xi_i\omega_i \dot{u}_i + \omega_i^2 u_i = P_i \]

According to Equations (14)-(16), the closed-form solution is obtained for the \( \varphi_i \) mode with changing the following parameters:

\[ m \rightarrow 1; \quad c \rightarrow 2\xi_i \omega_i; \quad P_{k+1} \rightarrow \alpha \ddot{v}_{gk+1} \]  \hfill (25)
It should be mentioned that $v_{gn-1} = \left(1/h^2 + 2\zeta_i \omega_i / 2h\right) u_i$ and $a$ are calculated only in the step $k = n-2$. Now, the solution of the structure is obtained as:

$$\omega_i = 2 \cos \left(\frac{(n-i+1)\pi}{2n+1}\right) \sqrt{\frac{k}{m}}, \quad y_i = \sum_{j=1}^{n} \varphi_{ij} u_j,$$

$$\varphi_{ij} = \sin \left(\frac{(2i-1)\pi}{2n+1}\right) \frac{2}{\sqrt{m(2n+1)}}$$  \hspace{1cm} (26)

All that should be done for the modal analysis of a structure is presented in Equations (16) and (26). As the degrees of freedoms and dimension of the corresponding matrices increase, the efficiency of the method is more apparent. This is because a usual solution, involved in eigensolution and matrix inversion with the computational complexities of $O(n^{2.373})$ and $O(n^3)$, respectively (Miller, 1975; Williams, 2011), is converted to the closed-form formula above with simple summation operations. It is achieved by converting an IVP to a BVP where the graph product rules can be applied to the obtained tri-diagonal and block tri-diagonal matrices.

The seismic analysis using the proposed modal solution is summarized as follows:

1. constructing dynamic matrix equation for a multi DOF system and changing the coordinate system to decompose the problem into $n$ single DOF problem (i.e. modal analysis) through Equations (17) and (18);
2. obtaining the eigenpairs using the closed-form relationships in Equation (26);
3. finding the closed-form modal loading in Equation (18) obtained through the eigenpairs and mathematical manipulations in Equations (21)-(24);
4. defining the matrix equation in Equations (14) and (25) associated with differential equation of $i$th mode (i.e. $i$th single DOF) using finite difference method;
5. calculating the components of unknown vector $u_i$ using Equations (16) and (25); and
6. finding the whole response under seismic loading through superposition of all modes in Equation (26).

5. Numerical example
Consider the schematic of a ten-story building shown in Figure 1 and its plan provided in Figure 2. Typical frame structures are shown in Figures 3 and 4. The structural system is a dual one composed of moment resisting and braced frames. The height of all stories is equal to 3.5 m. The dead load and live load are considered as 200 and 650 kg/m$^2$, respectively. The first four stories are chosen as type 1, the second three stories as type 2 and the third three stories as type 3. The stories of the same type are given same cross-sections during the design process (e.g. the beams of the stories 5-7 (defined as type 2) hold the same cross-sections). Eight cross-sections are chosen for columns as well as eight for beams and eight for braces. The cross-sections and their dimensions are shown in Figures 5 and 6 and Table I. The record of Imperial Valley ground motion is applied to the structure in the direction $X$. For optimizing the structural design, a genetic algorithm with 50 individuals and 60 generations are utilized.
Figure 1.
Schematic of a ten-story building

Figure 2.
The plan of the building

Figure 3.
The typical braced frame

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Figure 4. The typical moment resisting frame

Figure 5. The typical cross-section of beams

Figure 6. The typical cross-section of columns
The material properties of the frame are considered as:

\[ E = 2\varepsilon_8 \left( \frac{kN}{m^2} \right), \quad \rho = 76.82 \left( \frac{kN}{m^3} \right), \quad \text{and} \quad \nu = 0.3 \]

The following constraints are used in the genetic algorithm:

1. The constraints corresponding to the beams and columns include:

   \[
   \text{if } \frac{P_r}{P_n\phi_c} < 0.2 \Rightarrow \frac{P_r}{2P_n\phi_c} + \left( \frac{M_r}{\phi_b M_n^t} \right) \\
   \Rightarrow \frac{P_r}{P_n\phi_c} + 8 \left( \frac{M_r}{\phi_b M_n^t} \right) \leq 1
   \]

2. The constraints corresponding to the bracing members include:

   \[ \lambda \leq 4.23 \sqrt{\frac{E}{F_y}}; \quad P_{cr} \leq \phi_c, P_{ct}; \quad P_{tr} \leq \phi_t, P_{tn} \]

3. The constraints of the drift of stories and the roof include:

   \[
   \text{if } T \geq 0.7 \text{sec} \Rightarrow \overline{\Delta M} < 0.02h_s \quad \text{if } T < 0.7 \text{sec} \Rightarrow \overline{\Delta M} < 0.025h_s
   \]

See Kaveh et al. (2013b) for further details about the constraints.

Here, the adaptive coefficient is utilized for the penalty function introduced by Barbosa and Lemonge (2003). The fitness of this coefficient was shown in Goldberg (1989).

Because of 50 individuals and 60 generations, 3,000 time-history analyses should be performed. Time-history analyses can be very laborious even for only one analysis. In Kaveh et al. (2013b) neural networks were used to avoid time-history analyses, however, the method had its own problems like inaccurate answers in three-dimensional buildings. The importance of optimal analysis can be even seen in static analysis (Rahami et al., 2014). However, using the optimal seismic method, summarized in Equations (16) and (26), the analyses are efficiently performed. Since in the record of the earthquake \( \Delta t = 0.005 \text{sec} \), consequently the step size in the finite difference method is chosen as \( h = 0.005 \text{sec} \). By considering the initial values equal to

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<tbody>
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| Brace | 2UNP120 | 2UNP140 | 2UNP160 | 2UNP180 | 2UNP200 | 2UNP220 | 2UNP240 | 2UNP260 |

Table I. The cross-sections used in the genetic algorithm
zero \((u_1 = v_1 = 0)\) and \(\xi = 3\) percent for all modes, all parameters in Equations (16) and (26) are defined. The relationships corresponding to \(\omega_I\) and \(\varphi_{ij}\) in Equation (26) are valid for shear buildings with the same stiffness and mass in all stories as mentioned above. In the reference Kaveh (2013), it was shown that the relationships are applicable to shear buildings in which the stories do not have the same stiffness and mass and the error was less than 5 percent. However, the structure in this example is not a shear frame and therefore the relationships corresponding to \(\omega_I\) and \(\varphi_{ij}\) are not valid. Moreover, obtaining the eigenpairs for 3,000 structures using a traditional method is very time consuming. The weakness of the genetic algorithm is the numerous generations which should be solved. If we can take advantage of these generations in a way, the weakness is converted to a benefit. Here, an idea similar to Rayleigh-Ritz ratio is used in which instead of calculating the eigenpairs for all generations, the eigenpairs are only calculated in the first generation and the eigenpairs of the following generations are obtained via the previous generations.

Suppose in the \(N^{th}\) generation for a structure we have:

\[
K_N \varphi_N = \omega_N^2 M_N \varphi_N; \quad \omega_N^2 = \frac{\phi^t_N M^{-1}_N K_N \varphi_N}{\phi^t_N \varphi_N}
\]

Since little changes occur from a generation to another one, we can write:

\[
\omega_{N+1}^2 = \frac{\phi^t_{N+1} M^{-1}_{N+1} K_{N+1} \varphi_{N+1}}{\phi^t_{N+1} \varphi_{N+1}}
\]

Using this equation, eigenvalues of the new generation are obtained via utilizing the eigenvectors of the previous generation. Then the new eigenvectors are found using the following equation:

\[
M^{-1}_{N+1} K_{N+1} \varphi_{N+1} = \omega_{N+1}^2 \varphi_{N+1}
\]

The error obtained from this method is less than 4 percent.

Now, considering the first three modes and \(\xi = 3\) percent for all modes, the optimal designed structure is obtained as shown in Table II.

### 6. Conclusion

In this paper a combined graph products and finite difference method is developed for the solution of BVPs and IVPs. The boundary-value equations are directly solved in the present method, while the initial-value equations are solved through converting to the BVPs. Using the results of IVPs, an efficient solution is obtained for the seismic analysis of structures via modal analysis. The closed-form formulas obtained by this method can efficiently be solved, and the chosen time steps have a small effect on the computational time. This means by choosing smaller time steps one can gain more

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**Table II.** Cross-sections of the optimally designed structure
accurate results with no need to large amounts of computations. While the traditional methods of seismic analysis result in the laborious step by step solutions, the present method obtains all answers together.

References


Williams, V.V. (2011), *Breaking the Coppersmith-Winograd Barrier*, UC Berkeley, Berkeley, CA.


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