Estimation of Coda-Wave Attenuation in the Central and Eastern Alborz, Iran

by Mohsen Farrokhi, Hosseyn Hamzehloo, Habib Rahimi, and Mostafa Allameh Zadeh

Abstract The quality factor of coda waves $Q_c$ has been estimated by using single backscattering (SBS) and single isotropic-scattering (SIS) models. The earthquakes used were recorded by three permanent and one temporary network located in the central and eastern Alborz, Iran. The database was composed of 746 local earthquakes with local magnitude from 1.1 to 5.7. The estimated $Q_c$ has been found to be similar for lapse times greater than twice the $S$-wave travel time ($2t_S$) for both methods.

The estimated $Q_c$ for central frequencies $<1.0$ Hz shows less frequency dependency compared with the higher frequencies. By using a $Q_0 f^n$ relation, the average frequency dependence of $Q_c$ for the whole area has been estimated as $59 f^{1.03}$, $69 f^{0.97}$, $78 f^{0.97}$, $105 f^{0.93}$, $123 f^{0.89}$, $159 f^{0.79}$, and $203 f^{0.68}$ for central lapse times 30, 40, 50, 65, 85, 110, and 155 s, respectively. We found the value of $Q_0$ decreases at an average depth of 78 km, which may be the result of high-dissipating media at this depth. The average $Q_c$ values, estimated for central and eastern Alborz and their frequency-dependent relationships are similar to those of tectonically active regions.

Introduction

Seismic attenuation generally refers to the decrease in the amplitude of seismic waves with increasing distance from the source, due to elastic and anelastic properties of the medium. Attenuation due to the anelastic properties of media is generally explained as intrinsic absorption ($Q_i^{-1}$), in which the seismic energy dissipates due to the conversion of kinematic energy into heat. Anelastic properties depend on grain defect, grain boundary, internal friction, sliding processes across cracks, and thermoelastic effect (Jackson and Anderson, 1974). Attenuation due to scattering ($Q_{Sc}^{-1}$) is an elastic process in which the energy of seismic waves is not dissipated, but redistributed due to the reflection, refractions, and conversion of the incident wave to other waves. Continuous wave trains following the direct $P$ and $S$ waves, known as coda waves, are the results of seismic-wave scattering due to inhomogeneities that exist within the propagation media (Aki, 1969; Sato and Fehler, 2008; Sato et al., 2012). Total attenuation of seismic waves is generally expressed by the inverse of the quality factor ($Q_i^{-1}$) and is the result of the combined effects of scattering attenuation ($Q_{Sc}^{-1}$) and intrinsic absorption ($Q_i^{-1}$): $Q_i^{-1} = Q_{Sc}^{-1} + Q_i^{-1}$.

One of the most used methods for estimation of seismic-wave attenuation is based on the decay rate analysis of the coda wave envelope. The attenuation of coda waves ($Q_c^{-1}$) quantifies the average attenuation property inside a prolate spheroidal shell with source and receiver in its foci (Pulli, 1984; Sato et al., 2012). The $Q_c^{-1}$ represents the combined effect of intrinsic and scattering attenuation (Del Pezzo, 2008). Its value is close to the value of intrinsic attenuation (Gusev, 1995; Del Pezzo, 2008; de Lorenzo et al., 2013). Wennerberg (1993) noted that when $S$-wave attenuation ($Q_s^{-1}$) is dominated by intrinsic attenuation ($Q_i^{-1}$), and the $S$-wave amplitude to eliminate the influence of source and site effects. Wu (1985) introduced radiative transfer theory (RTT) into the field of seismology, in which both multiple and single scattering is taken into account. Zeng et al. (1991) proposed an exact solution for the radiative transfer equation (RTE) in three dimensions. Zeng (1991) obtained an approximation for RTE (Boltzmann equation) to make it simpler for calculation. According to RTT, Aki (1992) concluded that $S$-coda waves are mainly composed of scattering $S$ waves or conversion of $P$ to $S$-waves. Wennerberg (1993) introduced a simple method for separation of intrinsic and scattering attenuation by using Zeng (1991) approximation and the single backscattering (SBS) model of Aki and Chouet (1975). In research by Paaschens (1997),
a new approximation for RTE was introduced that gives very close results to the exact solution of RTE.

The attenuation of seismic waves is one of the basic physical parameters that is closely related to the seismicity and regional tectonic activity of a particular area. This is also important for seismic-hazard measurement. The scattering is produced by irregular topography, complex surface geology, and the heterogeneous elastic property of the rocks, fault, and cracks. The spatial variation of the regional coda quality factors has been utilized for a better understanding of tectonics, seismicity, seismic risk analysis, and engineering seismology (Singh and Herrmann, 1983; Jin and Aki, 1988). The attenuation parameter is very crucial for seismological studies, especially in active regions. The knowledge of the relative contribution of scattering and intrinsic attenuation effects is important so as to make a correct geological and tectonic interpretation (Mayeda et al., 1992; Bianco et al., 2002; Del Pezzo et al., 2006). Attenuation also depends on depth and lateral heterogeneity. Attenuation variations in the earth are usually greater than the velocity variations. A low value of $Q(Q < 200)$ is associated with the tectonically and seismically active regions, whereas a high value of $Q(Q > 600)$ is characteristic of seismically inactive and stable regions.

The seismically active region of central and eastern Alborz is the most populated region of Iran, with a population in excess of 20 million inhabitants. The seismic hazard of this region is very high (Building and Housing Research Center, 2007; Hamzehloo et al., 2012). The capital of Iran, Tehran, is located in this region, which has experienced eight large destructive earthquakes with magnitude greater than 7 from the fourth century B.C. to 1830 (Ambraseys and Melville, 1982). On 17 October 2009, a moderate earthquake $M_{L}$ 4.0 occurred in the vicinity of Tehran (Fig. 1). A temporary network (TN) was installed from central Alborz to the east of Alborz for a period of 14 months.

Attenuation of the Alborz region has been studied by several investigators using different dataset and methods (e.g., Motazedian, 2006; Rahimi et al., 2010; Motaghi and Ghods, 2012; Naghavi et al., 2012). In this study, we estimated attenuation of the coda wave by using the SBS (Aki and Chouet, 1975) and SIS models (Sato, 1977). Our new dataset provides a reliable estimation of the attenuation of the coda wave by considering 87 stations and 746 earthquakes. In this study, the attenuation property was investigated along central and eastern Alborz. The results provide data to fill an important gap in knowledge of the $Q$ factor, and these results are expected to make significant contributions to the understanding of the lithospheric structure of central and eastern Alborz.

**Tectonic-Setting and Seismic Activity**

The Alborz region, which is seismically active with an east–west-trending mountain belt, is a folded and faulted area that extends for a distance of 960 km across the northern part of Iran. The region is bounded to the north by the Caspian Sea block and to the south by the micro-plateau of central Iran. The Alborz region is formed as the result of a collision between the micro-plateau of central Iran and Eurasia, starting in the late Triassic (Stöcklin, 1974; Jackson and Mckenzie, 1984), and has undergone three major tectonic events. The first event was shortening due to collision, which led to thrusting and folding across northern Iran (Allen et al., 2003; Guest et al., 2006). Next, the central Alborz underwent extension, causing andesitic eruptions. The Damavand volcano provides evidence of this event (Davidson et al., 2004). The third tectonic event is the middle Miocene-to-recent collision-related compression, which affects northern Iran and the Caspian block (Guest et al., 2007).

The recent Cenozoic evolution has been explained as a result of strain partitioning between left-lateral strike slip and thrust faults parallel to the mountain belt (Allen et al., 2003). North–south shortening of 8 ± 2 mm/yr has been reported across the Alborz range between central Iran and the southern Caspian shore (Vernant et al., 2004). Its total shortening since the early Pliocene is estimated to be 30 km at the longitude of Tehran (Allen et al., 2003). The strikes of the Alborz range vary from N110° E in the western part to N80° E in the eastern part, and existing faults of the region are parallel to the range (Allen et al., 2003), making a V-shaped structure for the Alborz mountains. Hollingsworth et al. (2008) proposed that the strain partitioning in Alborz is associated with the southwestward motion of the South Caspian Sea block. Ritz et al. (2006) mentioned the same result for the clockwise rotation for the Caspian block. This motion, accompanied by north–south convergence between central Iran and the Caspian block, causes the trend of the tectonic regime in the Alborz mountain ranges.

The distribution of historical and instrumental earthquakes (Fig. 1) shows that the region has experienced eight large destructive earthquakes with magnitudes greater than 7 from the fourth century B.C. to 1830 (Ambraseys and Melville, 1982). The most important instrumental earthquakes...
occurred in this region are the 1 September 1962 $M_w$ 7.2 Buin Zahra earthquake, the 22 June 2002 $M_w$ 6.5 Changleure (Avaj) earthquake, the 28 May 2004 $M_w$ 6.3 Firozabad Kojor earthquake, and the 18 June 2007 $M_w$ 5.9 Kahak-Qom earthquake. The 17 October 2009 magnitude 4.0 Tehran–Ray earthquake was the nearest instrumentally recorded event to Tehran.

Data

The attenuation properties of this region were investigated using events recorded by one temporary and three permanent networks. The TN had 43 seismic stations and was installed from 20 October 2009 to 22 December 2010 (Fig. 2) by the International Institute of Earthquake Engineering and Seismology (IIEES). The TN consisted of 38 Guralp CMG-6TD velocity seismometers (relatively flat response between 0.1 and 50 Hz), and five Guralp CMG-5TD accelerometers. The TN recorded continuously with a sampling rate of 100 Hz.

Additional data were obtained from permanent networks in the study area to increase the ray coverage. The waveforms of 22 short-period seismometers of the Iranian Seismological Network (Iranian Seismological Center [IRSC]; Ghods and Sobouti, 2005), 9 broadband seismometers of IIEES, and 13 stations of the Tehran Disaster Municipality Management Center (TDMMC) were added to the database. Figure 2 shows a map of the study area, including the locations of stations of temporary and permanent networks and of the earthquakes used in this study. The data gathered from the IRSC network were recorded and digitized with 50 samples/s.

The IIEES network instruments are Guralp CMG-3T with flat instrumental response between 120 s and 50 Hz and digitized with a sampling rate of 50 samples/s. The instruments of TDMMC are short-period seismometers (CK 1', Russia) with digitization sampling rate of 71.43 samples/s and natural frequency of 1 Hz.

The data processing can be summarized in three steps. First, the short-term average/long-term average method was used to extract the events, then all of the extracted events were merged, preprocessed, and located using HYPOCENTER subroutine (Lienert and Havskov, 1995). More than 3000 events were recorded by the TN. Last, we selected 746 local earthquakes, which were located using more than eight stations and epicentral error of less than 10 km. The selected earthquakes with local magnitude $M_L$ from 1.1 to 5.7 were considered for estimation of coda-wave attenuation. The coda $Q_c$ has been estimated for hypocentral distances in the 1.5–200 km range. The magnitude–distance distribution of our dataset is shown in Figure 3. The maximum hypocentral distance of 160 km has been considered for events with magnitudes $<3$ (Fig. 3).

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**Figure 2.** (a) Locations of seismic networks used in this study: the black triangles are from the Iranian Seismological Center (IRSC), the inverted triangles are the broadband seismic stations of the International Institute of Earthquake Engineering and Seismology (IIEES), the black squares are the stations of Tehran Disaster Municipality Management Center (TDMMC), and the white triangles are the stations of the temporary network (TN). (b) Location of selected earthquakes used in this study. Magnitudes are represented by circles of different sizes.

$Q_c$ Estimation

The SBS model (Aki and Chouet, 1975) and SIS model (Sato, 1977) were used to estimate coda-wave attenuation, respectively.

**SBS Method**

The SBS model is generally used for describing the behavior of the coda waves from local earthquakes (Aki and Chouet, 1975). According to Aki and Chouet (1975), the coda waves are interpreted as backscattered body waves generated by randomly distributed heterogeneities in the Earth’s crust and upper mantle. In this method, source and receiver are supposed to be at the same point.

The amplitude of the coda wave envelope $A_c(f, t_c)$ in central frequency ($f$) at lapse time ($t_c$) (measured from the origin time of the seismic event) can be shown as

$$A_c(f, t_c) = I(f)A_0(f)t_c^{-\alpha} \exp\left(-\frac{\pi f t_c}{Q_c}\right),$$

(1)
in which $I(f)$, $A_0(f)$, and $Q_c$ are instrumental response, source function at frequency $f$, and coda-wave quality factor, respectively. It is well known that the decay rate of coda wave envelope of small earthquakes at local distances is independent of epicentral distance and earthquake size. On the other hand, it depends on lapse time from the origin time of the earthquake (Aki and Chouet, 1975; Sato, 1977; Phillips and Aki, 1986). The parameter $\alpha$ is the geometrical spreading factor and is considered to be 1 for body waves. Equation (1) can be written as

$$\ln[A_c(f, t_c)A_0(f)] - \frac{\pi f}{Q_c} t_c,$$

in which $Q_c$ for a specific frequency and central lapse time can be estimated using the linear regression of logarithmic amplitude of coda-wave decay rate versus lapse time ($t_c$) (Rautian and Khalturin, 1978).

### Data Processing

SBS and SIS methods were used to estimate $Q_c$ values. The velocity waveforms, which were provided by horizontal components, were used in the processing procedure. The coda $Q$ was individually estimated by using each source–station waveform and SIS and SBS methods. The average estimated values of $Q_c$ were considered as the $Q_c$ at each frequency bands.

Each waveform was considered 30 s before and 240 s after the $P$-wave arrival time. The waveform was corrected for baseline correction, then a cosine taper with width of 5% of the data length was applied to both ends of each waveform. Out of 87 stations, five stations were acceleration seismometers, so we converted the data recorded at these five stations to velocity records. Each waveform was filtered using the Butterworth band-pass filter with central frequencies 0.375, 0.75, 1.5, 3, 6, 12, and 24 Hz (Table 1).

The amplitude of the coda wave envelope was calculated using the formula

$$\ln\left[\frac{A_c(r, f, t_c)}{\sqrt{K(a)r}}\right] = \ln[I(f)A_0(f)] - \frac{\pi f}{Q_c} t_c,$$

in which $t_S$ is the arrival time of the $S$ wave. The $Q_c$ is estimated in a distinct frequency band and for lapse times greater than twice the $S$-wave travel times ($2t_S$) by using linear regression of logarithmic amplitude of coda-wave decay rate versus lapse time ($t_c$) (Rautian and Khalturin, 1978).

### SIS Method

The general model for excitation of coda waves, proposed by Sato (1977) and which assumes isotropic-scattering and a nonzero source-to-receiver distance ($r$), is also used in this study to investigate the method dependency of the derived coda-$Q$ values. In this method, the coda wave envelope is estimated using

$$\ln\left[\frac{A_c(r, f, t_c)}{\sqrt{K(a)r}}\right] = \ln[I(f)A_0(f)] - \frac{\pi f}{Q_c} t_c,$$

in which $K(a)$ is expressed as

$$K(a) = \frac{1}{a} \ln\left[\frac{a+1}{a-1}\right]$$

and

$$a = \frac{t_c}{t_S} > 2t_S,$$$$

in which $t_S$ is the arrival time of the $S$ wave. The $Q_c$ is estimated in a distinct frequency band and for lapse times greater than twice the $S$-wave travel times ($2t_S$) by using linear regression of logarithmic amplitude of coda-wave decay rate versus lapse time ($t_c$) (Rautian and Khalturin, 1978).

### Table 1

<table>
<thead>
<tr>
<th>Frequency Band (Hz)</th>
<th>Central Frequency (Hz)</th>
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<tbody>
<tr>
<td>0.25–0.50</td>
<td>0.375</td>
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<tr>
<td>0.50–1.0</td>
<td>0.75</td>
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<tr>
<td>1.0–2.0</td>
<td>1.5</td>
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<tr>
<td>2.0–4.0</td>
<td>3.0</td>
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<td>4.0–8.0</td>
<td>6.0</td>
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<tr>
<td>8.0–16.0</td>
<td>12.0</td>
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<td>16.0–24.0</td>
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</table>
Results

In this study, $Q_c$ was evaluated at seven central lapse times for each waveform. For this purpose, 26,000 horizontal waveforms of 746 local earthquakes have been processed.
We estimated $Q_c$ values for seven frequency bands in central lapse times of 30, 40, 50, 65, 85, 110, and 155 s using SBS and SIS methods (Table 2). We only considered $R^2$ of the linear regression greater than 0.8. The average of the $Q_c$ values was considered for each central lapse time and frequency band.

The estimated $Q_c$ values were nearly constant at frequencies $< 1$ Hz (Fig. 5). The $Q_c$ values manifest a dependence on frequency at frequencies $> 1$ Hz, which follows the law by Müller (1983),

$$Q(f) = Q_0(f_t = 1) \left( \frac{f}{f_t} \right)^n,$$

in which $Q_0$ is the value of $Q_c$ at a reference frequency $f_t$ is at 1 Hz, and $n$ is a power ($n \geq 0$). We made a regression analysis assuming a power law of equation (8) for frequencies $> 1$ Hz. The frequency-dependence relation has been determined for central and eastern Alborz for $Q_c$ values estimated using SBS and SIS methods (Table 2). Figure 6 shows the mean value of $Q_c$ as a function of frequency for different lapse times.

**Discussion**

In this article, SBS and SIS methods were applied to estimate attenuation properties of coda waves using 26,000 horizontal components from 746 earthquakes. The frequency-dependent $Q_c$ relationship for central and eastern Alborz was determined, and the relation shows low $Q_0$ values and high frequency dependence ($n$). The derived results are discussed below.

**Comparison of SBS and SIS Method**

In this study, two methods of SBS (Aki and Chouet, 1975) and SIS (Sato, 1977) were used to estimate coda-$Q$ values using equations (2) and (3), respectively. The left side of equations (2) and (3) is composed of two parts: the first part $A_c(f, t_c)$ is the amplitude of coda waves, which is calculated using equation (7). The second part is the geometrical-spreading correction ($GsC$) of coda-wave amplitudes, which are $t$ and $r/\sqrt{K(a)}$ for SBS and SIS methods, respectively. The first part is identical for both SBS and SIS methods; therefore, the estimated $Q_c$ value is only dependent on the calculated $GsC$ values by SBS and SIS methods. The amplitudes of $GsC$ are shown for both methods at different lapse times in Figure 7.

The slope of geometrical correction has an impact on determination of coda $Q$. A higher slope of geometrical correction gives higher coda-$Q$ values. The normalized amplitude (NA; equation 9) of geometrical correction is shown for each central lapse times in Figure 7. For short lapse times ($t_c < 85$ s), the NA values of the SIS method have slightly higher slopes compared with the SBS method, which gives higher values for estimated $Q_c$.

$$NA(t_c) = \log \left( \frac{GsC(t_c)}{GsC(t_1)} \right).$$

As the slope of the NA values of the two methods is nearly comparable for higher lapse times ($t_c > 85$ s), we
expect to estimate the same values for $Q_c$. The estimated $Q_c$ values, which were obtained using SBS and SIS methods, show high correlation for all lapse times (Fig. 8). Sato et al. (2012) reported the coda-$Q$ values derived by the SIS method converge to those of the SBS method for long lapse times, whereas we observed the same approach at lapse times $> 2t_S$ ($t_c > 30$ s).

**Attenuation Variation with Depth**

The value of $Q_c$ increased with lapse time (Fig. 6). The estimated $Q_c^{-1}$ at different lapse times indicates the average attenuation properties of seismic waves at different depth of crust and lithosphere. The responsible scatterers for generation of coda waves are generally assumed to be distributed over the surface area of a prolate spheroid. Scattered seismic

**Figure 6.** The variation of $Q_0^{-1}$ and $n$ for central and eastern Alborz (left) and variation of $Q_c$ versus frequency (right).

**Figure 7.** Geometrical correction criteria for the SBS and SIS methods for different lapse times.
waves sample the attenuation property of a prolate spheroidal-shaped area with the station and hypocenter at its foci (Pulli, 1984). The large semiaxis of this spheroid is $a_1 = c t/0.136$, in which $c$ is the average $S$-wave velocity of the crust ($c = 3.5$ km/s) and $t$ is the central lapse time (calculated from the origin time). The small semiaxis of spheroid is calculated as $a_2 = (a_1^2 - \Delta^2)^{0.5}$, in which $\Delta$ is epicentral distance. The average depth of the spheroid can be determined by $h = h_\text{av} + a_2$. The parameter $h_\text{av}$ is the average focal depth of events (Pulli, 1984; Havskov et al., 1989; Canas et al., 1995). The average penetration depth for each waveform was calculated using the Pulli (1984) method, and that depth was assigned as the mean penetration depth of coda waves at each lapse time. The values of $Q_0$ and average penetration depth versus lapse times are plotted in Figure 9. The value of $Q_0$ increases as lapse time increases, which shows that dissipation of energy decreases as depth increases.

The trend of $Q_0$ versus lapse time shows a slight decrease for lapse time 50 s (average depth of 78 km; Fig. 9), which indicates the presence of a highly dissipating media at this depth and may be indicative of a shallow lithosphere–asthenosphere boundary (LAB). Our results are in agreement with the results of other research showing a low-velocity zone beneath the Moho-depth of the Alborz Mountains (Maggi and Priestley, 2005; Shad Manaman et al., 2011) and $Q_c$ estimation of Alborz and the central Iran regions (Rahimi et al., 2010). Rahimi et al. (2010) reported the same change in the trend of $Q_0$ at lapse times around 50 s. They related this to the existence of a low-$Q$ layer beneath Moho depth. Sodoudi et al. (2009) have estimated the Moho depth to be 46 km and introduced the depth of 90 km for LAB by using the receiver function method. Rahimi (2010) reported two layers with low values of the shear-wave quality factor ($Q_\beta$) for the Alborz region, extending from 66 to 126 km and from 126 to 206 km, respectively, based on the surface-wave study for the Iran plateau.

New investigations reported that coda waves are more sensitive to the change in elastic properties of the media (increase in temperature, stress, etc.) than are the first arrivals of seismic waves (Grêt, 2004; Grêt et al., 2006). The decrease in the estimated value of $Q_c$ at depth of ~78 km may be related to a change in the elastic properties of the region. This low-$Q$-value may be related to partial melting of the oceanic crust of the South Caspian basin (SCB). The SCB is supposed to be overthrusted by the Alborz Mountains (Priestley et al., 1994; Jackson et al., 2002). This highly dissipating layer

Figure 8. Correlation between the values of $Q_c$ estimated using SBS and SIS methods.
Estimation of Coda-Wave Attenuation in the Central and Eastern Alborz, Iran

The RTE, which was introduced into the field of seismology by Wu (1985), considered single- as well as multiple-scattering processes. Zeng (1991) proposed a hybrid approximation for the exact solution of RTE in three dimensions. Paasschens (1997) produced an approximation for RTE (Boltzmann equation) for 1, 2, 3, and 4 dimensions and heuristically proposed a better approximation of RTE, based on the interpolation of exact solution of RTE for 2 and 4 dimensions. The new approximation has been used in attenuation determination of S waves (Ugalde et al., 2010), separation of intrinsic and scattering attenuation (Abubakirov, 2005), and seismic moment determination (Sens-Schönfelder and Wegler, 2006). We used Paasschens (1997) approximation to test the reliability of our processing for both SBS and SIS methods. This approximation of RTE for intensity is

\[ P(r, t, Q_i, Q_{Sc}, f) \approx \frac{1}{4\pi r^2} \delta(\nu t - r) U(t, Q_i, Q_{Sc}, f) + \frac{(1 - \frac{r^2}{v^2 t^2})^{1/8}}{(2\nu^2 Q_{Sc} t/3)^{3/4}} U(t, Q_i, Q_{Sc}, f) \]

\[ G(x) \approx \exp(x) \sqrt{1 + 2.026/x}, \]

where \( Q_i \), \( Q_{Sc} \), \( v \), \( r \), and \( f \) are intrinsic absorption, scattering attenuation, velocity, hypocentral distance, and frequency, respectively. \( P \) is the intensity of the radiated wave energy from the source. In this case, \( H \) and \( \delta \) represented the Heaviside and delta function, respectively.

The synthetic envelope of seismograms has been calculated at different hypocentral distances with different attenuation properties \((Q_i^{-1}, Q_{Sc}^{-1}, f)\) using equation (10). We selected the input parameters according to de Lorenzo et al. (2013; Table 1): \( f = 6.3 \text{ Hz, } Q_i^{-1} = 1/220 \) and \( Q_{Sc}^{-1} = 1/523 \), \( t_c = 40 \text{ s, } Q_s = 235 \). We considered an impulse force and half-space media with average shear-wave velocity of 3.5 km/s. The synthetic envelope versus lapse was shown for an earthquake with hypocentral distance and central frequency equal to 40 km and 6.3 Hz, respectively (Fig. 10a).

We found two important results based on the synthetic seismogram envelopes. First, the results of SBS and SIS methods show that the SIS method estimates slightly higher coda-\( Q \) values for lower lapse times. These results are comparable at longer lapse times \((t_c > 65 \text{ s; Fig. 10b})\). Second, the estimated value of \( Q_c \) converges to \( Q_i \) at longer lapse times.

Comparison of Results with Other Regions

Coda-\( Q \) values and its frequency-dependent relation were determined using equation (8) for different lapse times (Table 2). Low \( Q_0 \) values \((Q_0 < 200)\) and high frequency-dependent power \((n)\) have been reported by different investigators for tectonically active regions (Roecker et al., 1982; Hatzidimitriou, 1993; Akinci et al., 1994; Baskoutas, 1998; Giampiccolo et al., 2002; Rahimi and Hamzehloo, 2008; Ma’hood and Hamzehloo, 2009; de Lorenzo et al., 2013) and high \( Q_0 \) values \((Q_0 > 600)\) with low-frequency dependence \((n)\) for tectonically stable regions (Singh and Herrmann, 1983; Pulli, 1984; Pujades et al., 1991; Ramakrishna et al., 2007). As shown in Figure 11, some studies reported moderate \( Q_0 \) values (between tectonically active and stable regions) with high frequency dependence (Scherbaum and Kisslinger, 1985; Kvanme and Havskov, 1989; Kumar et al., 2005). The quality factor \( Q_0 \) and frequency dependence \( n \) estimated in this study for central and eastern Alborz correlate well with tectonically active regions (Fig. 11). We concluded that the \( Q_s \) estimations of this study \((78 f^{0.97})\) are greater than the estimated \( Q_s \) for the Zagros region \((99 f^{0.84}; Rahimi and Hamzehloo, 2008)\) but lower than those of central Iran \((101 f^{0.94}; Ma’hood and Hamzehloo, 2009)\).

\[ U(t, Q_i, Q_{Sc}, f) = \exp\left(-2\pi f t \left[ \frac{1}{Q_i} + \frac{1}{Q_{Sc}} \right] \right), \]

in which \( Q_i^{-1} \), \( Q_{Sc}^{-1} \), \( v \), \( r \), and \( f \) are intrinsic absorption, scattering attenuation, velocity, hypocentral distance, and frequency, respectively. \( P \) is the intensity of the radiated wave energy from the source. In this case, \( H \) and \( \delta \) represented the Heaviside and delta function, respectively.

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We found two important results based on the synthetic seismogram envelopes. First, the results of SBS and SIS methods show that the SIS method estimates slightly higher coda-\( Q \) values for lower lapse times. These results are comparable at longer lapse times \((t_c > 65 \text{ s; Fig. 10b})\). Second, the estimated value of \( Q_c \) converges to \( Q_i \) at longer lapse times.

Comparison of Results with Other Regions

Coda-\( Q \) values and its frequency-dependent relation were determined using equation (8) for different lapse times (Table 2). Low \( Q_0 \) values \((Q_0 < 200)\) and high frequency-dependent power \((n)\) have been reported by different investigators for tectonically active regions (Roecker et al., 1982; Hatzidimitriou, 1993; Akinci et al., 1994; Baskoutas, 1998; Giampiccolo et al., 2002; Rahimi and Hamzehloo, 2008; Ma’hood and Hamzehloo, 2009; de Lorenzo et al., 2013) and high \( Q_0 \) values \((Q_0 > 600)\) with low-frequency dependence \((n)\) for tectonically stable regions (Singh and Herrmann, 1983; Pulli, 1984; Pujades et al., 1991; Ramakrishna et al., 2007). As shown in Figure 11, some studies reported moderate \( Q_0 \) values (between tectonically active and stable regions) with high frequency dependence (Scherbaum and Kisslinger, 1985; Kvanme and Havskov, 1989; Kumar et al., 2005). The quality factor \( Q_0 \) and frequency dependence \( n \) estimated in this study for central and eastern Alborz correlate well with tectonically active regions (Fig. 11). We concluded that the \( Q_s \) estimations of this study \((78 f^{0.97})\) are greater than the estimated \( Q_s \) for the Zagros region \((99 f^{0.84}; Rahimi and Hamzehloo, 2008)\) but lower than those of central Iran \((101 f^{0.94}; Ma’hood and Hamzehloo, 2009)\).
results seem to be comparable with Rahimi et al. (2010), whose results for central Iran and Alborz ($79^\circ f^{1.07}$) have higher lapse times.

Coda waves are a combination of scattered seismic waves produced by the existing heterogeneities within the media with different wavelength and different scattering strength. Del Pezzo (2008) mentioned that differences in the attenuation patterns of high and low frequencies can be explained in terms of scale length of heterogeneities; therefore, a high value of $n$ can be related to heterogeneity of media. Our results also indicate the Alborz region is a tectonically active heterogeneous media.

**Conclusions**

In this study, coda $Q$ was estimated for central and eastern Alborz by using the SBS and SIS methods. Based on the analysis, the following conclusions emerged:

- The $Q_c$ estimation using SBS and SIS methods for real and synthetic data shows high correlation for lapse times $> 2t_S$.
- The trend of $Q_0$ shows a slight decrease at average depth of 78 km, which may be the result of high-dissipating media at this depth. The low $Q_0$ value may be related to the relatively highly dissipative layer beneath the lithosphere at a shallow LAB.
• The average $Q_c$ values, estimated for central and eastern Alborz and their frequency-dependent relationships show that the medium is tectonically active and highly heterogeneous.

Data and Resources

The seismograms used in this study were collected from the International Institute of Earthquake Engineering and Seismology (IIAES) for providing the required waveforms and instrumentally and financially supporting this research work, and Tehran Disaster Management Center on the basis of official requests.

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