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S. A. Tabatabaei a, K. Abrinia a, M. K. Besharati Givia a, P. Karami a & M. Mosavi Mashhadi a

a School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran


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Application of the Equi-Potential Lines Method in Upper Bound Estimation of the Extrusion Pressure

S. A. Tabatabaei, K. Abrinia, M. K. Besharati Givia, P. Karami, and M. Mosavi Mashhadi

School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran

Forming pressure plays a key role in extrusion process and widely affects product quality. Therefore, prediction of the optimal pressure is the main objective for effective extrusion process. In this study, the notion of Equi-Potential Lines (EPLs) was applied to pressure estimation in extrusion process for the first time. To implement analogy in the extrusion, the initial billet and square die were considered and two different potentials were assigned to them, and then EPLs were drawn between two shapes that show the minimum work path between the entry and exit cross sections. The drawn EPLs were connected to each other to build up a 3D-die. In the following, cubic polynomial curve was fitted between the initial and final shapes in the designed die. The constructed curve was the accurate assumption for deformation zone in the extrusion that was later used in a written code and upper bound formulation for pressure estimation. Effectiveness of the proposed method was examined via theory and experiment.

From the results, there was noticeable agreement between the theory and experiment. As a result, application of the EPLs method in upper bound formulation can lead to accurate results from theory.

Keywords: EPLs; Experiment; Extrusion.

INTRODUCTION

Extrusion process has been widely used to form different cross sections, and improve the quality of products. To achieve an optimal extrusion pressure it is essential to have a good knowledge about the actual metal flow properties inside the die. Some researchers have been done for the prediction of the actual metal flow characteristics inside the 3D die. Hoshino et al. [1] used the upper bound method in the three-dimensional (3D) extrusion process of circular billet inside the converging square die. Kiuchi et al. [2] used the upper bound method for extrusion of square, rectangular, hexagonal, and asymmetric billets. Park et al. [3] applied the 3D finite element analysis for helical extrusion of twisted sections, such as clover, and trocoidal gear sections. They used curved dies with continuous functions from entry to exit sections. In addition, they investigated the work-hardening effect, distortion of flow pattern, and distribution of the effective strain. Nagpal and Altan [4] extracted the velocity filed in 3D-extrusion process of rectangular and round billets inside the rectangular and elliptical dies, respectively. Juneja and Prakash [5] used the upper bound method to investigate the flow of material in drawing/extrusion process of round billets through converging polygonal die. They considered a constant frictional stress between the die and material and predicted the minimum drawing/extrusion stress at die surface.

Yang and Lee [6] proposed a new analysis for the extrusion of arbitrary shaped sections through curved die profiles. They found the kinematically admissible velocity field using the conformal mapping method. Finally, they applied the upper bound method to find the extrusion pressure of the rigid-perfectly plastic material through curved die profiles.

Yang et al. [7] used the conformal mapping technique in extrusion of round billets inside the 3D-shaped dies. Following, Yang and Lange, [8] applied the conformal mapping method combined with upper bound technique, in the internal metal deformation analysis. In addition, they calculated the energy dissipation in 3D hydro film extrusion. Chitkara et al. [9] used the upper bound method to investigate the 3D off-centric extrusion of arbitrarily shaped sections from arbitrarily shaped billets inside the curved dies. Abrinia et al. [10] proposed a new upper bound method for the extrusion of nonsymmetrical complex shapes from round billets.

In this study, the notion of Equi-Potential Lines (EPLs) combined with the upper bound method was applied for the first time, to estimate pressure in the extrusion process. The notion was initially used by Lee et al. [11] in the forging process. They used the EPLs concept in preform shape design between the initial billet and final die. In addition, they used the artificial neural networks to find the range of initial volume and potential value of the electric field. However, they did not discuss the possibility of applying EPLs to other forming methods. Wang and Li [12] used the 3D-electrostatic field simulation and geometric transformation method in preform shape design of super alloy disks. However, their method was limited to symmetrical shapes.

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Address correspondence to S. A. Tabatabaei, School of Mechanical Engineering, University of Tehran, Tehran, Iran; E-mail: S.A. Tabatabaei@ut.ac.ir

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EPLs method was used for preform shape design in tube hydro forming process by Tabatabaei et al. [13]. It was considered a tube inside a square die and its preform shape from EPLs method was extracted, and then its forming pressure from finite element modeling was used in an actual tube hydro forming process. However, they did not investigate the application of EPLs method in 3D complex dies.

In the present research, the idea of EPLs was used for the first time in 3D-die design and pressure estimation in extrusion of cylindrical billet through square die. In the following, a polynomial curve was fitted between the initial and final shapes in the constructed (3D) die, which then was used in upper bound method for prediction of extrusion pressure. Finally, the effectiveness of the proposed method was examined theoretically, and experimentally. The results, shown that, by using the curve obtained from EPLs method for deformation zone (in upper bound formulation), it is possible to predict the extrusion pressure accurately.

**Theory**

The governing equation of the electric filed \( \mathbf{E} \) containing charge density of \( \rho \) is expressed in the following form [11–14]:

\[
\nabla \cdot \mathbf{E} = 4\pi \rho, \tag{1a}
\]

\[
\nabla \times \mathbf{E} = 0. \tag{1b}
\]

If a charge is put in this field, it moves along a certain path and the potential value \( \phi \) at a point is obtained as follows:

\[
\mathbf{E} = -\nabla \phi \tag{2a}
\]

\[
\nabla^2 \phi = -4\pi \rho. \tag{2b}
\]

If there is no charge, the governing equation is represented by Laplace’s relation:

\[
\nabla^2 \phi = 0. \tag{3}
\]

From Cai et al. [14], when the physical object, like force field, magnetic field, or thermal flow field, has the same field equation in form as the electrostatic field equation, it can be simulated in the same way as the electrostatic field. If the material volume remains constant and the velocity field is nonspinning, the following equation holds true [14]:

\[
\nabla^2 \beta = \frac{\partial^2 \beta}{\partial x_1^2} + \frac{\partial^2 \beta}{\partial x_2^2} + \frac{\partial^2 \beta}{\partial x_3^2} = 0 \tag{4}
\]

Where \( \beta \) is a velocity potential function of movement or strain and \( x_i \) (\( i = 1, 2, 3 \)) is the coordinates.

To implement the analogy in the extrusion process, a typical cylindrical billet in a square die was considered. Two potentials were assigned to the initial billet and its final shape, and EPLs were drawn between these two shapes (potentials). It should be noted that the equi-potential lines generated between two conductors of different voltages show similar trends for minimum work paths between the undeformed and deformed shapes [11]. In the extrusion process, the initial billet is formed in the die cavity, thus the forming path is from outside of the billet to the inside of the die. Therefore, assuming that the billet was inserted into the die cavity, voltages of 0 and 1 were assigned to the outer side of the billet and the inner side of the die, respectively. Then the Laplace relation was solved between the two conductors (outer side of the initial billet and inner side of the die) and points of equal voltages were connected to create same-voltage contours.

In this article, *Matlab* code and finite element method was used for solving the Laplace’s relation and drawing the equi-potential lines. Each of these contours represents an intermediate shape between the initial shape and the final shape (die). Using EPLs method, the intermediate shapes in the extrusion process from round billet to square die were built. Then a polynomial curve was fitted through the initial, intermediate, and final shapes.

The polynomial curve was the accurate assumption for the deformation zone in the extrusion process, used in upper bound calculations of the extrusion pressure [15].

The estimated pressure using upper bound formulation, and, polynomial curve (from the EPLs method) was compared with that of bilinear curve [15]. Moreover, an experimental test was conducted to verify the results of bilinear and polynomial curves used in pressure estimation.

The results showed that the extrusion pressure using the EPLs method was so close to the experimental result that confirmed the capability of proposed method in this research for pressure estimation.

**Experiment**

In order to validate the results obtained from the theory, a lead cylindrical billet with a 25 mm diameter having mechanical properties shown in Table 1 was formed in a square die. The VCN-150 die, which was used in this experiment, had 17.16 mm side length and a 0.1 mm corner radius. The \( Ra \) (reduction of area) and \( L/R \) (ratio of the die length to the radius of the billet) was taken as 60% and 0.9, respectively in the experiments.

The billet material was modeled as \( \sigma = 15 + 14.18 \varepsilon^{0.19} \) (K. \( \varepsilon^n \)), where \( K \) and \( n \) are strength coefficient and strain hardening exponent, respectively. Moreover, the

<table>
<thead>
<tr>
<th>Modulus of elasticity</th>
<th>Poisson’s ratio</th>
<th>Density</th>
<th>Ultimate tensile strength</th>
<th>( K ) (MPa)</th>
<th>Compressive yield strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 GPa</td>
<td>0.35</td>
<td>11340 kg/m(^3)</td>
<td>25 MPa</td>
<td>14.18</td>
<td>0.19</td>
</tr>
</tbody>
</table>

**Table 1.**—Mechanical properties of the lead material.
compressive yield strength of lead, obtained from a standard compression test. An Instron 4028 hydraulic press was used to perform the tests.

RESULTS AND DISCUSSION

Figure 1 shows the application of the EPLs method in performing (intermediate) shape design for a cylindrical billet inside the square die. Ten preforms were considered between the two shapes. The intermediate shapes were assumed as a contour with 0(V) between the zero voltage of the initial billet and 1(V) of the final shape. In addition, in Fig. 1b, the intermediate shapes are shown in X-Y plane and 2-D view.

Coordinates of the intermediate shapes were used to build up the 3D die. Figure 2 shows the configuration of EPLs based die from different views.

From our investigation, bilinear, third- and fifth-order Bezier/polynomial curves were the most common curves used to define the deformation zone in upper bound method [15–18].

In the above-mentioned curves, the intermediate control points were defined using the same functions of the entry and exit cross sections with some arbitrary coefficients to adjust the position of intermediate control points. As a result, the curves were a rough assumption for the deformation path from the initial billet to the final shape. In this case, to find the minimum extrusion pressure, it was needed to use the optimization methods for finding the optimal variables that was a time-consuming process [15,16]. However, this research opens a new avenue for pressure estimation, which uses the accurate deformation curve with the known control points in predicting the extrusion pressure. Since ten intermediate shapes were considered between the initial and final shapes, it was possible to fit any polynomial curve from order one to nine or any other kind of curve. In this research, for the sake of simplicity, the cubic polynomial curve was used to define the deformation zone between the initial and final shapes. Matlab cftool was used for fitting the curve through the extracted points from EPLs method and then the polynomial cubic curve was used in a written code and upper bound formulation for prediction of the extrusion pressure (Table 3).

Figure 3 shows the experimental setup, die, and extruded part.

Finally, optimal theoretical and experimental results for the extrusion pressure using bilinear, third- and fifth-order Bezier curves, and accurate curve (from EPLs method) were compared (Table 2). Figure 3 shows the experimental setup, die, and extruded part.

As can be seen, in the bilinear curve there is 2.5% deviation from the experiment for theory. However, the difference between theoretical and experimental results using third- and fifth-order Bezier curves are 1.73% and 1.49%, respectively. Finally, the cubic polynomial curve predicts the extrusion pressure precisely with 0.085% deviation from the experiment, which can

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Equi-potential lines between the billet and square die: (a) 3-D view; (b) 2-D view (color figure available online).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Die configuration (from EPLs method) (color figure available online).}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Method & Theory & Experiment \\
\hline
Bilinear & 1.73% & 1.73% \\
Third-order & 1.49% & 1.49% \\
Fifth-order & 0.085% & 0.085% \\
\hline
\end{tabular}
\caption{Comparison of theoretical and experimental results for extrusion pressure using different methods.}
\end{table}

\textsuperscript{1}Curve fitting toolbox.
be attributed to the accurate estimation of the deformation path using the EPLs method.

To sum up, application of the equi-potential lines method in extrusion process opens a new avenue in pressure estimation which is simple and straightforward. Using the EPLs method enables us to predict the intermediate shapes from the entry to exit. In this case, it is possible to define the deformation path by fitting any curve between the initial, intermediate, and final shapes. Since the geometry of the intermediate shapes only depend on the geometrical shapes of the initial and final cross sections, the proposed method can be used in complex configurations without any limitation.

**Conclusion**

By defining an accurate streamline in deformation zone, it is possible to achieve acceptable results from upper bound method. Because accuracy of the results is related to the definition of a real velocity field, which in turn originates from a proper definition of the deformation zone. By using the EPLs method, it is possible to have a reliable control on the particle position in the deformation zone. Moreover, it is possible to predict the intermediate shapes from inlet to outlet sections.

The method proposed in this research can be used in prediction of the deformation curve from the initial to final shapes in asymmetric and complex dies as well.

**References**


