Constitutive modeling and prediction of hot deformation flow stress under dynamic recrystallization conditions

Hamed Mirzadeh *

School of Metallurgy and Materials Engineering, College of Engineering, University of Tehran, P.O. Box 11155-4563, Tehran, Iran

**Abstract**

Simple modeling approaches based on the Hollomon equation, the Johnson–Cook equation, and the Arrhenius constitutive equation with strain-dependent material’s constants were used for modeling and prediction of flow stress for the single-peak dynamic recrystallization (DRX) flow curves of a stainless steel alloy. It was shown that the representation of a master normalized stress–normalized strain flow curve by simple constitutive analysis is successful in modeling of high temperature flow curves, in which the coupled effect of temperature and strain rate in the form of the Zener–Hollomon parameter is considered through incorporation of the peak stress and the peak strain into the formula. Moreover, the Johnson–Cook equation failed to appropriately predict the hot flow stress, which was ascribed to its inability in representation of both strain hardening and work softening stages and also to its completely uncoupled nature, i.e. dealing separately with the strain, strain rate, and temperature effects. It was also shown that the change in the microstructure of the material at a given strain for different deformation conditions during high-temperature deformation is responsible for the failure of the conventional strain compensation approach that is based on the Arrhenius equation. Subsequently, a simplified approach was proposed, in which by correct implementation of the hyperbolic sine law, significantly better consistency with the experiments were obtained. Moreover, good prediction abilities were achieved by implementation of a proposed physically-based approach for strain compensation, which accounts for the dependence of Young's modulus and the self-diffusion coefficient on temperature and sets the theoretical values in Garofalo's type constitutive equation based on the operating deformation mechanism. It was concluded that for flow stress modeling by the strain compensation techniques, the deformation activation energy should not be considered as a function of strain.

1. Introduction

Since the computer simulation of metal forming processes is used increasingly in the industry, a proper flow stress description is the preliminary requirement (Gronostajski, 2000). This is especially the case for hot forming processes, which are usually more complex in terms of occurrence of additional microstructural phenomena such as dynamic recrystallization (DRX). Poliak and Jonas (1996) and Mirzadeh and Najafizadeh (2010a) have studied the critical conditions for initiation of DRX. Hot working is an important step in production of materials with required shape, microstructure, and mechanical properties. Therefore, a proper understanding of the microstructural evolution during hot deformation and characterizing the constitutive behavior describing material flow are inevitable (McQueen and Ryan, 2002).
As a result, considerable research has been carried out to model the flow stress of metals and alloys. Liang and Khan (1999) have reviewed the constitutive models for BCC and FCC metals over a wide range of strain rates and temperatures. Mirzadeh et al. (2011a) have proposed a simple physically-based approach for constitutive analysis in hot working as an alternative for the conventional apparent approach. Lin and Chen (2011) have published a review paper on the constitutive descriptions for metals and alloys in hot working. Mirzadeh et al. (2012) have critically discussed the efficiency of artificial neural network, Arrhenius equation, and a proposed formula in prediction of DRX flow curves. Bhattacharya et al. (2014) have developed some constitutive equations for AZ31 alloy under DRX conditions. Huh et al. (2014) have evaluated the dynamic hardening models of metallic materials for various crystalline structures. Finally, Mirzadeh (2014) has developed a useful approach for comparative hot working and alloy development studies based on constitutive analysis. Generally speaking, the accuracy of a model depends on both the mathematical structure of the model and the proper experimental determination of the material parameters used in the model (Gronostajski, 2000).

A constitutive relationship is a mathematical representation of flow stress of the material as a function of deformation temperature, strain rate, strain, and other factors. A useful form of the constitutive equations can be expressed as $\sigma = f(T, \dot{e}, \epsilon)$. Based on the work of Zener and Hollomon (1944), the coupled effect of temperature and strain rate is incorporated in a temperature-compensated strain rate parameter of the form $Z = \dot{e} \exp(Q/RT)$. The basic approach for the modeling of flow stress is the implementation of the classical Hollomon equation by Eq. (1) (Hollomon, 1945) and Ludwink equation by Eq. (2) (Ludwik, 1909):

$$\sigma = Ke^n$$  
(1)

$$\sigma = \sigma_0 + Ke^n_{\text{plastic}}$$  
(2)

where $n$ is called the work-hardening coefficient, $K$ is the stress coefficient, $\sigma_0$ is the yield stress, $\dot{e}$ is the true total strain and $\dot{e}_{\text{plastic}}$ is the true plastic strain. This response is usually called parabolic hardening (Meyers and Chawla, 2009). Obviously, the effects of temperature and strain rate on work hardening and stress level should be incorporated in $n$ and $K$ (Sheng and Shippur, 2006) or should be incorporated by adding additional terms (Takuda et al., 2005). A famous form of the latter approach by consideration of the effect of strain-rate hardening has been proposed by Fields and Backofen (1957) in the form $\sigma = Ke^{m\dot{e}}$, where $m$ is known as the strain rate sensitivity. However, the most common form of the latter approach has been proposed by Johnson and Cook (1983) as shown below:

$$\sigma = (\sigma_0 + Ke^n_{\text{plastic}}) \times \left(1 + C \ln \frac{\dot{e}}{E_T}\right) \times \left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^{q}\right)$$  
(3)

where $\dot{e}$ and $T_r$, $T_m$, and $\sigma_0$ are the reference strain rate, reference temperature, the melting point of the material, and the yield stress at reference temperature and strain rate, respectively (Milani et al., 2009). In the Johnson–Cook equation, the three groups of terms in parentheses represent work-hardening (based on the constant $n$), strain rate (based on the constant $C$), and thermal (based on the constant $q$) effects, respectively. It should be noted that the flow softening resulted from the occurrence of DRX has not been considered in simple expressions of Eqs. (1–3).

Sellars and McTegart (1966) have proposed another widely applied method in the literature for constitutive analysis, which is based on the expressions which relate $Z$ to the flow stress as shown in Eq. (4). In this equation, $Q$ is the hot deformation activation energy, $\dot{e}$ is the strain rate, $T$ is the absolute temperature, and finally $A^r$, $A^n$, $A^\beta$, $n$, $\beta$, $n$, and $\alpha$ are the material’s parameters. The power law is preferred for relatively low stresses. Conversely, the exponential law is suitable for high stresses. Finally, the hyperbolic sine law can be used for a wide range of $Z$ parameters. The stress multiplier $\alpha$ is an adjustable constant which brings $\alpha$ into the correct range that gives linear and parallel lines in $\ln \dot{e}$ versus $\ln [\sinh (\alpha \sigma)]$ plots and it can be estimated by $\alpha \approx \beta/n'$ (Mirzadeh, 2015a).

$$Z = \dot{e} \exp \left(\frac{Q}{RT}\right) = \left\{\begin{array}{ll}
A^r \dot{e}^{n} \\
A^n \exp(\beta \sigma) \\
A[\sinh(n' \sigma)]^\alpha
\end{array}\right.$$

Conventionally, the material’s parameters are considered to be apparent ones, which makes it impractical to conduct the comparative studies to elucidate the effects of alloying elements (Mirzadeh, 2014) or second phases (Mirzadeh, 2015b). Recently, Mirzadeh et al. (2011a) showed that when the deformation mechanism is controlled by the glide and climb of dislocations, a constant hyperbolic sine power of $n = 5$ and self diffusion activation energy ($Q_{sd}$) can be used to describe the appropriate stress. This is possible by taking into account the dependences of Young’s modulus ($E$) and self-diffusion coefficient ($D$) on temperature in the hyperbolic sine law. Accordingly, the unified relation can be expressed as $\dot{e}/D = B[\sinh(\alpha \sigma/E)]^\beta$. In this equation, $D = D_0 \exp(-Q_{sd}/RT)$, where $D_0$ is a pre-exponential constant. The constants $\alpha$ and $B$ are the modified stress multiplier and the modified hyperbolic sine constant, respectively. The consideration of hyperbolic sine power of 5 and self diffusion activation energy gives a physical and metallurgical meaning to this equation and also reduces the number of unknown parameters and constant to 2 ($\alpha$ and $B$). However, Mirzadeh et al. (2011a) showed that the occurrence of dynamic recrystallization (DRX) generally exerts an influence on the value of $n$. Therefore, the unified physically-based relation in its general form can be expressed as $\dot{e}/D = B[\sinh(\alpha \sigma/E)]^\beta$ and the physically-based equations based on the power, exponential, and hyperbolic sine laws can be summarized as follows:

$$\dot{e}/D = \left\{\begin{array}{ll}
B'(\sigma/E)^n \\
B^n \exp(\beta' \sigma/E) \\
B[\sinh(\alpha \sigma/E)]^\alpha
\end{array}\right.$$

In these expressions, $B'$, $B''$, $B'$, $n'$, $\beta'$, $n$, and $\alpha'$ are the material’s parameters. In Eqs. (4) and (5), the flow stress
is related to both the deformation temperature and strain rate. However, the description of flow stress by the expressions of Eqs. (4) and (5) is incomplete, because no strain for determination of flow stress is specified. Therefore, characteristic stresses that represent the same deformation or softening mechanism for all flow curves, such as steady state stress \( \sigma_0 \) or peak stress \( \sigma_P \), should be used. In general, the peak stress is the most widely accepted one in order to find the hot working constants of different materials such as magnesium alloys (Mirzadeh, 2014, 2015a), steels (Mirzadeh et al., 2009, 2011b), and composites (Mirzadeh, 2015b). To make it possible to model the whole flow curve, the conventional approach is to express the constants of the hyperbolic sine equation (in Eq. (4)) as functions of strain using the experimental data (strain compensation), which has been successfully applied for a variety of engineering materials such as stainless steels (Mirzadeh and Najafizadeh, 2010b), steels (Mirzadeh et al., 2012; Lin et al., 2008), magnesium alloys (Yu, 2013), aluminum alloys (Li et al., 2013), titanium alloys (Cai et al., 2011), intermetallics (Khamei and Dehghani, 2010), and composites (Wang et al., 2011).

In the current work, the applicability of the Hollomon equation, the Johnson–Cook equation, and the strain compensation by the Arhenius equation will be evaluated for prediction of hot flow stress and subsequently some straight-forward modified approaches will be proposed to achieve acceptable results.

2. Experimental details

The 17-4 PH stainless steel (AISI 630) with chemical composition of 0.03 wt% C–15.14 wt% Cr–4.53 wt% Ni–3.4 wt% Cu–0.25 wt% Nb, which has a single-phase austenitic structure and exhibits DRX at hot working conditions, was used for constitutive analysis. Cylindrical specimens were prepared, which were 11.4 mm in height and 7.6 mm in diameter for hot compression tests. Tantalum foils and boron nitride solution were used to reduce friction between the anvils and the specimen. The specimens were austenitized at 1100 °C for 15 min and cooled down at a rate of 1.5 °C/s to deformation temperature and held there for 5 min before hot compression tests. Single-hit hot compression tests were performed at temperatures between 950 °C and 1100 °C under true strain rates ranging from 0.0001 s\(^{-1}\) to 0.1 s\(^{-1}\) using an Instron deformation machine up to true strains of ~0.8, where the displacements were acquired based on the cross-head data. The resultant flow curves (Fig. 1) were divided into two groups: (1) the modeling set for finding the constants of constitutive equations and (2) the prediction set to evaluate the generalization ability of the developed models for unseen deformation conditions.

3. Results

3.1. Basic constitutive analysis

The hot compression flow curves of Fig. 1 exhibit typical single peak dynamic recrystallization (DRX) behavior with a broad peak followed by a gradual fall toward a steady state stress.

By consideration of the values of peak stress, based on the power, exponential, and hyperbolic sine laws of Eq. (4), the slopes of the plots of \( \ln \sigma \) vs. \( \ln \sigma_P \) \( (n' = \frac{\partial \ln \sigma}{\partial \ln \sigma_P}) \), \( \ln \sigma \) vs. \( \sigma \) \( (\beta = \frac{\partial \ln \sigma}{\partial \sigma_P}) \), and \( \ln \sigma \) vs. \( \ln(\sinh(\alpha \sigma_P)) \) \( (n = \frac{\partial \ln \sigma}{\partial \ln(\sinh(\alpha \sigma_P))}) \) can be used for obtaining the values of \( n' \), \( \beta \), and \( \alpha \), respectively. The required plots for obtaining \( n' \) and \( \beta \) are shown in Fig. 2a and the subsequent linear regression of the data resulted in the average value of \( \alpha \approx \beta \approx n' \approx 0.011 \) MPa\(^{-1}\). Now, based on the hyperbolic sine law, the average value of \( Q \) can be determined based on \( Q = \frac{Rn[\partial \ln(\sinh(\alpha \sigma_P))]}{\partial(1/T)} \), Therefore, the slope of the plot of \( \ln(\sinh(\alpha \sigma_P)) \) against \( 1/T \) was used for obtaining the value of \( Q = 417.6 \) kJ/mol, as shown in Fig. 2b. The slope and the intercept of the plot of \( \ln Z \) against \( \ln(\sinh(\alpha \sigma_P)) \) was used for obtaining the final values of \( n = 5.18 \) and \( \ln A = 32.65 \) (Fig. 2c). Moreover, based on the power law equation shown in Eq. (4), the formula for the peak stress can be expressed as \( \sigma_P = \beta Z^n \). So, the slope and the intercept of the plot of \( \ln \sigma_P \) against \( \ln Z \) was used for obtaining the

![Fig. 1. The obtained flow curves at different deformation conditions, which were subsequently used for modeling of flow stress (based on the modeling set) and prediction of flow curves (based on the prediction set).](image-url)
values of $p = 0.143$ and $\ln k = -0.322$ (Fig. 2d). The similar procedure for the peak strain based on $\varepsilon_p = B Z^p$ was used to find the values of $p = 0.16$ and $\ln B = -6.081$ (Fig. 2d). The resulting equations can be summarized as follows:

$$Z = \dot{\varepsilon} \exp(417600/RT)$$

$$= 685.53 \{\sinh(0.011 \times \sigma_p)\}^{5.18} \quad (6)$$

$$\sigma_p = 0.72Z^{0.143} \quad (7)$$

$$\varepsilon_p = 0.0023Z^{0.16} \quad (8)$$

3.2. Modeling by fitting a polynomial function

It was found by regression analysis that a six order polynomial can appropriately model the flow curves for different deformation conditions (different $Z$ values). Therefore, seven constants will be obtained for each $Z$. It is expected to be possible to find some relations between these constants and $Z$. However, it is obvious that this method is not a good approach for modeling the flow curves and does not have a scientific value. To address this issue, an innovative idea was implemented, in which the normalized stress...
\( (\sigma /\sigma_P) \) was plotted versus the normalized strain \( (\varepsilon /\varepsilon_P) \) for the flow curves of the modeling set and a six order polynomial was fitted to the data. This is shown in Fig. 3a. It can be seen that almost all the data can be fairly represented by the fitted curve. This curve can appropriately model the change in the shape of flow curves by increasing strain. The main advantage of this method is behind the incorporation of peak stress and peak strain into the constitutive equation, as they bring the effect of deformation temperature and strain rate based on their relation with \( Z \) as shown in Eqs. (7) and (8). The resulting constitutive equation can be summarized as follows:

\[
\begin{align*}
\frac{\sigma}{\sigma_P} &= -0.0173(\varepsilon /\varepsilon_P)^6 + 0.1901(\varepsilon /\varepsilon_P)^5 - 0.8211(\varepsilon /\varepsilon_P)^4 \\
&+ 1.7877(\varepsilon /\varepsilon_P)^3 - 2.1137(\varepsilon /\varepsilon_P)^2 + 1.3099(\varepsilon /\varepsilon_P) \\
&+ 0.662 \\
\sigma_P &= 0.722Z^{0.143} \\
\varepsilon_P &= 0.0023Z^{0.16}
\end{align*}
\]  

(9)

Fig. 3b shows the comparison between the experimental and calculated flow stress for some representative curves of the modeling set using Eq. (9). As can be seen in this figure, the calculated flow stresses are fairly consistent with the experimental values in many cases.

Despite the visual examination, the ability of the model can be better evaluated by calculating the root mean square error (RMSE) and the percentage of the average relative absolute error (AAE) using the following formulae:

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2}
\]  

(10)

\[
\text{AAE} = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{t_i - y_i}{t_i} \right| \times 100
\]  

(11)

where \( t_i \) is the target output and \( y_i \) is the model output. The average RMSE and AAE for the modeling set are 10.88 MPa and 10.72%, respectively. In the case of prediction set (Fig. 3c), the average RMSE and AAE were determined as 8.81 MPa and 6.11%, respectively. These error values show that this approach is relatively suitable for prediction of flow curves.

Since this method is based on a master normalized curve, some level of error is inevitable. However, the merit of this method is in the assumption that the DRX flow curves of a specific material follow the same trend, which can be represented by a six order polynomial function. This technique can also be used for modeling of dynamic recovery (DRV) flow curves, for which, another master normalized curve with another six order polynomial should be considered. Moreover, the peak stress and the peak strain should be replaced by the saturation stress and the highest value of strain, respectively.

### 3.3. Modeling by the Hollomon equation

Taking natural logarithm from both sides of the Hollomon equation of the form \( \sigma = K\varepsilon^n \) (Eq. (1)) results in \( \ln \sigma = \ln K + n \ln \varepsilon \). Therefore, if the plotted flow curves in log–log scale results in nearly straight lines, it can be concluded that the Hollomon equation is suitable for modeling of flow curves. A representative flow curve has been plotted in log–log scale in Fig. 4a. It can be seen that the degree of work-hardening (the slope) varies depending on the strain range but the data represent straight lines. Therefore, the Hollomon equation can be used for the modeling purposes. By neglecting the very low strain range (<0.04), two work-hardening coefficients \( (n) \) should be considered. The first one should be used for flow hardening stage before the peak point and the second one should be used for flow softening stage after the peak point. Therefore, \( n \) is a positive value for \( \varepsilon < \varepsilon_P \) and it is a negative value for \( \varepsilon > \varepsilon_P \).

Since good results were obtained from the approach of the previous section based on the master normalized curve, this curve was used for the calculation of \( n \) and \( K \) before and after the peak point. Therefore, the effect of
deformation temperature and strain rate is considered in the resulting equations based on the relation of the peak stress and the peak strain with $Z$. The required plots are shown in Fig. 4b and c and the resulting constitutive equations can be summarized as follows:

\[
\begin{align*}
\sigma /\sigma_P &= 1.020 \times (\varepsilon /\varepsilon_P)^{-0.1022} \iff \varepsilon < \varepsilon_P \\
\sigma /\sigma_P &= 1.037 \times (\varepsilon /\varepsilon_P)^{-0.1288} \iff \varepsilon > \varepsilon_P \\
\sigma_P &= 0.72Z^{0.143} \\
\varepsilon_P &= 0.0023Z^{0.16}
\end{align*}
\]

(12)

Fig. 5 shows the comparison between the experimental and calculated flow stress for some representative curves of the modeling and prediction sets using Eq. (12). As can be seen in this figure, the calculated flow stresses are fairly consistent with the experimental values in many cases. The average RMSE (AAE) for the modeling and prediction sets are 7.74 MPa (7.82%) and 8.69 MPa (6.03%), respectively. These error values are slightly lower than those determined for the Modeling by fitting a polynomial function, which shows that this approach is slightly better for modeling and prediction of flow curves.

The Fields and Backofen equation of the form $\sigma = Ke^{n}t^{m}$ considers the effect of strain rate by the strain rate sensitivity ($m$). Therefore, for including the thermal effects, the dependence of the constants ($n$, $m$, and $K$) on temperature should be investigated. This can result in the complexity of the formula (Takuda et al., 1998). Therefore, the approaches based on the coupled effect of temperature and strain rate by incorporating the Zener–Hollomon parameter into the formula are preferred. In the present work, this was achieved indirectly and appropriately by utilization of the peak stress and the peak strain.

3.4. Modeling by the Johnson–Cook equation

In the Johnson–Cook (JC) equation (Eq. (3)), the three groups of terms in parentheses represent work-hardening, strain rate, and thermal effects and hence the values of $K$, $n$, $C$, and $q$ should be calculated for modeling of flow stress. The melting temperature of AISI 630 stainless steel is $T_m = 1695$ K (Mirzadeh et al., 2011a). Based on the available flow curves, $T_r$ and $\dot{\varepsilon}_r$ were considered as 1223 K (950 °C) and 0.01 s$^{-1}$, respectively. Since the parameter $q$ is normally a decimal number (Liang and Khan, 1999), the lowest available temperature (1223 K in the case of the present investigation) should be considered as $T_r$ to make the JC equation valid ($T \geq T_r$). Moreover, since the true total strain is used for the modeling of flow curves, the Ludwik form in the first parentheses (based on the true plastic strain) can be replaced with the Hollomon form (based on the true total strain). As a result, the useful form of the Johnson–Cook (JC) equation in this case can be expressed as follows:

\[
\sigma = (K\dot{\varepsilon}^n) \times (1 + C \ln(\dot{\varepsilon} / 0.01)) \times \left(1 - \left(\frac{T - 1223}{472}\right)^q\right)
\]

(13)

Since Eq. (13) simplifies to $\sigma = Ke^{n}$ at reference temperature and strain rate, the reference flow curve (950 °C–0.01 s$^{-1}$) should be used for determination of $K$ and $n$. Therefore, the slope and the intercept of the plot of $\ln \sigma$ against $\ln \varepsilon$ (Fig. 6a) was used for obtaining the values of $n = 0.111$ and $\ln K = 5.0534$ ($K = 156.55$). It can be seen that only one pair of $n$ and $K$ values was determined for flow curve of 950 °C–0.01 s$^{-1}$. However, Eq. (12) shows that for the modeling of hot deformation flow curves of this material, two pairs of $n$ and $K$ values should be considered. Therefore, it can be concluded that availability of a good reference flow curve is essential for modeling by the JC model.

The flow curves at temperature of 950 °C for strain rates of 0.001 s$^{-1}$, 0.01 s$^{-1}$, and 0.05 s$^{-1}$ can be used for determination of $C$. In this case, Eq. (13) can be simplified as $\sigma / \{156.55\varepsilon^{0.111}\} - 1 = C \ln(\dot{\varepsilon} / 0.01)$. Therefore, the slope of the plot of $\sigma / \{156.55\varepsilon^{0.111}\} - 1$ against $\ln(\dot{\varepsilon} / 0.01)$ using an equation of the form of $y = ax + b$ (intercept of zero) can be used for obtaining the value of $C$. This was performed for different strains (from 0.1 to 0.7 with an increment of 0.2) as shown in Fig. 6b. By fitting a straight line with an intercept of zero to these data, the average value of $C$ was determined as 0.1185. However, Fig. 6b shows that $C$ clearly depends on strain. This is a common problem with the JC model (Lin et al., 2012), which can be ascribed to the main idea behind the JC model, i.e. dealing separately with the
strain, strain rate, and temperature effects (Shin and Kim, 2010). This problem can be solved by consideration of coupled effects of temperature and strain rates, as it was achieved by consideration of Zener–Hollomon parameter for the modeling by the Hollomon equation in Section 3.3.

The flow curves for strain rate of 0.01 s⁻¹ at temperatures of 1000 °C and 1100 °C can be used for determination of q. In this case, Eq. (13) can be simplified as
$$1 - \frac{\sigma}{(156.55 s^{0.111})} = \left(\frac{T - 1223}{472}\right)^{q}. $$
Therefore, the slope of the plot of ln (1 - σ/(156.55 s^{0.111})) against ln [(T - 1223)/472] using an equation of the form of \( y = ax + 0 \) (intercept of zero) can used for obtaining the value of q. This was performed for different strains (from 0.1 to 0.7 with an increment of 0.2) as shown in Fig. 6c. By fitting a straight line with an intercept of zero to these data, the average value of q was determined as 0.48. Again, the consistency of the JC model with the experimental data is not good and it can be seen that q depends on strain. It should be noted that only two temperatures of 1000 °C and 1100 °C were available for calculation of q but even more experimental data cannot solve the problem of strain dependency of q. The resulting constitutive equations can be summarized as follows:

$$\begin{align*}
\sigma &= (156.55 s^{0.111}) \times \left(1 + 0.1185 \ln \left(\frac{\dot{\varepsilon}}{0.01}\right)\right) \\
& \times \left(1 - \left(\frac{T - 1223}{472}\right)^{0.48}\right)
\end{align*} \tag{14}
$$

The three flow curves at temperature of 950 °C for strain rates of 0.001 s⁻¹, 0.01 s⁻¹, and 0.05 s⁻¹ and two additional flow curves for strain rate of 0.01 s⁻¹ at temperatures of 1000 °C and 1100 °C consist the flow data for the modeling set of the JC model and all other flow curves can be considered as the flow curves of the prediction set. Fig. 7 shows the comparison between the experimental and calculated flow stress for some representative curves of the modeling and prediction sets using Eq. (14). As can be seen, the calculated flow stresses are relatively consistent with the experimental values for the curves of the modeling set, which shows that the obtained values of K, n, C, and q have sufficient accuracy. However, the calculated curves deviate largely for the flow curves of the prediction set, which can be related to both the inability of the model and the ignorance of flow softening. The average RMSE (AAE) for the modeling and prediction sets are 7.83 MPa (6.67%) and 14.72 MPa (15.02%), respectively. The large error values for the prediction set show that the original JC model is not an optimized one for modeling and prediction of hot deformation flow curves.

3.5. Modeling by the conventional strain compensation approach

In order to obtain Eq. (6), the peak flow stresses were used to determine the hot working behavior of the material. However, for calculation of strain-dependent material’s constants (a, Q, n, and A), the flow stresses corresponding to different strains ranging from 0.05 to 0.8 with an increment of 0.05 were obtained from the flow curves (modeling set) and the material’s constants at each given value of strain were computed using the flow stresses obtained at different deformation conditions by the methods described in Section 3.1. The results are shown in Fig. 8, in which the solid curves represent the best fitted polynomials to the data. The obtained equations from these regression analyses were subsequently used for flow stress modeling. The hyperbolic sine equation can be used to predict the flow stress by consideration of a, Q, n, and A as strain-dependent variables.

Therefore, based on the hyperbolic sine law, the flow stress of the material at a given strain can be expressed as follows:

$$\begin{align*}
\sigma &= \frac{1}{\alpha} \left\{ \sinh^{-1} \left(\frac{Z}{A}\right)^{1/n} \right\} \\
& = \frac{1}{\alpha} \ln \left\{ \left(\frac{Z}{A}\right)^{1/n} + \left(\frac{Z}{A}\right)^{2/n} + 1\right\}^{1/2}
\end{align*} \tag{15}
$$

In this equation, the effect of temperature and strain rate has been considered in the Z parameter and the
material’s constants \((A, n, a, \text{and } Q)\) as functions of strain bring the effect of strain into the flow stress equation. Fig. 9 shows the comparison between the experimental and calculated flow stress for some representative curves of the modeling and prediction sets using Eq. (15). As can be seen in this figure for the modeling set, the calculated flow stresses are fairly consistent with the experimental values in some cases but deviate considerably in other cases. However, the calculated curves deviate largely for the flow curves of the prediction set. The average RMSE (AAE) for the modeling and prediction sets are 10.76 MPa (9.41%) and 13.78 MPa (9.91%), respectively. The large error values for the prediction set show that the conventional strain compensation approach is not an optimized one for modeling and prediction of hot deformation flow curves.

3.6. Modeling by the simplified strain compensation approach

Modeling based on the conventional strain compensation approach has an important issue, which is the change in the microstructure at a given strain for different deformation conditions. For instance, as shown in Fig. 1, while the flow curve at 1100 °C under 0.0001 s\(^{-1}\) reaches its peak stress at strain of \(\sim 0.13\), the flow curve at 950 °C under 0.05 s\(^{-1}\) is in the work-hardening stage. Therefore, for each given strain, the hyperbolic sine equation will be applied to the flow stress data determined from the different microstructures (e.g. different stages of DRX or different substructures). This is the main reason that the characteristic stresses that represent the same deformation or softening mechanism for all flow curves, such as steady state or peak stress, should be used in constitutive analysis to propose a valid constitutive equation like Eq. (6) (Mirzadeh, 2014). Moreover, this method needs time-consuming calculations. For calculation of \(\alpha\), both \(n\) and \(A\) should be calculated at each given strain and for calculation of \(Q\), the plots of \(\ln e\) against \(\ln(\sinh(\alpha r))\) and \(\ln(\sinh(\alpha r))\) against \(1/T\) should be used for each strain. Finally, the analysis similar to those performed for Fig. 2c should be used to find the final values of \(A\) and \(n\) at each given strain.

To solve the problems of this approach, a modified approach can be used in which the values of \(\alpha\) and \(Q\) can be taken from the analysis based on the peak stress (Section 3.1) and only \(n\) and \(A\) can be considered as strain-dependent parameters. This bypasses the indicated shortcoming and also simplifies the calculations. The
important step in the utilization of the hyperbolic sine equation to find the material’s constants is the determination of $a$ and $Q$. The modified approach uses the peak stress values for these calculations, which does not suffer from the differences in the microstructure at various $Z$ values. Moreover, the modified approach removes the need of heavy and time-consuming calculations for the determination of $a$ and $Q$ at each strain and only analyses similar to those performed in Fig. 2c should be used to find the final values of $A$ and $n$ at each given strain. Therefore, the modified approach can also be called the "simplified strain compensation approach". Therefore, using the values of $a = 0.011$ and $Q = 417.6$ kJ/mol, the values of $A$ and $n$ for each given strain were determined. The results are shown in Fig. 10a, in which the solid curves represent the best fitted polynomials to the data that were subsequently used for flow stress modeling. Fig. 10b and c show the comparison between the experimental and calculated flow stress by the simplified strain compensation approach. The average RMSE (AAE) for the modeling and prediction sets are 7.95 MPa (6.72%) and 7.22 MPa (6.32%), respectively. It is apparent from Fig. 10 and also from the error values that the simplified strain compensation approach results in a better fit to experimental data. This improvement is evident from the predicted curves (Fig. 10c), which shows that the correct implementation of the Arrhenius constitutive equation can result in a good prediction ability for unseen deformation conditions.
3.7. Constitutive analysis based on the physically-based approach

In Eq. (5), the values of $D_R$ and $Q_{SD}$ can be taken from the tables of the work of Frost and Ashby (1982). In these tables, the dependence of the shear modulus ($G$) on temperature in the form of $G/G_0 = 1 + \eta (T - 300)/T_M$ is also available. Here, $G_0$ is the shear modulus at 300 K, $T_M$ is the melting temperature of the material, and $\eta = (T_M/G_0)dG/dT$ shows the temperature dependence of the shear modulus. According to the relation of $E = 2G(1 + \nu)$, the values of $E$ can be estimated ($\nu$ is usually taken as 0.3). Using the available data for the most similar materials to AISI 630 stainless steel (which are AISI 304 and AISI 316 stainless steels) as shown in Table 1, the following expressions can be derived for $D$ and $E$:

$$D = 3.7 \times 10^{-5} \times \exp(-280000/RT)$$

(16)

$$E = 210600 \times (1 - 0.85(T - 300)/1680)$$

(17)

It can be deduced that there are three unknown parameters ($B$, $n$, and $x'$). It follows from the modified power and exponential laws based on Eq. (5) that the slope of the plot of $\ln(\dot{\varepsilon}/D)$ against $\ln(\sigma_p/E)$ and the slope of the plot of $\ln(\dot{\varepsilon}/D)$ against $\sigma_p/E$ can be used for obtaining the values of $n'$ and $\beta'$, respectively. These plots are shown in Fig. 11a. The linear regression of these data resulted in the average value of $x' = \beta'/n' = 1212.8$.

A visual comparison between Figs. 11a and 2a shows that in the physically-based approach (Eq. (5)), unlike the apparent approach (Eq. (4)), there is no need to consider separate regression analysis for each deformation temperature in order to find the value of the modified stress multiplier $x'$. This shows that the consideration of the dependences of Young’s modulus and self-diffusion coefficient on temperature and inserting these parameters into the constitutive equations significantly simplifies further regression analysis.

According to the hyperbolic sine law of Eq. (5), the slope and the intercept of the plot of $\ln(\dot{\varepsilon}/D)$ against $\ln(\sinh(x'\sigma_p/E))$ was used for obtaining the values of $n = 4.99$ and $\ln B = 29.8$ (Fig. 11b). It can be seen that the value of $n$ is close to 5 as discussed by Mirzadeh et al. (2011a) for the cases that the deformation mechanism is controlled by the glide and climb of dislocations (climb controlled) and the observed small deviation can be ascribed to the occurrence of dynamic recrystallization until the peak point of the flow curves. In summary, the resultant constitutive equation can be expressed as follows:

$$\dot{\varepsilon} = 387.615 \times (\sinh(1212.8 \times \sigma_p/E))^{4.9}$$

(18)

Or equivalently,

$$Z' = \dot{\varepsilon} \exp(280000/RT)$$

$$= 50.35^5 \times (\sinh(1212.8 \times \sigma_p/E))^{4.9}$$

(19)

where $Z' = \dot{\varepsilon} \exp(280000/RT)$ is an alternative expression for the Zener–Hollomon parameter, which can be easily determined by consideration of lattice self-diffusion activation energy as the hot deformation activation energy.

3.8. Modeling by the physically-based strain compensation approach

To make it possible to model the whole flow curve, the hyperbolic sine law of Eq. (5) can be applied to flow stresses that were taken at a given strain but at various $Z$ values to find the material’s constants at that particular strain. This procedure should be repeated for many strains to find the values of material’s constants ($x'$, $B$, and $n$) as functions of strain. Note that based on the physically-based approach, the value of $Q$ is equal to the lattice self-diffusion activation energy, which has been considered in $D$ by Eq. (16). Therefore, based on the hyperbolic sine law of Eq. (5), there will be three strain-dependent parameters. The flow stresses corresponding to different strains ranging from 0.05 to

### Table 1

Data required for characterizing the temperature dependence of the lattice self-diffusion coefficient and the shear modulus.

<table>
<thead>
<tr>
<th>Material</th>
<th>$D_R$ (m$^2$/s)</th>
<th>$Q_{SD}$ (kJ/mol)</th>
<th>$\eta$</th>
<th>$G_0$ (MPa)</th>
<th>$T_M$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 304 and 316 stainless steel</td>
<td>$3.7 \times 10^{-5}$</td>
<td>280</td>
<td>-0.85</td>
<td>81000</td>
<td>1680</td>
</tr>
<tr>
<td>$\gamma$-Iron</td>
<td>$1.8 \times 10^{-5}$</td>
<td>270</td>
<td>-0.91</td>
<td>81000</td>
<td>1184–1665</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$1.7 \times 10^{-4}$</td>
<td>142</td>
<td>-0.50</td>
<td>25400</td>
<td>933</td>
</tr>
<tr>
<td>Magnesium</td>
<td>$1.0 \times 10^{-4}$</td>
<td>135</td>
<td>-0.49</td>
<td>16600</td>
<td>924</td>
</tr>
<tr>
<td>Copper</td>
<td>$2.0 \times 10^{-5}$</td>
<td>197</td>
<td>-0.54</td>
<td>42100</td>
<td>1356</td>
</tr>
<tr>
<td>Nickel</td>
<td>$1.9 \times 10^{-4}$</td>
<td>284</td>
<td>-0.64</td>
<td>78900</td>
<td>1726</td>
</tr>
</tbody>
</table>
0.8 with an increment of 0.05 were obtained from the flow curves (modeling set) and the material's constants at each given value of strain were computed using the flow stresses obtained at different deformation conditions by the methods described in Section 3.7. The results are shown in Fig. 12a and b, in which the solid curves represent the best fitted six order polynomials to the data. The obtained equations from these regression analyses were subsequently used for flow stress modeling by equation of the form $\sigma = \frac{1}{C_0} \sinh \left( \frac{\varepsilon}{B} \right)^{1/n} \times E/\varepsilon^r$. In this equation, the effect of temperature has been considered in the $E$ and $D$ parameters and the material's constants ($B$, $n$, and $a_0$) as functions of strain bring the effect of strain into the flow stress equation. Fig. 12c and d show the comparison between the experimental and calculated flow stress by the physically-based approach. The average RMSE (AAE) for the modeling and prediction sets are 8.29 MPa (7.02%) and 8.41 MPa (6.66%), respectively.

It is apparent from Fig. 12 and also from the error values that the physically-based approach results in a good fit to experimental data. These results are consistent with the work of Samantaray et al. (2011) on a ferritic steel and Wei et al. (2014) on microalloyed steels. The physically-based approach partially solves the mentioned problems of the conventional strain compensation approach (Section 3.6) by consideration of the lattice self-diffusion activation energy as the hot deformation activation energy ($Q$). It is evident from Fig. 12 that this consideration has resulted in a significantly better prediction ability of the model. However, the physically-based approach, itself, suffers from the abovementioned shortcomings in calculation of $\varepsilon'$ for different strain values. Therefore, the physically-based approach should be simplified to appropriately address the mentioned problems.

3.9. Modeling by the simplified physically-based strain compensation approach

To solve the encountered problem of the original physically-based approach with $\varepsilon'$, the value of $\varepsilon' = 1212.8$ was taken from the analysis based on the peak stress (Section 3.7) and only $n$ and $B$ were considered as strain-dependent parameters. Therefore, the values of $B$ and $n$ at each given strain were determined. The results are shown in Fig. 13a, in which the solid curves represent the best fitted six order polynomial to the data that were subsequently used for flow stress modeling.

Fig. 13b and c show the comparison between the experimental and calculated flow stress by the simplified physically-based approach. Moreover, the average RMSE (AAE) for the modeling and prediction sets were determined as 8.32 MPa (7.20%) and 8.37 MPa (6.58%), respectively. These error values are comparable to those determined for the original physically-based approach and Fig. 13 shows that the prediction ability of this technique is good. Therefore, since the simplified physically-based approach is easier to use, it is suggested to model the flow curves by this approach.

4. Discussion

Table 2 compares the error values of the seven employed approaches. It can be seen that modeling by the Hollomon equation, the simplified strain compensation approach,
and the two physically-based strain compensation approaches are better than other methods in term of the error values for both the modeling and prediction sets. However, the appropriateness of the modeling by the Hollomon equation (and also modeling by fitting a polynomial function) highly depends on the ability of Eq. (8) in prediction of the value of the peak strain, which is usually a major problem in hot deformation studies as shown in Fig. 2d and other relevant research works (Mirzadeh et al., 2010, 2011b). This resulted in high error values for deformation conditions of 950 °C–0.05 s⁻¹ and 950 °C–0.01 s⁻¹ in Fig. 5. As shown in Fig. 14, if the experimentally determined value of peak strain for the 950 °C–0.05 s⁻¹ flow curve is used, a significantly better consistency with the experimental flow curve will be resulted.

The basic form of the Johnson–Cook model is well suited for computations and it is widely used in large-scale deformation codes because it uses variables that are readily available in most of the applicable computer codes (Liang and Khan, 1999). However, it was found in the present study and other relevant works (Vural and Caro, 2009).
that the original JC model cannot adequately represent the hot flow behaviors of many materials, mainly because it deals with temperature, strain rate, and strain, separately. Moreover, it was indicated in Section 3.4 that the availability of a good reference flow curve is essential for modeling by the JC model to be able to appropriately represent the hardening and softening stages.

The modeling approach based on the application of Hollomon equation to the master normalized flow curve can be considered as the simplest constitutive equation, in which by consideration of the coupled effect of temperature and strain rate (in the form of the Z parameter) through incorporation of the peak stress and peak strain into the formula, good prediction abilities can be easily achieved. The good results obtained from the modeling by the Hollomon equation and modeling by fitting a polynomial function imply that the idea of implementation of a master normalized curve can be used in future works for development of constitutive equations based on other approaches.

Another commonly used method for modeling of high temperature flow curve is the conventional strain compensation approach. In the present work, it was shown that this approach suffers from the change in the microstructures at different Z parameters for each given strain, which also resulted in high error values. The simplified strain compensation approach, by taking the stress multiplier and the deformation activation energy from the peak stress analysis, can be considered as an alternative one, which is simpler and results in the significantly better results. Similarly, good prediction abilities were achieved by implementation of the proposed physically-based approach for strain compensation, which accounts for the dependence of Young's modulus and the self-diffusion coefficient on temperature. The results of the four employed strain compensation approaches show that the main problem of strain compensation by the Arrhenius-type constitutive equation is with the deformation activation energy (Q) and this parameter should not be considered as a function of strain. Moreover, by setting the lattice self-diffusion activation energy in the physically-based strain compensation approach or by setting the apparent deformation activation energy from the analysis based on the peak stress in the simplified strain compensation approach, good prediction ability or fit to experimental data can be achieved.

It is interesting to note that the error values for the simplified strain compensation approach are slightly lower than those obtained for the physically-based approach. This can be ascribed to the fact that the deformation activation energy in the physically-based approach has a theoretical pre-defined value and it cannot be varied to give a better fit to the experimental data. Conversely, the activation energy for the simplified strain compensation approach is determined by the constitutive analysis based on the available data.

5. Conclusions

A comparative study was carried out on the appropriateness of the Hollomon equation, Johnson–Cook equation, and strain compensation (using the Arrhenius equation) in modeling and prediction of hot flow stress. The following conclusions can be drawn from this study:

1. The representation of a master normalized stress-normalized strain flow curve by simple constitutive equations is successful in modeling of high temperature flow curves, in which the coupled effect of temperature and strain rate in the form of the Zener–Hollomon parameter is considered through incorporation of the peak stress and the peak strain into the formula. However, the appropriateness of the modeling highly depends on the ability of the obtained formula for the peak strain in prediction of the correct value of the peak strain.

2. The Johnson–Cook equation failed to appropriately predict the hot flow stress, which was ascribed partly to the fact that it deals separately with the strain, strain rate, and temperature effects. Moreover, the availability of a good reference flow curve was found to be essential for modeling by the JC model to represent correctly the hardening and softening stages.

3. The conventional strain compensation approach is based on the equation of the type $Z = \dot{\varepsilon} \exp(Q/RT) = A[\sinh(\alpha \sigma)]^n$ with strain dependent material's parameters. It was shown that the change in the microstructure of the material at a given strain for different deformation conditions is responsible for the failure of this approach. Subsequently, a simplified approach was employed, in which by correct implementation of the hyperbolic sine law (taking $\alpha$ and $Q$ from the peak stress analysis), significantly better prediction abilities were obtained and the required calculations were simplified significantly.

4. Good prediction abilities were achieved by implementation of the proposed physically-based approach for strain compensation, which accounts for the dependence of Young's modulus ($E$) and the self-diffusion coefficient ($D$) on temperature in the hyperbolic sine equation of the form $\dot{\varepsilon}/D = B[\sinh(\alpha \tau \sigma/E)]^n$.

5. Based on the different techniques used in the present work, it can be concluded that for flow stress modeling through strain compensation by $Z = \dot{\varepsilon} \exp(Q/RT) = A[\sinh(\alpha \sigma)]^n$, the deformation activation energy should not be considered as a function of strain and only expressing the hyperbolic sine power ($n$) and the hyperbolic sine constant ($A$) as functions of strain is recommended. Moreover, for the physically-based approach, taking the modified stress multiplier ($\alpha'$) from the peak stress analysis is the recommended practice.

References
