Research article

An analytical study of the acoustic force implication on the settling velocity of non-spherical particles in the incompressible Newtonian fluid

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ABSTRACT: In this paper, the effect of the acoustic radiation force on the acceleration motion of a vertically falling non-spherical particle in the incompressible Newtonian fluid is investigated. The using of the acoustic radiation force for increasing the falling velocity of particles (especially non-spherical particles) has the most important implication on increasing the efficiency of particulate control devices such as gravity settling chambers. In this study, the motion of the non-spherical particles in the fluids such as water can be described by the force balance equation (Basset–Boussinesq–Ossen equation) plus the acoustic radiation force term. The main difficulty in the solution of this equation lies in the nonlinear term due to the nonlinearity nature of the drag coefficient. The settling velocity was calculated by using the differential transformation method, which is an analytical solution technique. The effect of some parameters such as the particle’s sphericity (0.3, 0.5, and 0.7), the amplitude of the acoustic pressure wave (200, 400, and 600 KPa), the distance from the acoustic pressure node (0.2, 0.4, and 0.6 m), and the acoustic source frequency (20, 40, and 60 KHz) on the settling velocity as a function of time was fully studied in this paper. © 2014 Curtin University of Technology and John Wiley & Sons, Ltd.

KEYWORDS: solid non-spherical particles; Basset-Boussinesq-Ossen equation; acoustic radiation force; differential transformation method

INTRODUCTION

The studying of the acceleration motion of particles in the different media has the most important implication on the selection of the best equipment for pollution control. Therefore, the researches and investigations in this field have been increased, and several studies have been carried out.\cite{1-4}

Several studies that have investigated the separation of particles utilizing the acoustic radiation force have previously been presented. Theoretical studies on the forces, known as acoustic radiation forces, date back to King in 1934. These forces were used for separating incompressible particles suspended in a viscous fluid. In 1955, other scientists such as Yosioka and Kawasima expanded the analysis to use acoustic radiation forces for the separation of the suspended particles. These results were organized by Gorkov in 1962, but limited to viscous fluids and particles smaller than the acoustic wavelength.

With recent developments in technology allowing for integration of the ultrasound sources with particle control devices, the acoustic radiation force has found a new role as a contact-free procedure to control particles.\cite{5}

Many empirical studies have proven that the application of the acoustic forces increases the settling velocity of particles, and therefore the collection efficiency of the conventional control devices will be improved.\cite{6-8}

These studies illustrate the fact that the acoustic forces move particles toward the pressure nodes or the pressure antinodes, depending on the density and the compressibility of the particles and the fluids. Identification of all parameters, which affect the motion and the settling of particles in the incompressible fluids and derive the differential equation, which can define the effect of these parameters have the most important implications on the development and the presentation of new devices for pollution control in the field of environmental engineering and open up new vision for construction advanced systems to control the emissions of particles.\cite{9-17}

There are two important parameters to introduce the settling motion of non-spherical particles. First of all
is the drag force, which is introduced by the drag coefficient, and the second is determination of a parameter, which can define the influence of different shapes of particles. This recent parameter is called sphericity, $\phi$, and the degree of it is described as the following equation\[1\]:

$$\varphi = \frac{A_s}{A_{as}}$$  \hspace{1cm} (1)

Where $A_s$ is the surface of a sphere, which has the same volume as the particle, and $A_{as}$ is the actual surface area of the particle. According to the investigation by Chien, the drag coefficient is illustrated by Eqn (2). In this equation, Re is Reynolds number. The Equation is valid in the range of $0.2 < \varphi < 1$ and $Re < 5000$.\[18\]

$$C_D = \frac{30}{Re} + 67.289\varphi^{(-5.03\Phi)}$$   \hspace{1cm} (2)

Reynolds number is introduced as Eqn (3) where $\mu$, $\rho$ are the viscosity of a fluid and the density of a fluid and $u$ and $D$ are the settling velocity and the diameter of a particle.\[1\]

$$Re = \frac{upD}{\mu}$$  \hspace{1cm} (3)

Previous studies was concentrated on the motion of a particle in a fluid described by a force balance equation called Basset–Boussinesq–Oseen (BBO) equation and solved the nonlinear ordinary differential equation by several analytical methods such as variational iteration method and differential transformation method (DTM). In this study, the influence of an external acoustic radiation force on the motion of a particle in the incompressible Newtonian fluid is investigated. For solving this problem, a term related to the acoustic radiation force is added to BBO equation, then it is solved by DTM analytical method to calculate the falling velocity of a particle in the acoustic field.\[1-4\]

**PROBLEM STATEMENT**

In this study, a non-spherical particle with sphericity of $\phi$ falling in a Newtonian medium under the effect of the gravity and the acoustic radiation force is considered. In this situation, a particle will accelerate until the drag force becomes equal to the gravity and the acoustic radiation force. This settling velocity is called terminal velocity. The motion of a particle for these assumptions is illustrated by the equation called BBO equation. For dropping a dense particle in the light liquids, an assumption is used, which says that

the density of fluid is negligible in comparison with the density of particle.\[1,2,9\]

According to the acoustic force theory presented by Yosioka and Kawasima, the force on a particle ($F_a$) can be illustrated in the following equation\[1,2,19\]:

$$F_a = -\left(\frac{\pi P_0^2 V p \beta_f}{2l}\right) \times \left(\frac{5\rho_s - 2\rho}{2\rho_s + \rho - \beta_p}{\beta_f}\right) \times \sin(2kx)$$  \hspace{1cm} (4)

Therefore BBO equation plus the acoustic radiation force is illustrated as follows\[1,2,19\]:

$$m \frac{du}{dt} = mg \left(1 - \frac{p}{\rho_s}\right) - \frac{1}{8} \pi D^2 \rho C_D u^2$$  \hspace{1cm} (5)

$$- \frac{1}{12} \pi D^3 \rho \frac{du}{dt} - \left(\frac{\pi P_0^2 V p \beta_f}{2l}\right) \times \left(\frac{5\rho_s - 2\rho}{2\rho_s + \rho - \beta_p}{\beta_f}\right) \times \sin(2kx)$$

In the aforementioned equation $\rho_s$, $m$ and $D$ are illustrated as the density of particle, the mass of particle, and the diameter of particle, respectively. Other parameters such as $P_0$, $Vp$, $\beta_p$, $B_p$, $\lambda$ and $\beta$ are defined as the pressure amplitude of the acoustic waves, the particle volume, the compressibility of fluid, the compressibility of particle, the acoustic wavelength, and the distance from the acoustic pressure node, respectively. The parameter called $k$ is defined as $2\pi/\lambda$.\[18\]

After combination three equations (2, 3, and 5), Eqn (5) will be arranged as follows\[1,2,19\]:

$$\left(m + \frac{1}{12} \pi D^3 \rho\right) \frac{du}{dt} + 3.75 \pi D \mu u$$

$$+ \frac{67.289\varphi^{(-5.03\Phi)}}{8} \pi D^2 \rho u^2 - mg \left(1 - \frac{p}{\rho_s}\right)$$

$$+ \left(\frac{\pi P_0^2 V p \beta_f}{2l}\right) \times \left(\frac{5\rho_s - 2\rho}{2\rho_s + \rho - \beta_p}{\beta_f}\right) \times \sin(2kx) = 0$$  \hspace{1cm} (6)

Equation (6) has five terms, which are introduced by the mass + added mass term, the linear drag term, the nonlinear drag term, the gravity–buoyancy term, and the acoustic radiation force term, respectively.

The parameter called the acoustic contrast factor determines the direction of the force on particles. A positive acoustic factor moves those particles to a pressure node, on the other hand, a negative acoustic factor moves those particles to a pressure antinode. Acoustic contrast factor is introduced as follows\[18\]:

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Acoustic factor

\[ Acousticfactor = \left( \frac{5\rho_s - 2\rho - \beta_p}{2\rho_s + \rho - \beta_f} \right) \]  

The most important aspect of the acoustic contrast factor is the possible sign change depending on the densities and the compressibilities, which determines the direction of the acoustic force, and thus whether the particle will move toward the standing wave pressure node or the antinode. Generally, solid particles in aqueous media are moved toward a pressure node while gas bubbles are moved toward the pressure antinode. Because of the acoustic contrast factor, even particles that are neutrally buoyant can experience an acoustic force as long as the compressibility differs from the surrounding medium.\(^{[17]}\)

For simplification of the nonlinear ordinary differential equation, some constants are introduced as follows\(^{[1,2,19]}\):

\[ a = \left( m + \frac{1}{12} \pi D^3 \rho \right) \]  
\[ b = 3.75 \pi D \mu \]  
\[ c = \frac{67.289 \rho^{(-5.03)} \pi D^2 \rho}{8} \]  
\[ d = mg \left( 1 - \frac{\rho}{\rho_s} \right) \]  
\[ e = -\left( \frac{\pi\rho_0^2 V \beta_f}{2\lambda} \right) \times \left( \frac{5\rho_s - 2\rho - \beta_p}{2\rho_s + \rho - \beta_f} \right) \times \sin(2kx) \]  

(12)

By using the constant parameters, which have been defined in the Eqns (8) to (12), the final nonlinear ordinary differential equation is introduced as follows\(^{[1,2,19]}\):

\[ a \frac{du}{dt} + bu + cu^2 - (d + e) = 0 \]  
\[ u(0) = 0 \]  

(13)

The initial condition for solving the nonlinear ordinary differential equation is considered zero.

\[ U(5) = \frac{1}{5!} \times \frac{(d + e)(b^4 - 22b^2c(d + e) + 16c^2(d + e)^2)}{a^5} \]  

Table 1. The fundamental operations of the differential transform method.\(^{[2]}\)

<table>
<thead>
<tr>
<th>Original function</th>
<th>Transformed function</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ x(t) = a(t) \pm \beta g(t) ]</td>
<td>[ X(k) = aF(k) \pm \beta G(k) ]</td>
</tr>
<tr>
<td>[ x(t) = \frac{df(t)}{dt} ]</td>
<td>[ X(k) = (k + 1)F(k + 1) ]</td>
</tr>
<tr>
<td>[ x(t) = \frac{d^2f(t)}{dt^2} ]</td>
<td>[ X(k) = (k + 1)(k + 2)F(k + 2) ]</td>
</tr>
<tr>
<td>[ x(t) = \exp(\lambda t) ]</td>
<td>[ X(k) = \frac{\lambda^k}{k!} ]</td>
</tr>
<tr>
<td>[ x(t) = f(t)g(t) ]</td>
<td>[ X(k) = \sum_{l=0}^{k} F(l)G(k - l) ]</td>
</tr>
</tbody>
</table>

**SOLUTION WITH DIFFERENTIAL TRANSFORMATION METHOD**

The difficulty in the solution of the Eqn (13) lies in the third term called the nonlinear drag term. In this regard, the analytical method called DTM is used for solving this equation. In this method, fundamental operations are carried out on original functions. These operations are illustrated in Table 1. After using these operations, the final nonlinear ordinary differential equation is converted to following equation.\(^{[20-24]}\)

\[ a(k + 1)U(k + 1) + bU(k) + c \left( \sum_{l=0}^{k} U(l)U(k - l) \right) - (d + e)\delta(k) = 0 \]  

(14)

The \(U(k+1) (k=0, 1, 2, 3, 4,...)\) is calculated from Eqn (14) as the following\(^{[20-24]}\).

\[ U(1) = \frac{(d + e)}{a} \]  
\[ U(2) = \frac{1}{2} \times \frac{b(d + e)}{a^2} \]  
\[ U(3) = \frac{1}{3!} \times \frac{(d + e)(b^2 - 2c(d + e))}{a^3} \]  
\[ U(4) = \frac{1}{4!} \times \frac{b(d + e)(b^2 - 8c(d + e))}{a^4} \]  

(17)  

(18)

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The calculation for the velocity of a non-spherical particle in any time Eqn (23) is illustrated as following\textsuperscript{[20–24]}:

\[ u_i(t) = \sum_{k=0}^{n} \left( \frac{t}{H_i} \right)^k U_i(k), 0 \leq t \leq H_i \] (23)

Where \( H \) is a constant, \( u(t) \) is the velocity of a non-spherical particle, and \( U(k) \) is the transformed velocity function. After substituting Eqns (15) to (22) in Eqn (23), the velocity of non-spherical particles is calculated as the following\textsuperscript{[20–24]}:

\[ u_i(t) = U(1)t + U(2)t^2 + U(3)t^3 + U(4)t^4 + U(5)t^5 + U(6)t^6 + U(7)t^7 + U(8)t^8 + \ldots \] (24)

**RESULTS AND DISCUSSION**

A single aluminum particle which has 3 mm equivalent diameter was considered to fall in a medium of water. The effect of some parameters such as the sphericity (0.3, 0.5, and 0.7), the amplitude of the pressure wave (200, 400, and 600 KPa), and the acoustic frequency (20, 40, and 60 KHz) on the falling velocity of an Aluminum particle are illustrated in this section. The density and the compressibility of Aluminum and water in standard temperature is 2702 kg/m\(^3\), 13.3 \times 10^{-12} Pa^{-1}, 996.51 kg/m\(^3\), and 4.58 \times 10^{-12} Pa^{-1}, respectively. In water, sound approximately travels 1497 m/s at 25°C.\textsuperscript{[12,19]}

In this study, the implication of a particle’s sphericity, the frequency of the acoustic radiation source, the distance from the acoustic pressure node, and the amplitude of it on the falling velocity of an Aluminum particle in water medium as the incompressible Newtonian fluid have been investigated and analyzed. The effect of these parameters on the acceleration motion of an Aluminum particle is illustrated in Figs 1 to 5.

The effect of the particle’s sphericity (0.3, 0.5, and 0.7) on the falling velocity of it without the acoustic force in comparison with the presence of the acoustic force is negligible and the curves overlap each other (Fig. 1). The acoustic radiation force has the important implication on increasing the settling velocity of particles, but the magnitude of the nonlinear drag term

\[
U(6) = -\frac{1}{6!} \times \frac{b(d+e)(b^4 - 52b^2c(d+e) + 136c^2(d+e)^2)}{a^6}
\]

\[
U(7) = \frac{1}{7!} \times \frac{(d+e)(b^6 - 114b^4c(d+e) + 720b^2c^2(d+e)^2 - 272c^3(d+e)^3)}{a^7}
\]

\[
U(8) = \frac{1}{8!} \times \frac{b(d+e)(b^6 - 240b^4c(d+e) + 3072b^2c^2(d+e)^2 - 3968c^3(d+e)^3)}{a^8}
\]
including the drag coefficient (which has relationship with particle’s sphericity) in BBO equation in comparison with magnitude of the acoustic force term of that equation is negligible. Therefore, it is possible for engineers to carry out the acoustic force for separating or settling the wide range of particles with different sphericity factor.

In Fig. 2, the effect of changing the amplitude of the acoustic radiation source on the falling velocity of an Aluminum particle is illustrated. By increasing the acoustic amplitude (200, 400, and 600 KPa), the settling velocity of the particle is increased. The maximum terminal settling velocity in 200, 400, and 600 KPa is introduced 2.648, 3.065, and 3.760 m/s, respectively. The direct relationship between the acoustic radiation force and the power two of the source amplitude has the most important effect to increase the settling velocity of it. Therefore, using this property can help environmental engineers to design the devices with more performance in comparison with other conventional devices.

The effect of different frequencies of the acoustic source (20, 40, and 60 KHz) on the falling velocity of an Aluminum particle is shown in Fig. 3. By increasing the frequency of the acoustic radiation source and decreasing the wavelength of it, the settling velocity of the particle is extremely decreased. The maximum terminal settling velocity in 20, 40, and 60 KHz is introduced 3.760, 1.030, and 0.069 m/s, respectively. Therefore, it is better for engineers to use the acoustic radiation force with lower radiation frequency to have the best efficiency for separating and settling of particles in the different medium. Figs. 2 and 3 explain the fact that using the acoustic source with lower frequency for increasing the settling velocity of the particles is better than using the acoustic source with higher acoustic radiation strength. In this situation, energy depletion related to the acoustic radiation source is reduced and access to the same terminal settling velocity (3.760 m/s) is achievable.

Small particles can generally be trapped in an acoustic standing wave. The first criterion is that the particle diameter must be less than half the wavelength,
otherwise, the acoustic force will act in more than one direction on the particle. The second criterion is that the acoustic factor must not equal zero, otherwise there will be no net force exerted on the particles.[17]

The influence of the distance from the acoustic pressure node on the falling velocity of an Aluminum particle from 0.2 to 0.6 m is shown in Fig. 4. By increasing the distance from the acoustic pressure node, the settling velocity of an Aluminum particle is decreased. Therefore, the reduction this distance has the most important implication on increasing the falling velocity of particles and the efficiency of the particle control devices.

The acceleration motion variations of an Aluminum particle in the presence of the acoustic force (with different acoustic pressure amplitudes and the constant frequency) and without using it is illustrated in Fig. 5. The initial acceleration of the particle in 600 KPa has the highest value (6.219 m/s²) and after passing the time between 0.008 and 0.009 s, the acceleration values reach to zero. The initial acceleration of the particle without using the acoustic force has the lowest value (4.150 m/s²), and after passing more time, in comparison with the situation, using the acoustic force of the acceleration values decrease and reach to zero.[1,2]

According to the Eqn (7) and by using the density, the compressibility of Aluminum and water, it is illustrated that the acoustic contrast factor is positive, and an Aluminum particle moves to the pressure nodes of the acoustic radiation field.[19]

**CONCLUSIONS**

In this paper, the analytical method called DTM is applied to solve the nonlinear differential equation of the falling motion of a non-spherical particle. The nonlinear differential equation is derived by using BBO equation plus the term related to the acoustic radiation force. After changing some parameters such as the sphericity, the amplitude of the acoustic radiation source, the frequency of the acoustic source, and the distance from the acoustic pressure node, the following conclusions are illustrated:

1. Modification of BBO equation with adding a term related to the acoustic radiation force is very useful to define the acceleration motion of particles in the incompressible Newtonian fluid.
2. The influence of the particle’s sphericity in the presence of the acoustic force on the falling motion of particles is negligible.
3. Increasing the amplitude of the acoustic radiation source results to increase the initial acceleration of particles and the falling velocity of them.
4. Increasing the frequency of the acoustic source results to decrease the settling velocity of particles.
5. Increasing the distance from the acoustic pressure node results to decrease the settling velocity of particles.

**REFERENCES**