Abstract

During the past few years, integration of key decisions areas is regarded as an important ingredient for the success in marketing planning. In this paper, we developed an integrated model, which simultaneously determines optimal price, setup cost, service cost, lot size, Marketing expenditure, inventory holding cost and reliability of the production process for a manufacturer who produces a single product and competes in two different markets. In order to solve the resulted model, we developed a multi-objective geometric programming (GP) approach. In addition, we used compromise programming approach to transform the multi-objective into a single objective function. The study considers demand as a function of marketing expenditure, price, quality and service expenses. We also assume the unit production cost is affected by lot-size, product’s quality and process reliability. Finally we discuss different aspects of the results problem by performing sensitivity analysis.

Keywords: geometric programming, compromise programming, optimal Pricing, Customer service.

1. Introduction

During the past few years, management science literatures have clearly emphasized on collaboration and harmony among different functional areas in the formulation of an integrated strategy (Song et al. 1997; Crittenden et al. 1993; Ruekert and Walker 1987; Kahn and McDonough 1997; Olson et al. 2001; Rosenzweig et al. 2003; Wind 2005; Hutt and Speh 1984; Shapiro 1977). Many studies stated that aligning marketing and manufacturing strategies could increase firms’ capabilities to become more responsive on meeting customer demands (Ho and Tang 2004; Weir et al. 2000; Kahn and Mentzer 1998; Crittenden et al. 1993; Prabhaker 2001). According to porter, competitive advantage is obtained through the value a particular firm creates for its customers through key value adding activities such as logistics, operations, marketing and sales and service (Porter 1985). In this regard, as Bateson notes, this often prompts the traditional product- and production-oriented firms to shift the focus to a more customer-oriented technique (van Birgelen et al. 2003; Srivastava et al. 1984). In reviewing the literature on marketing, relationship marketing (RM) is considered as a vehicle to reach objective. It is not only highly applicable for all sorts of services, but it also provides input into a general paradigm for business of various industries, especially in manufacturing industry (Gummesson 1994, 1998). RM is the union of customer service, quality and marketing (Christopher et al. 1991). In respect of quality, in the literature of production/operations management, there have been several ways to define the term of quality (Chen 2000). But the definition highlighted in the RM strategy is Customer Perceived Quality (CPQ) (Grönroos 1997), which can briefly be described as percentage of customers that are fully compliant with the purchase product (Sadjadi et al. 2012). In respect to customer service, there have been several studies among which (Youngdahl and Kellogg 1997; Coelho and Henseler 2012; Watson 1987) are representative. Especially, Wouters (Wouters 2004) described that the customer service capability of organization varies and the results indicate that the impact of customer service performance varies among customers and situations. Accordingly, combining supplier customer service capability and buyer customer service sensitivity provides a basis for customer service strategy taxonomy.
Since the publication of seminal studies like Shapiro (Shapiro 1977), the problem of integration the key decision areas, has received much attention in the literature. In this direction, Geometric programming (GP) modeling studies is part of the literature. The GP theory for first time was proposed in 1961 (Duffin et al. 1967), which is a type of modeling used in the literature. For example, Lee and Kim (Lee and Kim 1993) investigated the effect of integrating production and marketing decisions for a short and medium range planning horizon in profit maximizing profit. Lee (Lee 1993) presented two models and analyzed the issue of collaboration and non-collaboration between marketing and production departments. Kim and Lee (Kim and Lee 1998) developed a general optimal coordination approach based on a marginal analysis of the profit function. Jung and Klein (Jung and Klein 2001) analyzed two economic order quantity based inventory models under total cost minimization and profit maximization via GP technique and the problem was analyzed based on sensitivity analysis. Sadjadi et al. (Sadjadi et al. 2005) formulated a profit maximizing GP model for optimal production and marketing planning. Jung and Klein (Jung and Klein 2005) investigated three GP models which in each of these models a different definition of cost function is presented. Mandal et al. (Mandal and Roy 2006; Mandal et al. 2006) solved some inventory problems using GP method. Fathian et al. (Fathian et al. 2009) solved a pricing problem for electronic products. Some researchers have considered the issues of process reliability, flexibility (set-up) (Leung 2007; Islam and Roy 2007; Diaby et al. 2013; Panda and Maiti 2009; Sadjadi et al. 2010) and quality in GP problems (Chen 2000). Recently Sadjadi et al. (Sadjadi et al. 2012) developed an economic production quantity model considering flexibility and reliability of production process.

This paper presents a new model to determine optimal prices and customer service costs influencing on company’s revenues in multiple competition markets. Also we assume the demand as a function of price, marketing expenditure, customer service expenditure and CPQ of the products. The resulted problem is formulated as multi-objective GP model based on global criterion method.

The remainder of the paper is organized as follows. In the next Section, we present the problem statement and underlying assumptions for developing the profit maximization GP model for the problem under consideration. Mathematical formulation of the GP model is presented in Section 3. Solution approach, followed by a numerical example for illustrative purposes, is the subject of Section 4. Sensitivity analysis of the optimal solution is presented in section 5. Finally, conclusions are made in Section 6.

2. Problem statement and assumptions

Consider a manufacturer who produces a product and competes in several markets. Manufacturer to meet any market demand; generates manufacturing sites within each market. But all marketing activities and services in different markets will be accomplished in marketing department as centralized approach. Because of the increased competitive environment in the market, the manufacturer must simultaneously consider several facts. First, due to budgetary constraints, the proposed model should select the most appropriate decisions so that the profits are optimized. On the other hand, in order to fulfill the demands of clients as the most important factor in determining the CPQ to make appropriate decisions on capital allocation (investment) in field of quality and other fields. Finally, in order to achieve manufacturing flexibility and reliability in the production process forced to make a series of investments in personnel training, producing technology and maintaining tools. The point is that decisions in each of these areas must consider the condition and to be integrated into other areas.

A integrated model is developed under following assumptions and notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i$</td>
<td>Annual demand in market $i=1,...,I$.</td>
</tr>
<tr>
<td>$C_{2i}$</td>
<td>Production cost per unit in production site $i=1,...,I$.</td>
</tr>
<tr>
<td>$f$</td>
<td>percent inventory holding cost (per unit per unit time) $(0 &lt; f &lt; 1)$</td>
</tr>
<tr>
<td>$w$</td>
<td>space requirement for each item</td>
</tr>
<tr>
<td>$W_i$</td>
<td>total space available for holding produced items in site $i=1,...,I$.</td>
</tr>
<tr>
<td>$B$</td>
<td>total budget available to the marketing and customer service.</td>
</tr>
<tr>
<td>$N_i(r_i)$</td>
<td>maintenance costs per production cycle in site $i=1,...,I$.</td>
</tr>
<tr>
<td>$Y_i(C_{ii},r_i)$</td>
<td>total cost of interest and depreciation for the production process in each cycle in production site $i=1,...,I$.</td>
</tr>
</tbody>
</table>
Marketing costs share for market $i=1,...,I$.

**Decision variables**

$p_i$  
Selling price per unit in market $i=1,...,I$.

$M_j$  
Volume of investments in marketing method $j=1,...,J$, per unit time in market $i=1,...,I$.

$S_l$  
Volume of investments in customer service method $l=1,...,L$, per unit time in market $i=1,...,I$.

$q_i$  
Economic production quantity in production site $i=1,...,I$.

$q$  
Quality level of the product from the customer's point of view.

$r_i$  
Reliability level of the production process (percent of non-defective items in a batch) in production site $i=1,...,I$.

$C_{ii}$  
Set-up cost (representing process flexibility) in production site $i=1,...,I$.

The following assumptions are used in this paper:

I: Replenishment is instantaneous.

II: No excess stock is carried, and no shortage and lost sales are allowed.

III: The production quantity is produced in batches (lots).

IV: All batches are subject to a 100% inspection policy and all defective items are discarded.

The following power function relations are defined for our model:

$$D_i(p_i, M_j, q, S_l) = k_i p_i^{-\alpha_i} \prod_{j=1}^{J} M_j^{\beta_{ij}} \prod_{l=1}^{L} S_l^{\tau_{il}} q^{\gamma_i}$$ (1)

$$C_{ii} Q_i(r_i) = b_i Q_i^{-\lambda_i} q^{r_i} r_i$$ (2)

$$Y_i(C_{ii}, r_i) = c_i C_{ii}^{\phi_i} r_i$$ (3)

$$N_i(r_i) = m_i r_i$$ (4)

In Eq. (1), demand per unit time is defined as a decreasing power function of price per unit, and increasing power function of marketing expenditure, customer service expenditure and quality. $M_j = (M_1, M_2,...,M_J)$ and $S_l = (S_1, S_2,...,S_L)$ respectively are vectors representing volume of investments in different marketing methods (channels) and customer service types in several market. Here, $k_i$ is scaling constant ($k_i > 0$), $\alpha_i$ is price elasticity to demand ($\alpha_i > 1$), $\beta_{ij}$, $\tau_{il}$ respectively are marketing and customer service elasticity of demand with respect to expenditures in market $i$, ($0 < \beta_{ij}, \tau_{il} < 1$), and $\sigma_i$ is quality elasticity of demand in market $i$ ($0 < \sigma_i < 1$).

In Eq. (2), unit production cost is defined as a decreasing power function of lot size, and increasing power function of both product’s quality and process reliability. In this function, $b_i$ is scaling constant ($b_i > 0$), $\lambda_i$ is elasticity of unit production cost with regard to lot size in production site $i$, ($0 < \lambda_i < 1$), $\psi_i$ is quality elasticity of unit production cost in production site $i$ ($\psi_i \geq 1$), and $\varphi_i$ is reliability elasticity of unit production cost in production site $i$ ($\varphi_i > 1$).

In Eq. (3), interest and depreciation cost is defined as a decreasing power function of set-up cost, and increasing power function of process reliability. In this function, $\gamma_i$ is a scaling constant ($\gamma_i > 0$), $\delta_i$ is elasticity of interest and depreciation cost with regard to set-up cost in market $i$, and $\Theta_i$ is elasticity of interest and depreciation cost with regard to reliability in market $i$.

In Eq. (4), maintenance cost is defined as a decreasing power function of process reliability, where $m_i$ is a scaling constant ($m_i > 0$), and $\gamma_i$ is elasticity of maintenance costs with regard to reliability in production site $i$ ($0 < \gamma_i < 1$). This is generally true, since by increasing the reliability of the process the less failure-prone the machinery will become which, in turn, results in decreasing of maintenance costs.
More or less similar structures for the above power functions are proposed by other researchers (Fathian et al. 2009; Sadjadi et al. 2010; Islam and Roy 2007; Lee and Kim 1993; Chen 2000; Jung and Klein 2005).

3. Mathematical model

In this part, consider the fact that a market condition differs from other market conditions and the market reaction to such advertising, product price, product quality and service is different and also taking into account the assumptions stated in Section 2, we have tried to present an integrated model for profit maximization.

In this model the manufacturer profit for each cycle of market \((i=1,\ldots, I)\) shown as follows:

\[
\text{profit}(i) = \frac{r_i Q_i}{C_{i1}} - \frac{C_{2i}}{2} - f_i C_{2i} r_i Q_i T_i - Y_i(C_{i1}, r_i) - N_i(r_i) - \left(\sum_{j=1}^{I} M_j \right) - e_i T_i \times \left(\sum_{i=1}^{I} S_i\right)
\]

In Eq. \(5\) inventory holding cost in per cycle

\[
= f_i C_{2i} \int_{0}^{T} q(t) dt = f_i C_{2i} \int_{0}^{T} (r_i Q_i - D_i t) dt = f_i C_{2i} (r_i Q_i T_i - \frac{D_i T^2}{2}) = f_i C_{2i} \frac{1}{2} r_i Q_i T_i
\]

In this model \(q(t)\) is inventory level in time \(q(t) = \begin{cases} r_i Q_i & \text{at } t = 0 \\ 0 & \text{at } t = T_i \end{cases}\) and \(T_i = \frac{r_i Q_i}{D_i}\), is the cycle length.

This model is similar to those proposed by (Sadjadi et al. 2012; Sadjadi et al. 2010).

Based on the Eq. \(5\) and the number of cycles \(\frac{D_i}{r_i Q_i}\) Annual earnings per manufacturer market include:

\[
\pi(p_i, M_i, S_i, Q_i, r_i, q, C_{i1}) = \frac{1}{T} \left[ p_i r_i Q_i - C_{i1} - C_{2i} Q_i - f_i C_{2i} \frac{1}{2} r_i Q_i T_i - Y_i(C_{i1}, r_i) - N_i(r_i) - T_i \times \left(\sum_{j=1}^{I} M_j + \sum_{i=1}^{I} S_i\right) \right] = \left[ k_i p_i \prod_{j=1}^{\gamma} M_j^{\beta_j} \prod_{i=1}^{\lambda} S_i^{\alpha_i} q^{\sigma_i} - k_i C_{i1} p_i \prod_{j=1}^{\gamma} M_j^{\beta_j} \prod_{i=1}^{\lambda} S_i^{\alpha_i} q^{\sigma_i} r_i^{\beta_i} - k_i b_i Q_i^\lambda q^{\omega_i + \eta_i} r_i^{\beta_i} - p_i \prod_{j=1}^{\gamma} M_j^{\beta_j} \prod_{i=1}^{\lambda} S_i^{\alpha_i} q^{\sigma_i} \right.
\]

\[
\left. + k_i m_i r_i^{\gamma_i} q^{\omega_i + \eta_i} p_i \prod_{j=1}^{\gamma} M_j^{\beta_j} \prod_{i=1}^{\lambda} S_i^{\alpha_i} Q_i^{\lambda_i} q^{\gamma_i} \right]
\]

In maximaizing Eq. \(6\), it is well understood that a manufacturer in the world is faced with a series of restrictions, Restrictions that we have used in this model are as follows:

\[
\sum_{j=1}^{I} M_j + \sum_{i=1}^{I} S_i \leq B, \quad \forall i
\]

\[
\sum_{i=1}^{I} D_i \geq \rho_i P_i, \quad \forall i
\]

\[
\eta_i Q_i \leq R_i, \quad \forall i
\]

\[
q \leq 1,
\]
\begin{align*}
    r_i & \leq 1, \quad \forall i \\
    w r_i Q_i & \leq W_i, \quad \forall i \\
    p_i & > 0, q > 0, C_i > 0, Q_i > 0, M_i > 0, S_i > 0, r_i > 0, \forall i, j, l
\end{align*}

In explaining the constraints that producers are faced, we can say that Constraint (7) indicates the limit of budgets available for marketing and servicing for all methods and types. Constraints (8) represent the manufacturer's production capacity for the available resources. \( R \) represents the limits of available resources, and \( \eta \) represents the resources needed to produce each item. Constraint (9) implies that the amount of covered demand by the manufacturer in each market should not be less than the proportion of target market \( (\rho) \) to total market \( (\rho) \). Constraint (10) indicates that the product quality is associated with the customer demands can not exceed than 1. Here 1 indicates that the customer is 100% satisfied with the product. Constraint (11) indicates that reliability of the production process can not be greater than 1. Constraint (12) is the limitation of storage space for acceptable products. Finally, constraint (13) indicates that all decision variables should be positive.

The maximizing Eq(6) is a signomial GP that can be converted into the posynomial GP from with one additional variable \((z)\) and constraint (see appendix A).

4. Solution approach and numerical experiment

4.1.a numerical example

For better understanding the model, consider that a manufacturer produces a product at two sites in different geographic regions. Marketing operations and services in the two markets are concentrated in the central department (see Fig.1). The marketing department uses two methods to advertising the products for example TV and Radio. The results of the questionnaire survey conducted shows that one market (market customers), which items provided by the Site 1, are more sensitive to the quality and sales service in comparison to market 2.

also, the price in market 2 in customer view have high importance and advertising is more effective than market 1.

![Fig.1 general schema for model configuration](image)

Accordingly, the central office with regard to all aspects decided to decarded 55% and 45% costs of advertising and services given to customer of sales revenue of market 1 and market 2 respectively. Total budget of central department for advertising and customer service is $65,000. The required parameters to build the model are set after market research studies. these parameters are as follows:

<table>
<thead>
<tr>
<th>Market 1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_i = 1.25 \times 10^4 )</td>
<td>( \alpha_i = 1.72 )</td>
</tr>
<tr>
<td>( \sigma_i = 0.68 )</td>
<td>( \beta_i = 0.038 )</td>
</tr>
<tr>
<td>( P_i = 4 \times 10^{-4} )</td>
<td>( \beta_{i1} = 0.073 )</td>
</tr>
<tr>
<td>( \rho_i = 0.21 )</td>
<td>( \tau_{i1} = 0.045 )</td>
</tr>
<tr>
<td>( \psi_i = 1.2 )</td>
<td>( \tau_{i2} = 0.071 )</td>
</tr>
<tr>
<td>( \lambda_i = 0.03 )</td>
<td>( \varphi_i = 1.9 )</td>
</tr>
<tr>
<td>( \phi_i = 1.9 )</td>
<td>( \delta_i = 2.2 )</td>
</tr>
<tr>
<td>( \theta_i = 1.15 )</td>
<td>( \gamma_i = 0.27 )</td>
</tr>
<tr>
<td>( \Psi_i = 6000 )</td>
<td>( \psi_{i1} = 15 )</td>
</tr>
<tr>
<td>( \lambda_i = 0.03 )</td>
<td>( \psi_{i2} = 1400 )</td>
</tr>
</tbody>
</table>
Based on this information of any competitive markets, the function of profit maximization is defined as:

\[
\max \pi_1(p_1, M_j, S_1, q, Q_1, C_{11}, r_1) = \left[ p_1 D_1 - C_{11} D_1 r_1^{-1} Q_1^{-1} - C_{22} D_1 r_2^{-1} - 0.5 f(C_2, r_1 Q_1) - Y_1(C_{11}, r_1) D_1 r_1^{-1} Q_1^{-1} - N_1(r_1) D_1 r_1^{-1} Q_1^{-1} - e_1 \left( \sum_{j=1}^{2} M_j + \sum_{l=1}^{2} S_l \right) \right]
\]

subject to the same constraints as in Eqs. (7-13).

(14)

\[
\max \pi_2(p_2, M_j, S_1, q, Q_2, C_{22}, r_2) = \left[ p_2 D_2 - C_{22} D_2 r_2^{-1} Q_2^{-1} - C_{22} D_2 r_2^{-1} - 0.5 f(C_2, r_2 Q_2) - Y_2(C_{22}, r_2) D_2 r_2^{-1} Q_2^{-1} - N_2(r_2) D_2 r_2^{-1} Q_2^{-1} - e_2 \left( \sum_{j=1}^{2} M_j + \sum_{l=1}^{2} S_l \right) \right]
\]

subject to the same constraints as in Eqs. (7-13).

(15)

One can view the summation arguments in (14) and (15) in two ways: as transformations of the original objective function, or as components of a distance function that minimizes the distance between the solution point and the utopia point in the criterion space. Consequently, global criterion methods are often called utopia Point methods or compromise programming methods, as the decision-maker usually has to compromise between the final solution and the utopia point (Marler and Arora 2004). The weighted global criterion for minimizing distances is stated as

(Miettinen 1999). minimize

\[
L_F(f(x)) = \left( \sum_{i=1}^{f} \omega_i \left| f_i(x) - f_i(x^*) \right|^F \right)^{\frac{1}{F}}
\]

Subject to \( x \in X \) for \( 1 \leq F \leq \infty \),

where, \( \omega \) is a vector of weights typically set by the decision maker such that \( \sum_{i=1}^{f} \omega_i = 1 \) and \( \omega > 0 \). Generally, the relative value of the weights reflects the relative importance of the objectives. \( F \) is a parameter \( (1 \leq F \leq \infty) \). Generally, \( F \) is proportional to the amount of emphasis placed on minimizing the function with the largest difference between \( f(x) \) and \( f(x^*) \).

In our model, \( \pi_1 \) and \( \pi_2 \) are the utopia objective values of \( z_1 \) and \( z_2 \), respectively. utopia objective values can be obtained by using GP method (Appendix A).

Hence, weighted \( L_F \) problem for our model is as follows,

\[
\min T_F(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C_{11}, C_{12}, r_1, r_2, z_1, z_2)
\]

\[
= \left( \omega_1 (z_1^{-1} - \pi_1^{-1})^F + \omega_2 (z_2^{-1} - \pi_2^{-1})^F \right)^{\frac{1}{F}}
\]

subject to the same constraints as in Eqs. (7-13).

(17)

For \( F = 1 \) metric, the best compromise solution is as follows,

\[
\min T_1(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C_{11}, C_{12}, r_1, r_2, z_1, z_2)
\]

\[
= \omega_1 (Z_1^{-1} - \pi_1^{-1}) + \omega_2 (Z_2^{-1} - \pi_2^{-1})
\]

(18)
subject to the same constraints as in Eqs.(7-13).

Since \( \omega_1, \omega_2, \pi_1 \) and \( \pi_2 \) are independent of the decision variables, so it is enough to solve the following problem:

\[
\begin{align*}
\text{minimize } & U(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C, r_1, r_2, z_1, z_2) \\
\text{subject to the same constraints as in Eqs.(7-13).}
\end{align*}
\]

(19)

Where

\[
\begin{align*}
& T_i(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C, r_1, r_2, z_1, z_2) \\
& = U(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C, r_1, r_2, z_1, z_2) - (\omega_1\pi_1^{-1} + \omega_2\pi_2^{-1})
\end{align*}
\]

(20)

For \( F = 2 \) metric, the best compromise solution is as follows,

\[
\begin{align*}
\text{minimize } & U(p, p, M, M, S, S, q, Q, Q, C, C, r, r, z, z) \\
\text{subject to the same constraints as in Eqs.(7-13).}
\end{align*}
\]

(21)

The closest answer to the minimization problem is as follows.

\[
\begin{align*}
& z^* = \omega_1 z_1^{-2} + \omega_2 z_2^{-2}
\end{align*}
\]

(22)

\[
\begin{align*}
& T_i(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C, r_1, r_2, z_1, z_2) \\
& \leq w_1 z_1^{-2} + w_2 z_2^{-2}
\end{align*}
\]

(23)

The closest answer to the minimization problem is as follows.

\[
\begin{align*}
& \text{min}(p_1, p_2, M_1, M_2, S_1, S_2, q, Q_1, Q_2, C, r_1, r_2, z_1, z_2) = w_1 z_1^{-2} + w_2 z_2^{-2}
\end{align*}
\]

(24)

\[
\begin{align*}
& w_1 z_1^{-2} + w_2 z_2^{-2} \leq 1,
\end{align*}
\]

(25)

\[
\begin{align*}
& w_2 z_2^{-2} \leq 1,
\end{align*}
\]

(26)

\[
\begin{align*}
& (\sum_{j=1}^2 M_j + \sum_{j=1}^2 S_j) B^{-1} \leq 1,
\end{align*}
\]

(27)

\[
\begin{align*}
& r_1 \leq 1,
\end{align*}
\]

(28)

\[
\begin{align*}
& r_2 \leq 1,
\end{align*}
\]

(29)

\[
\begin{align*}
& \eta_1 Q_1^{-1} \leq 1,
\end{align*}
\]

(30)

\[
\begin{align*}
& \eta_2 Q_2^{-1} \leq 1,
\end{align*}
\]

(31)

\[
\begin{align*}
& q \leq 1,
\end{align*}
\]

(32)

\[
\begin{align*}
& p_1 > 0, q > 0, C_1 > 0, C_2 > 0, M_1 > 0, S_2 > 0, r_1 > 0, \forall i, j, l
\end{align*}
\]

(33)

Model (22)-(33) is a very difficult GP problem with eighteen degrees of difficulty (Duffin et al. 1967). Therefore, in order to find the optimal solution of the problem the CVX Modeling System (Boyd and Michael 2009), which can be implemented on MATLAB software is used.

For the above example, utopia objective values are \( z_1 = \$45030589 \) and \( z_2 = \$40403341 \). Also, the results cvx of compromise programming method for the following \( (\omega_1 = 0.5, \omega_2 = 0.5) \) will be:

\[
\begin{align*}
& (\omega_1 = 0.5, \omega_2 = 0.5)
\end{align*}
\]

458
For $F = 1$

$$z_1^* = 42997161, \quad z_2^* = 37920062, \quad p_1^* = 15, \quad p_2^* = 22, \quad M_1^* = 11342, \quad M_2^* = 21932$$
$$S_1^* = 12216, \quad S_2^* = 19509, \quad q^* = 0.7, \quad Q_1^* = 466, \quad Q_2^* = 433$$
$$r_1^* = 0.69, \quad r_2^* = 0.62, \quad C_{11}^* = 2.6, \quad C_{12}^* = 2.7$$

For $F = 2$

$$z_1^* = 4258706, \quad z_2^* = 38212763, \quad p_1^* = 15.3, \quad p_2^* = 22.7, \quad M_1^* = 11353, \quad M_2^* = 21980$$
$$S_1^* = 12183, \quad S_2^* = 19481, \quad q^* = 0.73, \quad Q_1^* = 466, \quad Q_2^* = 433$$
$$r_1^* = 0.67, \quad r_2^* = 0.61, \quad C_{11}^* = 2.6, \quad C_{12}^* = 2.7$$

5. Sensitivity analysis

In this section, according to assumptions of $\omega_1 = 0.5, \omega_2 = 0.5$ and $F = 2$, a sensitivity analysis is performed on the examples described in previous section to analyze the effects of parameter changes on the optimal solution. Table 3 shows the results of the sensitivity analysis on the example. In this table, changes of the parameters for the first and the second markets are examined as follows:

$$\alpha_i = \alpha_{i0} + (0.01 \times H), \quad \beta_{11i} = \beta_{110} (r_{1i}) + (2 \times 5^{-4}) \times H, \quad \beta_{21i} = \beta_{210} (r_{2i}) - (2 \times 5^{-4}) \times H, \quad \sigma_i = \sigma_{i0} + (7 \times 5^{-4}) \times H, \quad \lambda_i = \lambda_{i0} + (2 \times 5^{-4}) \times H \quad \text{ (34) } \quad \psi_i = \psi_{i0} + (6 \times 5^{-3}) \times H, \quad \text{ (39) }$$

The following conclusions can be drawn from Table 1:

- We can observe, that with the increase in $\alpha$ in each the market, the company for maintaining its competitive position has to decrease its product price . As a result, the total profit also decrease. Therefore, the company has to reduce its total cost such as maintenance cost and quality products cost, and according to marketing and customer service budgets limitations, it also has to invest in the effective methods (tayps) in order to alleviate huge losses in total profit.
- The effects of changes in $\beta_{jl}$ and $r_{jl}$ (j, l, i = 1, 2) on the optimal solutions are investigated in situations that $\beta_{jl}$ and $r_{jl}$ are increased, while simultaneously $\beta_{ji}$ and $r_{ji}$ are decreased. This investigation shows that when $\beta_{jl}$ and $r_{jl}$ get closes to each other, the product price, and total annual profit decrease. Hence the company has to manage its costs with the same intensity.
- The changes in $\sigma_i$ shows that when the demand of customer are highly sensitive to the level of product’s quality, generally, the sensitive of customer to the price of product decreases, therefore, the company can increases its selling price. Also, as a result of this increase in the quality level, on the one hand, the model shows that the customer service has more practical impact on the demand of product in comparison to advertising. On the other hand, the model forces the set-up cost and the process reliability to decrease in order to hedge against losses in the profit due to increased costs.
- According to changes of $\lambda_i$ (increases), the optimal price, set-up cost, total profit, reliability of process and optimal quality are highly sensitive to the changes of parameter $\lambda_i$. Generally, when the unit production cost is more sensitive to lot-size, the lot-size should be increases to decreases total cost. Lot size in this example due to the limited production capacity, is not possible to be increased. For example, if the source increases (for example, the machining time) at the second site from 1300 to 1350, Lot size in the second site will increase from $Q_2^* = 433$ to $Q_2^* = 450$.
- From changes of $\psi_i$, it can be seen that as $\psi_i$ increases, the optimal quality and optimal selling price decreases, while the optimal reliability of process increases. Since unit production cost has a positive correlation
with the quality, the company has to reduce its product’s quality level in order to minimize its total cost. Hence
the company to compensate negative effects of this reduction such as loss of market share has to decreases sales
price, increases reliability of process, and investment on the appropriate options ($M_i$ and $M_j$) to attract
customers.

- According to changes of $\phi_i$, with increases in $\phi_i$, the model forces the optimal price and quality to
increases, and the process reliability and set-up cost to decreases in order to reach the optimal condition.

- Also, it is seen that the optimal solutions are slightly sensitive to the changes in the values of $\delta_j$ and
$\theta_j$ except set-up cost that with as increases in this parameters are increased.

- According changes of $\gamma_i$, the more $\gamma_i$ increases, the more process reliability will increases. Also,
increasing the parameters $\gamma_i$ will lead to price reduction, which will be obvious with regards to maintenance
cost reduction.

- The behavior of the total annual profit ($z_i^*$) in each sites with regard to changes in the values of $\omega_j$ are
shown in Figs.2 and 3. Also Fig. 4 shows that when total annual profit of company reaches its peak that weights
of markets are $\omega_1 = 0.6$ and $\omega_2 = 0.4$. The optimal solutions of these situations for $F = 2$ are as follows:

\[
\begin{align*}
    z_i^* &= $43614507, & z_j^* &= $37393453, & p_i^* &= $14.6, & p_j^* &= $21, & M_i^* &= $11293, \\
    M_j^* &= $21824, & S_i^* &= $12286, & S_j^* &= $19596, & q_i^* &= 0.64, & Q_i^* &= 350, \\
    Q_j^* &= 325, & r_i^* &= 0.71, & r_j^* &= 0.64, & C_{1i}^* &= $2.6, & C_{1j}^* &= $2.7
\end{align*}
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**Fig. 2.** Behavior of $z_i^*$ for different value of $ω_i$ in $F=1$

**Fig. 3.** Behavior of $z_i^*$ for different value of $ω_i$ in $F=2$
6. Conclusions

In this paper we introduced an integrated decision model which can help managers to make decisions in different situations. The weighted compromise method is used to converting the model to a solvable model. Then, GP techniques is used to find optimal values to decision variables subject to some side constraints. Since the resulting GP model is very complex, with the eighteen degrees of difficulty, it is very hard to derive a closed-form solution for this problem. Thus, the CVX MATLAB toolbox, is applied in order to solve this nonlinear hard problem. A numerical example is used to demonstrate the applicability of the proposed production-inventory model. Sensitivity analysis on the parameter changes is also performed. Future research can be done to consider other kinds of uncertainty environments.

Appendix A

Here, it is illustrated that how the signomial model transformed to the posynomial model in order to derive the utopia objective values for each market \( i \).

The objective function of general model is:

\[
\max \pi_i (p_i, M_{ji}, S, q, Q_i, C_i, r_i) = p_i D_i - C_{ji} D_i r_i^{-1} Q_i^{-1} - C_{2j} D_i r_i^{-1} - 0.5 f_i C_{2j} r_j Q_i - Y_i (C_{li}, r_i) D_i r_i^{-1} Q_i^{-1} - N_i (r_j) D_i r_i^{-1} Q_i^{-1} - e_i \left( \sum_{j=1}^{2} M_j + \sum_{l=1}^{2} S_l \right)
\]

subject to the same constraints as in Eqs.(7-13) for market \( i \).

The model is equivalent to the following model, after defining another auxiliary variable (\( z \)) and constraint:

\[
\max z_i
\]

subject to the same constraints as in Eqs.(7-13) for market \( i \).

Obviously, the above model can be written in the form

\[
\min z_i^{-1}
\]

subject to the same constraints as in Eqs.(7-13) for market \( i \).
Now, this equation is a GP in standard posynomial form after rearranging constraints to become in standard form. Note that inequality constraints can only have the form of a posynomial less than or equal to one (Boyd et al. 2007). Consequently, the utopia objective values can be obtained by solve each problem separately.

Reference


