On the complexity of recognizing tenacious graphs

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Abstract

We consider the relationship between the minimum degree $\delta(G)$ of a graph and the complexity of recognizing if a graph is $T$-tenacious. Let $T \geq 1$ be a rational number. We first show that if $\delta(G) \geq \frac{Tn}{T+1}$, then $G$ is $T$-tenacious. On the other hand, for any fixed $\epsilon > 0$, we show that it is $NP$-hard to determine if $G$ is $T$-tenacious, even for the class of graphs with $\delta(G) \geq \left(\frac{T}{T+1} - \epsilon\right)n$.

Keywords: $NP$-complete problem, tenacity, tenacious, $NP$-hard.

1. Introduction

We consider only graphs without loops or multiple edges. Our terminology will be standard except as indicated; a good reference for any undefined terms is [2]. We use $V(G)$, $\alpha(G)$, and $\omega(G)$ to denote the vertex set, independence number and number of components in a graph $G$, respectively. We consider only finite undirected graphs without loops and multiple edges. Let $G$ be a graph. We denote by $V(G)$, $E(G)$ and $|V(G)|$ the set of vertices, the set of edges and the order of $G$, respectively. The concept of tenacity of a graph $G$ was introduced in [4,5], as a useful measure of the "vulnerability" of $G$. In [5] Cozzens et al. calculated tenacity of the first and second case of the Harary Graphs but they didn’t show the complete proof of the third case. In [18] we showed a new and complete proof for...


