Lattice Boltzmann model for thermal behavior of a droplet on the solid surface

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Abstract

In this work, a thermal lattice Boltzmann model is applied to simulate thermal behavior of a droplet on the solid surface. For this reason Lee's model which uses the Cahn–Hilliard diffuse interface theory, is utilized to capture the droplet interface. Also contact angle between solid, liquid and gas phases has been considered in simulations. To take into account the thermal effects, passive scalar model is conjugated with the Lee's method. Since in this model the solution of temperature distribution is independent of flow field, application of the Boussinesq approximation couples the energy and momentum equations. Numerical results for two thermal boundary conditions; constant wall temperature and constant heat flux on the wall, are presented and results have been compared with previous numerical results. Results show that by increasing the Prandtl number ratio between droplet and its surrounding, thermal diffusion within the droplet will be delayed and this causes reduction in the droplet average temperature. Also it was shown that the wall heat flux is concentrated around the droplet, while that in the gas phase is negligible.

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1. Introduction

Investigation of heat transfer in two-phase flows is of great interest because of its large application in industry and scientific researches. Pasandideh-Fard et al. [1] provided an experimental and a numerical study to investigate droplet impact on a hot stainless steel surface. They utilized Volume of Fluid (VOF) to capture the fluids interactions. Also the substrate temperature varies in a range beyond droplet evaporation. Their results were validated with experimental observations and indicated that the droplet spreading on a substrate will increase heat flux only a small amount. Díaz and Ortega [2] presented an experimental and numerical investigation of water droplet which is carried by gas stream impinging on a heated surface. Computational simulations were conducted using a conjugate solution and the VOF model. They have concluded that the heat transfer process during the early stage of impact was mainly dominated by diffusion. Another experimental study by Negeed et al. [3] studied the influence of oxide layer over the hot surfaces on the behavior of single droplets impacting the high temperature surfaces using high speed camera. Strotos et al. [4] studied the impact of 2D water droplets onto a heated non-isothermal surface using the VOF methodology and assuming axisymmetric computational domain. Also in their work, wall temperatures are low enough to prevent the onset of nucleate boiling. They proposed, a formula for maximum droplet spreading which accounts for droplet heating during spreading. Kompinsky et al. [5] focused on dynamics of a binary fuel droplet impinging on a heated surface and have shown experimentally that the impact regimes obtained with binary fuel droplets are significantly different from those obtained with single component droplets, especially when surface temperatures exceeded the boiling point of n-hexane, the more volatile component in the mixture. Also the effects of surface roughness amplitude and surface oxide layer thickness on the dynamic behavior of micrometric single water droplets during collision with surfaces at high temperature have been studied experimentally by Negeed et al. [6].

Among all these researches, recently, Lattice Boltzmann Method (LBM) is considered as a most capable numerical one, which simplifies interface capturing and applying interface forces such as wettability effects. Against the traditional Computational Fluid Dynamics (CFD) methods, which solve the macroscopic governing equations of mass, momentum, and energy, LBM models the fluid consisting of fictive particles, and such particles
perform consecutive propagation and collision processes over a
discrete lattice mesh. Lattice Boltzmann models vastly simplify
Boltzmann’s original conceptual view by reducing the number of
possible particle spatial positions and microscopic momenta
from a continuum to just a handful and similarly discretizing
time into distinct steps. Lee [7] suggested a two-distribution
function LBM in which the incompressibility is enforced by the
pressure evolution equation. As long as the intermolecular force
is expressed in the potential form, the incompressible LBM for
binary fluids is able to eliminate parasitic velocities. To model the
contact line dynamics on partially wetting surfaces T. Lee and L.
Liu [8] developed Lee’s method for incompressible binary fluids.
For taking into account the thermal behavior in the LBM frame
work, several investigations in single-phase flows have been done [9–12]. He et al. [13] proposed a novel and more stable
thermal lattice Boltzmann model (TLBM) to simulate thermo-
hydrodynamics in incompressible flows. They introduced an
internal energy density distribution function to simulate the tem-
perature field. The macroscopic density and velocity fields are
still simulated using the density distribution function. Their
model is numerically more stable compared with the multi-
speed thermal lattice Boltzmann models. Attar and Korner [14]
developed an algorithm to use the Lattice Boltzmann method
for solving free surface thermal flow problems with solid/liquid
phase changes. They applied a multi-distribution function model
to simulate hydrodynamic flow and the coupled thermal
diffusion-convection problem. For the first time, Chang and
Alexander [15] extended the lattice Boltzmann method to
include the effects of interfacial tension and its dependence on
temperature. They also considered the problem of buoyancy-
driven flow in a non-isothermal two-phase system. Dynamic
behavior of a droplet in presence of solid bodies is investigated
by Taghiolu and Rahimian [16]. They handled the Lee’s method to
model the two-phase dynamics and imposed the wetting
boundary conditions on the solid surfaces. Haghani et al. [17]
presented a numerical simulation of droplet dripping through a
hole which is located on the horizontal plate. They considered
wettability effect by imposing contact angle between solid–
liquid and gas phases using Cahn’s wetting theory.

In this work hydro-thermal behavior of the droplet impinge-
ment with solid surface at specific contact angle is studied using
lattice Boltzmann method. For this reason, passive scalar model
with two thermal boundary conditions namely; constant wall
temperature and constant heat flux are considered and conju-
gated with the Lee’s model. Code accuracy is verified by
comparing the results with previous numerical works. Results
show that, by increasing the Prandtl number ratio between
droplet and its surrounding, thermal diffusion within the droplet
will be delayed and this causes reduction in the droplet average
temperature. Also variation of wall heat flux is plotted versus the
radius on the surface and it is found that the wall heat flux is
concentrated around the droplet, while that in the gas phase is
negligible.

2. Two-phase flow modeling

Using a method which applies the proper expression for the
pressure term, can be useful in stabilizing the simulations in classic
CFD and LBM. Lee in 2009 [7] presented a method for simulating
the binary fluid flow based on the Cahn–Hilliard diffuse interface
theory. In this model, one distribution function recovers pressure
and momentum, and the other distribution function recovers the
composition of species, C. The following are the Lattice Boltzmann
Equation (LBE) for each distribution function [16]:

\[
\mathsf{g}_a(x + e_a \delta t, t + \delta t) - \mathsf{g}_a(x, t) - \frac{1}{\tau + 0.5} \left( \mathsf{g}_a - \mathsf{g}_a^{eq} \right)(x, t) + \delta t (e_a - u) \left[ \nabla \rho \nabla^2 \left( I_a - I_a(0) \right) - C \left( \nabla \rho + \nabla \mu \right) \right] \bigg|_{(x, t)},
\]

(1)

\[
\mathsf{h}_a(x + e_a \delta t, t + \delta t) - \mathsf{h}_a(x, t) - \frac{1}{\tau + 0.5} \left( \mathsf{h}_a - \mathsf{h}_a^{eq} \right)(x, t) + \delta t (e_a - u) \left[ \nabla C - \frac{C}{\rho \nu C_v} \left( \nabla p + \nabla \mu \right) \right] \bigg|_{(x, t)} + \delta t (M \nabla \mu) I_a \bigg|_{(x, t)},
\]

(2)

where \( \mathbf{e}_a \) is the microscopic particle velocity in the \( a \) direction, and \( \rho_s \) is the constant speed of sound, \( u \) is the macroscopic velocity, \( \delta t \) refers to the time step, \( \tau \) is the dimensionless relaxation time, \( \rho \) expresses the mixture density, \( \mu \) stands for chemical potential, and \( M > 0 \) is the constant mobility. In the above equations, \( \mathsf{g}_a^{eq} \) and \( \mathsf{h}_a^{eq} \) are modified equilibrium distribution functions and given as:

\[
\mathsf{g}_a^{eq} = \mathsf{g}_a^{eq} \left( \frac{\delta t}{2} (e_a - u) \left[ \nabla \rho \nabla^2 \left( I_a - I_a(0) \right) - C \nabla \mu I_a \right] \right),
\]

(3)

\[
\mathsf{h}_a^{eq} = \mathsf{h}_a^{eq} \left( \frac{\delta t}{2} (e_a - u) \left[ \nabla C - \frac{C}{\rho \nu C_v} \left( \nabla p + \nabla \mu \right) \right] \right) \bigg|_{(x, t)},
\]

(4)

where equilibrium distribution function for \( \mathsf{g}_a \) and \( \mathsf{h}_a \) could be calculated by:

\[
g_a^{eq} = W_a \left[ p + \rho C_v \left( \frac{e_a u}{c_l^2} + \frac{(e_a u)^2}{2c_t^2} - \frac{\left( uu \right)}{2c_t^2} \right) \right],
\]

(5)

\[
h_a^{eq} = W_a \left[ 1 + \frac{e_a u}{c_l^2} + \frac{(e_a u)^2}{2c_t^2} - \frac{\left( uu \right)}{2c_t^2} \right] C,
\]

(6)

with

\[
I_a(u) = W_a \left[ 1 + \frac{e_a u}{c_l^2} + \frac{(e_a u)^2}{2c_t^2} - \frac{\left( uu \right)}{2c_t^2} \right].
\]

(7)

Finally the value of compositions, momentum and dynamic
pressure can be computed by taking the zero and first moments of
the modified particle distribution function:

\[
C = \sum_a \mathsf{h}_a,
\]

(8)

\[
\rho u = \frac{1}{c_l^2} \sum_a e_a \mathsf{g}_a - \frac{\delta t}{2} C \nabla \mu.
\]

(9)

\[
p = \sum_a \mathsf{g}_a + \frac{\delta t}{2} u \nabla \rho C_v^2.
\]

(10)

In our computations in this work all gradient and Laplacian
terms have been discretized using the compact relations which are
in second-order accurate as used by T. Lee et al. [18] as follows:

\[
\frac{\partial \rho \phi}{\partial x_i} = \sum_a W_a (e_a u)(\phi(x + e_a \delta t) - \phi(x - e_a \delta t)),
\]

(11)

\[
\frac{\partial^2 \phi}{\partial x_i \partial x_i} = \sum_a W_a (e_a u)(\phi(x + e_a \delta t) - 2\phi(x) + \phi(x - e_a \delta t)),
\]

(12)

Note that the discretization scheme in this way and use of
the Lee method can eliminate spurious velocities at the interface which
effectively performs the interfacial dynamics.
The density and dimensionless relaxation time are taken as linear functions of the composition as: \( \rho = \rho_i + (1 - \rho_i)\rho_g, \) 
\( \tau = \tau_i + (1 - \tau_i)\tau_g \) Here subscript \( l \) and \( g \) represent the droplet and surrounding fluid, respectively. Also the relation between kinematic viscosity and relaxation time is \( \nu = \tau / 3 \). Cahn and Hilliard [19] asserted that the mixing energy density for an isothermal system takes the following form:

\[
E_{\text{mix}}(C, \nabla C) = E_0(C) + \frac{k}{2} \nabla C^2,
\]

(13)

\( k \) is gradient parameter and \( E_0 \) denotes bulk energy by \( E_0(C) = \beta C^2(C - 1)^2 \), where \( \beta \) is a constant. The classical part of the chemical potential is also derived by derivative of \( E_0 \) with respect to \( C \), \( \mu_0 = \partial E_0/\partial C \). Equilibrium profile is obtained when the mixing energy is minimized and reads \( \mu = \mu_0 - k \nabla^2 C = \text{const.} \) The plane interface profile eqt is then given by:

\[
C(z) = \frac{1}{2} + \frac{1}{2} \tan h \left(\frac{2z}{D}\right),
\]

(14)

where \( z \) is the coordinate normal to the plane interface and \( D \) is the numerical interface thickness, which is chosen based on accuracy and stability. Having \( D \) and, \( k \) one can compute the gradient parameter \( k = \beta D^2 / 8 \) and surface tension \( \sigma_{bf} \) is calculated from \( \sigma_{bf} = \sqrt{2kb} / 6 \). A point should be cleared, since the equation of \( E_0 \) is usually valid in the near-critical state of a fluid, i.e. at low phase-density ratios, it is named as an artificial equation of state [20]. Thermodynamic pressure \( P_0 \) expresses by \( P = (C\mu_0 - E_0) \) which \( \mu_0 \) is the bulk chemical potential. This equation is the equation of state which is used by Lee and used in the whole computational domain. Both the bulk and the interfacial regions are governed by the same density dependent free energy given in \( E_0 \). Because of the bulk regions following an artificial EOS, Lee’s model yields unphysical dynamics in the presence of body forces. Since the LB method is inherently a quasi-compressible method, the presence of body forces in the domain yields a very small density gradient in the direction of the body force. With increasing time, this small density gradient is further enhanced in the Lee model due to the increased effective body force. The density gradient in the direction of the body force effectively provides an artificial force, which is continuously being added to the body force as time progresses, making the system more and more compressible. Because they are governed by the artificial EOS, bulk densities are unable to correct this temporally increasing compressibility error, and the simulation ultimately becomes numerically unstable because of the unusually large density ratios produced in the simulations.

With assumption of short-range effect of liquid—gas interaction with solid surface, the total free energy then takes the following form.

\[
\psi_b + \psi_s = \int \left( E_0(C) + \frac{k}{2} \nabla C^2 \right) dV + \int \left( \phi_0 - \phi_1 C_4 + \phi_2 C_s^2 \right. \\
\left. - \phi_3 C_4^2 + \cdots \right) dS.
\]

(15)

Here, \( C_4 \) is the composition at the solid surface which depends only on the composition at the surface \( C_4 \) and \( S \) is the surface bounding \( V \). Minimizing Eq. (15) based on the natural boundary conditions (see [21]) gives an equilibrium boundary conditions on the surface \( S \). The boundary condition is:

\[
\frac{\partial E_0}{\partial C} - k \frac{d^2 C}{d^2 z} = \mu_b \quad \text{for} \quad z > 0.
\]

(16)

Here, \( \mu_b \) and is the chemical potential in the bulk flow. And,

\[
\mathbf{n} \cdot \nabla C_{s} = -\frac{\phi_1}{k} \quad \text{for} \quad z = 0.
\]

(17)

Integrating of Eq. (16) yields:

\[
\frac{k}{2} \left( \frac{d C}{d z} \right)^2 = \beta C^2 (C - 1)^2 = W(C).
\]

(18)

Substituting Eq. (18) into Eq. (17) yields the \( C_4 \) as:

\[
\phi_1 = \pm \sqrt{2kW(C_4)}.
\]

(19)

Four solutions, \( C_1 > C_2 > C_3 > C_4 \) can be obtained from Eq. (19) if \( \phi_1 \) is smaller than the height of the double well function defined by \( \sqrt{2kW} \). The corresponding results are:

\[
C_1 = \frac{1}{2} - \frac{1}{2} \sqrt{1 + \Omega},
\]

(20)

\[
C_2 = \frac{1}{2} - \frac{1}{2} \sqrt{1 - \Omega},
\]

(21)

\[
C_3 = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \Omega},
\]

(22)

\[
C_4 = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \Omega},
\]

(23)

where \( \Omega \) is the wetting potential given by:

\[
\Omega = \frac{\phi_1}{\sqrt{2k\beta}}.
\]

(24)

The surface tension between wall and fluid, \( \sigma_{bf} \) is given by Ref. [22]:

\[
\sigma_{bf} = -C_4 \phi_1 + \int \sqrt{2kW} dC.
\]

(25)

For \( \phi_1 > 0 \), the minimized solutions \( C_2 \) if the fluid contacting the wall is a gas and \( C_4 \) if the fluid is a liquid. Sequentially, the following expressions for surface tensions are obtained [23,24]:

\[
\sigma_{bg} = -C_2 \phi_1 + \int \frac{C_2}{C_4} \sqrt{2kW} dC = \frac{\phi_1}{2} + \frac{\sigma_{bg}}{2} - \frac{\sigma_{bg}}{2} (1 - \Omega)^{1.5},
\]

(26)

\[
\sigma_{il} = -C_4 \phi_1 + \int \frac{C_4}{C_4} \sqrt{2kW} dC = \frac{\phi_1}{2} + \frac{\sigma_{il}}{2} - \frac{\sigma_{il}}{2} (1 + \Omega)^{1.5},
\]

(27)

where \( \sigma_{bg}, \sigma_{il}, \) and \( \sigma_{il} \) are the liquid—gas, solid—gas, and solid—liquid surface tensions. \( \Omega = \phi_1 / \sqrt{2k\beta} \) is the dimensionless wetting potential. The wetting angle is found by substituting Eqs. (26) and (27) into Young’s law (\( \cos \theta_{eq} = (\sigma_{bg} - \sigma_{il})/\sigma_{bg} \)). The result is:

\[
\cos \theta_{eq} = \frac{(1 + \Omega)^{1.5} - (1 - \Omega)^{1.5}}{2}.
\]

(28)

Eq. (28) is an equation for \( \theta_{eq} \) given a wetting potential \( \Omega \). A suitable inversion for the range \( 0 < \theta_{eq} \leq \pi \) is:
\[ \Omega = 2 \operatorname{sgn} \left( \frac{\pi}{2} - \theta_{eq} \right) \left[ \cos \left( \frac{\pi}{2} \right) \left( 1 - \cos \left( \frac{\pi}{3} \right) \right) \right]^{1/2}, \]  

where \( \alpha = \arccos(\sin^2 \theta_{eq}) \) and \( \operatorname{sgn}(x) \) gives the sign of \( x \). Therefore, by choosing a desired angle \( \theta_{eq} \) one can calculate the required wetting potential \( \Omega \).

3. Thermal Lattice Boltzmann

The Lee model is an isothermal model. In order to take thermal effects into account, the passive-scalar approach is used. If heat dissipation due to viscosity and compressibility are negligible, then the temperature field satisfies a much simpler passive-scalar equation:

\[ \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = -\nabla \cdot (\alpha \nabla T) + \psi, \]  

where \( \mathbf{u} \) is the macroscopic velocity, \( \alpha \) is the thermal diffusivity and \( \psi \) denotes the thermal sources. Eq. (30) can be solved in the LB framework by considering the single-phase LBM in which the thermal equilibrium distribution function \( f_{eq} \), evaluated by Ref. [25,26]

\[ f_{eq}^{\alpha} = Tw_{\alpha} \left[ 1 + \frac{3}{c_s^2} \mathbf{e}_n \mathbf{u} + \frac{9}{2c_s^2} (\mathbf{e}_n \mathbf{u})^2 - \frac{3}{2c_s^2} (\mathbf{u} \cdot \mathbf{u}) \right]. \]  

Also the collision operator satisfies the following equation

\[ f_i(x, t + \delta t) = f_i(x, t) - \frac{1}{\tau_f} \left[ f_i(x, t) - f_{eq}^{\alpha}(x, t) \right], \]

where \( \tau_f \) is the dimensionless single relaxation time for temperature and the summation of PDFs will give the temperature value. Hence thermal diffusion coefficient \( \alpha \) will be evaluated as \( \alpha = (\tau_f - 0.5)/3 \). Clearly the Prandtl number is expressed as \( Pr = \frac{\nu}{c_p} \) or \( \alpha = \tau_f (\tau_f - 0.5) \). Considering that in the passive scalar method the solution of the temperature field is independent of the flow field and the only factor which affects the flow field is the Boussinesq approximation. Under this approximation all fluid properties except density are considered independent of temperature. The density is obtained with linear assumption in density variation:

\[ \rho = \rho_0 [1 - \beta (T - T_0)]. \]  

\[ F = -\rho \beta (T - T_0) \mathbf{g}, \]  

where \( \mathbf{g} \) is the gravitational acceleration.

By adding single-phase heat transfer to the two-phase flow, the Lee model with heat transfer is introduced. As it was previously mentioned, in the present study passive scalar model is used to simulate heat transfer. Hence, if the solution of dynamical field is on hand, simulation of heat transfer can be done easily. Now it is critical to say that the stability restricts high density and viscosity ratio simulations. Also the obtained temperature field does not correspond for the adhesive force between fluid and solid surface.

3.1. Thermal boundary conditions

Two different thermal BCs named as isothermal wall and heat flux BCs have been used in this paper. Here we explain them in the context of a D2Q9 model.

3.1.1. Isothermal wall

Suppose that the bottom wall in Fig. 1 is fixed in a constant temperature, \( T_b \). After streaming, the values of \( f_2, f_3 \) and \( f_6 \) are unknowns. Assume these unknown PDFs equal their equilibrium distribution given by Eq. (31) with \( T \) replaced by some unknown temperature \( T' \). Summing these three PDFs together, we have [26]:

\[ f_2 + f_3 + f_6 = \frac{1}{6} T' \left( 1 + 3 u_y + 3 u_y^2 \right). \]  

where \( u_y \) is the velocity normal to the wall. If we know \( T' \), we will be able to solve for \( f_2, f_3, \) and \( f_6 \). Meanwhile, we notice that for the isothermal wall, \( \sum_{i=2}^{6} f_i = T_b \). Substituting Eq. (35) into this, \( T' \) can then be calculated as follows:

\[ T' = \frac{6}{1 + 3 u_y + 3 u_y^2} (T_b - f_0 - f_1 - f_3 - f_4 - f_5 - f_8). \]  

Finally, \( f_2, f_3, \) and \( f_6 \) can be obtained by substituting \( T' \) into Eq. (31).

3.1.2. Constant heat flux

After streaming, the temperature of the inner domain can be obtained. A second-order finite difference scheme is used to get the temperature on the wall, i.e., for the bottom wall at \( y = 0 \),

\[ \frac{\partial T}{\partial y}_{1,1} = \frac{4T_{1,3} - T_{1,1} - 3T_{1,1}}{2\Delta y}. \]  

After finding the wall temperature, the same procedure as described in the isothermal wall case is used to calculate the unknown PDFs [26].

4. Code verification

Dynamics and thermal behavior of a droplet are validated separately comparing with previous numerical works. Droplet contact angle and droplet spreading on the dry surface along with droplet splashing on a thin liquid film are compared with theoretical and numerical data. Also for the thermal validation, droplet impingement onto a hot surface is carried out and compared with François’s [27] results.

4.1. Droplet dynamics

In order to show the grid independency, domain is divided to \( 200 \times 100, 400 \times 200 \) and \( 600 \times 300 \) lattices and a droplet with radius of 20 \( l_u \) is located on the surface with contact angle of 60°. According to Fig. 2, it is concluded that the grid resolution has no substantial effect on the results. To ensure the accuracy of contact angles, different equilibrium contact angles are simulated and numerical results are compared with Eq. (28). As it is shown in Fig. 3, results are in good agreement with analytical ones. Fig. 4 shows three different equilibrium states, in contact angle of 30, 90 and 120°.

To be more specific on the contact angle behavior, the spread factor \( \zeta = d/D \) is defined as a ratio of wetted diameter on the solid surface (\( d \)) to initial diameter of the droplet (\( D \)). We put the density

![Fig. 1. Schematic of the thermal boundary conditions at the bottom wall.](image-url)
ratio, dynamic viscosity and equilibrium contact angle equal to 100, 10 and 90°, respectively. To continue, non-dimensional parameters, such as Reynolds number \( Re = \frac{\mu I U I D I}{\mu I} \), Weber number \( We = \frac{\mu I U I^2 D I}{\sigma} \) and non-dimensional time \( t^* = \frac{I U I}{D I} \) are defined. Where \( U I \) is droplet initial velocity, \( \mu I \) is droplet kinematic viscosity and \( I \) is the number of iterations. Droplet behavior after its collision with solid surface is plotted in Fig. 5 in different Reynolds and Weber numbers. According to this figure four phases appear when the droplet collides with solid surface: kinematic, spreading, relaxation and equilibrium phases.

When the Weber numbers becomes less than 5, droplet collision has no relaxation phase and reaches its equilibrium state after spreading phase. Gupta and Kumar [28] have proposed the equation of \( \zeta = 1.42 \sqrt{t^*} \) in kinematic phase. Fig. 6 shows the spread factor variation versus \( t^* \) for different Reynolds and Weber numbers. From this figure a good agreement can be seen between current work and given correlation.

As another test, droplet splashing on the thin liquid film test was performed and its results are compared with Lee’s work [18]. For this reason non-dimensional height of liquid film \( h^* = h/D I \) is defined. Where \( h \) stands for liquid film height on the surface. Numerical results for the case of \( Re = 500, We = 8000 \) and \( h^* = 0.15 \) are shown in Fig. 7. Comparison of current work (Fig. 7a) with [18]
Fig. 7. Droplet splashing on a thin liquid film, \( Re = 500, We = 8000 \) and \( h^* = 0.15 \)

a) current simulation

b) Lee [18].

Fig. 8. The isotherms during the impingement of a droplet on a constant temperature flat surface.

a) current work

b) François [27].
4.2. Thermal validation

Impingement of a droplet on a constant wall temperature is investigated as an accuracy test. In this problem, the bottom wall of the computational domain, where the droplet impacts, is kept at a constant temperature $T_b = 1.0$, and the top and side walls are adiabatic. Initially, both the droplet and surrounding phases are at $T_0 = 0$. For this case the numerical results of François [27] are used and results correspond for the case of $Re = 300$, $We = 60$, $Pr = 2$ and $\theta_{eq} = 60^\circ$. Other parameters such as conductivity, specific heat and diffusivity are chosen in a way that the thermal diffusivity of the gas is 50 times larger than droplet. Also in this case the droplet ratio is set to 50 against the François’ work which the density ratio corresponds for the water. This restriction arises from the numerical instability in high density ratios. The isotherms during the impingement of a droplet on a constant temperature flat surface are shown in Fig. 8. They develop more quickly in the gas phase than in the liquid droplet. This can be explained by the fact that the thermal diffusivity of the gas is 50 times larger than the one in the liquid.

Fig. 9. The wall heat flux versus the radial position at different times.

Fig. 10. Droplet interface and isotherms in different times, in two different Prandtl number ratios; $Re = 45$, $We = 75$. 
The heat flux which is defined by Fourier’s law is calculated on the bottom wall as \( q(t) = -k \frac{dT}{dx} \). The wall heat flux is plotted in Fig. 9 versus the radial position at different instants. From Fig. 9 one notices that the wall heat flux is concentrated around the liquid droplet, while that in the gas phase is negligible. High heat flux rates take place at the early stage of impingement, and decay with time.

5. Results and discussion

5.1. Constant temperature on the surface

In this section, hydro-thermal behavior of a droplet impacting onto a hot surface is investigated in different Prandtl number ratios. For this reason a 80 x 160 computational domain is considered. A 2-D droplet of radius 15 \( lu \) is placed tangent to the surface, once it hits the surface with specific velocity which is consistent with \( Re = 45 \) and \( We = 75 \). Temperature of the surface is set to \( T_b = 1 \) and temperature of field and droplet are initiated to 0, equilibrium.
Fig. 16. Droplet interface and isotherm contours in different times; $Pr = 5, q^* = 0.24$. 
contact angle equals with $\theta_{eq} = 60^\circ$ and density ratio is fixed at 10. The Prandtl number ratio between two phases is defined as a non-dimensional parameter by $Pr_r = Pr_f/Pr_g$. Also the buoyancy force effect is neglected. Consider the case which the ratio of momentum diffusivity to thermal diffusivity in two phases is equal ($Pr_r = 1$), this causes the similar temperature distribution in two phases. On the other hand by increasing the Prandtl number ratio, thermal diffusion in the droplet decreases. Hence, temperature distribution in the surrounding of droplet will grow rapidly. This fact is clearly shown in Fig. 10.

To demonstrate the effectiveness of the Prandtl number ratio, the average droplet temperature versus dimensionless time, $t^*$ is presented in Fig. 11. As it can be seen in this figure by increasing the Prandtl number ratio the average temperature of the droplet, reduces which indicates the role of the Prandtl number ratio in developing the thermal boundary layers. Figs. 12–15 represent the non-dimensional heat flux $q^* = \frac{q_f}{q_l} \times \frac{D_l}{T_{in}}$ versus dimensionless variable of $r/D_l$ in different Prandtl number ratios on the surface. Where $T_{in}$ corresponds for initial temperature of the domain and $r$ is distance from the surface. From Figs. 12–15, it is clear that the wall heat flux is concentrated around the droplet, while that in the gas phase is negligible. High heat flux rates take place at the early stage of impingement, and decay with time.

5.2. Constant heat flux on the surface

In this section impact of a cold droplet with the vertical surface with constant heat flux will be discussed. For this reason, a droplet with radius of 10 $lu$ is located tangent to a vertical surface in a $100 \times 250$ computational domain. Droplet is injected inclined at $45^\circ$ with $Re = 30$ and $We = 10$, which defined by horizontal velocity. Periodic boundary condition is applied on the bottom and upper walls and adiabatic boundary condition imposed on the right wall. Hydro-thermal behavior of the droplet in this condition is shown in Fig. 16. In this figure both droplet interface and isotherm contours are presented. From a physical point of view, a constant heat flux on the wall will increase the wall temperature. Hence impact of cold droplet on this wall, will decrease the temperature. Nevertheless after cooling the surface, the heat flux on the wall will increase the temperatures of the wall and droplet. Note that the droplet will fall down on the wall and the next cold droplet could be injected on it. Fig. 16 shows the droplet interface and isotherm contours in different times. The following parameters are set to: $Pr_r = 5$, $q_f = 0.24$, $T_{in} = 15$, $T_i = 10$ and $\theta_{eq} = 60^\circ$.

For determining the wall temperature variations, a non-dimensional temperature of $\theta^* = \frac{\left[T_{wall} - T_i\right]}{\left[T_{in} - T_i\right]}$ is introduced, where $T_{wall}$ is the average temperature of the wall which evaluated between two points located 10 $lu$ down and upper of initial droplet location. Fig. 17 shows the average wall temperature variation with the time for different Prandtl number ratios. As it is expected, wall temperature will increase by time increasing until droplet hits at $t^* = 12.5$. Immediately after the droplet impingement, wall temperature decreases. Since the constant heat flux is applied on the wall, its temperature begins to increase. It is worthwhile to say that by increasing the Prandtl number ratio, cooling effects magnifies. This means that the required time for temperature increment on the wall is larger in higher Prandtl number ratio. Also the average wall temperature variation is presented in Fig. 18 in different heat fluxes. It is obvious by increasing the heat flux, wall temperature increases at the same times.

6. Conclusions

The main aim of this work is devoted to applied thermal lattice Boltzmann model into the Lee’s model using passive scalar method. The Boussinesq approximation was handled to couple the thermal and hydrodynamic effects. Results have shown that by increasing the Prandtl number ratio between two phases thermal diffusion within the droplet will be delayed and this causes reduction in the droplet average temperature. Also it was shown that the wall heat flux is concentrated around the droplet, while that in the gas phase is negligible. At the case of cooling, impact of the cold droplet with heated surface was investigated and surface temperature variation was studied. According to the results it is inferred that by increasing the Prandtl number ratio, required time for temperature increment on the wall is larger in higher Prandtl number ratios. By the way, the current TLBM model suffers from some restrictions. Stability
restricts high density and viscosity ratio simulations, and temperature field was obtained without considering the adhesive force between fluid and solid surface.

References